Equations for Robot 3D Curve Determination Decisions

Robert W. Newcomb and Dimokritos A. Panagiotopoulos

Microsystems Laboratory
Electrical Engineering Department, University of Maryland
College Park, MD 20742

Abstract

Curvature and torsion are determined in terms of Euler angles from coordinates measured locally on a curve in three dimensional space and referred to a fixed robot frame. These serve to give equations for robot curve determination and are particularly organized for intelligent robot decision making.

Introduction

The field of robotics is being revolutionized by the presence of microprocessors which are paving the way to intelligent robots [1]. It is also clear that intelligent robots will need to make decisions concerning three-dimensional curves, for example for trajectory following or object recognition, inspection, and acquisition purposes [2, p. 43][3]. Consequently it will become important to have three-dimensional curve determation equations on hand and especially so in a form suitable for microprocessor use in intelligent robots. Since, as we show elsewhere [4], Euler angles are determinable from microprocessor controlled robot vision systems, we here use Euler angles as local coordinates established on a curve. Using these coordinates the curvature, $k(\cdot)$, and torsion, $\tau(\cdot)$, as functions of arc length s are calculated. Since, by a theorem of differential geometry [5, pp. 13-18], a three-dimensional curve is uniquely determined by its curvature and torsion, the desired curve determination equations are on hand.

The Euler Angles

Consider a fixed, (right-handed) rectangular coordinate system, for example its origin 0 could be in the base of the robot. We denote points measured in this system by x or sometimes to distinguish it as the base coordinate system by x_0, thus

$$\underset{\sim}{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \underset{\sim}{x}_0 \tag{1a}$$

In this system the coordinate axes unit vectors are

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{1b}$$

Next consider a curve in this three-dimensional Euclidean space and choose upon it a starting point from which arc-length s is measured. For convenience we assume that the curve has a single-valued, piecewise analytic description in s. At any point P on the curve we set up a local rectangular coordinate system with points measured in this system denoted by

$$\underset{\sim}{t} = \begin{bmatrix} t \\ n \\ b \end{bmatrix} \tag{2a}$$

This local coordinate system is set up following differential geometry techniques so that

$$\vec{t} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{n} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2b}$$

designate unit vectors, respectively, in the direction of the tangent to the curve (measured with respect to increasing s), in the direction of the principal unit normal to the curve, and in the perpendicular direction necessary to set up a right-handed system. By a translation of 0 to P we can identify the two coordinate origins and after that we can line up the coordinate axes by three rotations of Euler angles Θ_1, Θ_2, Θ_3 defined by

Θ_1 = rotation around the x axis = pitch (3a)
Θ_2 = rotation around the resulting y axis = roll (3b)
Θ_3 = rotation around the resulting z axis = yaw (3c)

The situation is shown in Fig. 1 where intermediate coordinate systems, of coordinates $\underset{\sim}{x}_1$, $\underset{\sim}{x}_2$ and $\underset{\sim}{x}_3 = \underset{\sim}{t}$, are given. Letting

$$C_i = \cos\Theta_i\ , \quad S_i = \sin\Theta_i, \quad i = 1, 2, 3 \tag{4a}$$

and using straightforward trigonometrical considerations the coordinates in the various frames are seen to be related by the transformations

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_1 & S_1 \\ 0 & -S_1 & C_1 \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}, \quad \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} C_2 & 0 & -S_2 \\ 0 & 1 & 0 \\ S_2 & 0 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \quad \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} C_3 & S_3 & 0 \\ -S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \tag{4b}$$

Denoting these coefficient matrices by T_1, T_2, T_3, respectively, we clearly have ($\tilde{}$ = transpose)

$$\underset{\sim}{t} = T_3T_2T_1\underset{\sim}{x}\ , \quad \underset{\sim}{x} = \tilde{T}_1\tilde{T}_2\tilde{T}_3\underset{\sim}{t} \tag{4c}$$

Thus we can find the coordinates of the unit vectors t, n, and b in the robots fixed frame (4c) as

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$$\underset{\sim}{x}_{\vec{t}} = \begin{bmatrix} C_2C_3 \\ C_1S_3 + S_1S_2C_3 \\ S_1S_3 - C_1S_2C_3 \end{bmatrix},$$

$$\underset{\sim}{x}_{\vec{n}} = \begin{bmatrix} -C_2S_3 \\ C_1C_3 - S_1S_2S_3 \\ S_1C_3 + C_1S_2S_3 \end{bmatrix}, \quad \underset{\sim}{x}_{\vec{b}} = \begin{bmatrix} S_2 \\ -S_1C_2 \\ C_1C_2 \end{bmatrix} \qquad (5)$$

