Computer networking capacity in robotic neural systems

Carol Niznik and Robert Newcomb* explore the analogy between the capacity of computer networks and robotic neural systems

This paper illustrates the mathematical and philosophical link between capacity measures developed in the mathematical theory of capacity, computer networking and robotic neural systems. The work of classical authors is founded upon concepts associated with physical circuit capacitors and Fourier series singularity points, and is developed through mathematical works originating with Gauss and Wiener. Three such mathematical measures of capacity are used here as a theoretical base: Newtonian capacity, Hausdorff measure capacity and analytic capacity. The linking of circuit and computer networking capacity measures, which equates to the capacity of robotic neural systems, is discussed by this means in two contexts, one in terms of the frequency of action potentials, and the other in terms of the frequency of bursts of action potentials in neural circuits. An all-or-none path model is also illustrated to indicate the position of coding in the robotic neural system.

Keywords: computer networks, capacity, robotics, neural networks, VLSI

Before the advent of computer system design and channel information flow, the term capacity was used in the circuitry and electromagnetic literature to indicate the ratio between charge on a conductor and the value of the potential of the conductor. This definition gives a capacity, often called capacitance, which is independent of the charge or voltage. Hence the capacity can be found to be equal to the charge when the potential is normalized to one. This notion of capacity long ago led directly to a mathematical theory of capacity. Among the mathematicians working on this problem was Gauss, who in 1840[1] was probably the first to discuss the problem of assigning potential to arbitrary sets. Much later, Wiener, following ideas he associated with Kellogg in 1924[2