Next we use the Frenet - Serret formulas [6, p.60]

$$\frac{d\vec{t}}{ds} = k(s)\vec{n}\ ,\ \frac{d\vec{n}}{ds} = -(k(s)\vec{t} + \tau(s)\vec{b}),\ \frac{d\vec{b}}{ds} = \tau(s)\vec{n} \qquad (6a)$$

which characterize the curve. Thus we can find the curature and torsion from

$$k(s) = \tilde{\vec{n}} \cdot \frac{d\vec{t}}{ds} \quad , \quad \tau(s) = \tilde{\vec{n}} \cdot \frac{d\vec{b}}{ds} \qquad (6b)$$

These calculations are straightforward. Writing $d\cdot/ds = {}'$ and using $C_i' = -S_i\Theta_i'$, $S_i' = C_i\Theta_i'$ we get

$$k(s) = S_2\Theta_1' + \Theta_3' \qquad (7a)$$

$$\tau(s) = -C_2C_3\Theta_1' - S_3\Theta_2' \qquad (7b)$$

There is also a constraint due to $\vec{b}$ and $\vec{n}$ being perpendicular which, from the first of (6a), is

$$\tilde{\vec{b}} \cdot \frac{d\vec{t}}{ds} = C_2S_3\Theta_1' - C_3\Theta_2' = 0 \qquad \Theta_1' = \frac{C_3}{C_2S_3}\Theta_2' \qquad (7c)$$

This is a constraint among the Euler angles to guarantee they are representing a curve. In other words when travelling on a curve the Euler angles as we have defined them are not independent, (7c) for example giving Θ_1 in terms of Θ_2 & Θ_3 via

$$\Theta_1(s) = \Theta_1(s_o) + \int_{s_o}^{s} \frac{1}{\cos \Theta_2(x)} \cdot \frac{\cos \Theta_3(x)}{\sin \Theta_3(x)} \cdot \frac{d\Theta_2(x)}{dx}\, dx \qquad (7d)$$

Since any two (analytic) curves having the same curvature and torsion are identical except possibly for orientation in space [6, p. 63], our curve is uniquely specified by equations (7). Consequently tracking of the Θ_i by a robot and formation of k(s) & τ(s) via (7) in the robot's microcomputer allows the robot to keep track of the curve for whatever purpose it may need.

Discussion

By indexing a one dimensional curve in three dimensional space by arc length s measured from a marked point $s = s_0 = 0$ on the curve, the curve can be parameterized via its three Euler angles determined locally as a function of s. Once these angles are known, say by a robot's vision and processing system, the curve can be identified by calculating its curvature, k(s), and torsion, τ(s), via (7a, b) taking into account the curve constraint, (7c), between Euler angles. Various decisions can then be made by an intelligent robot's processing system, for example to pursue the curve as an arm's trajectory or to acquire a part outlined by the curve. It should be observed that scaled curves are readily handled within the theory.

Since a curve is uniquely determined by its curvature and torsion and, as shown here, also by its locally measured Euler angles, it would be of interest to determine the Euler angles as explicit functions of k and τ. This is implicit in equations (7) but it remains an interesting open problem to make it explicit.

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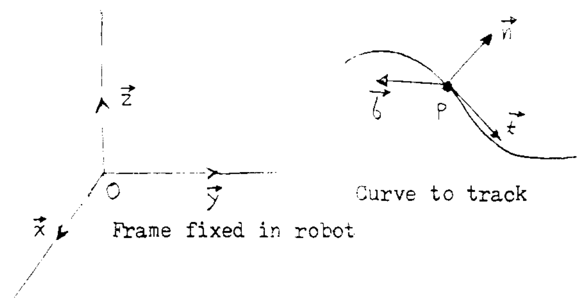

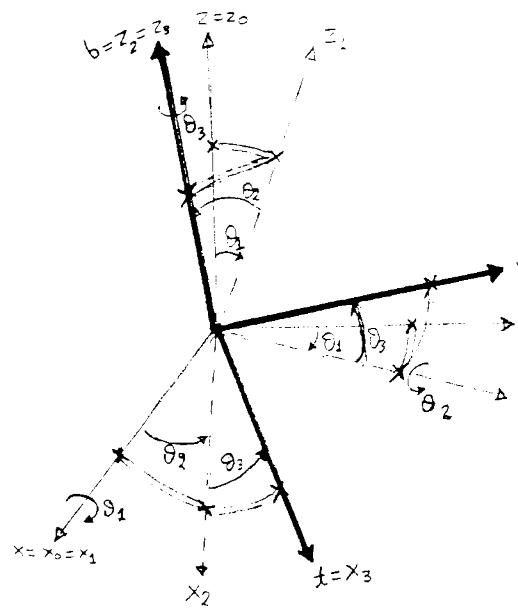

Figure 1. Euler Angles, O Translated to P.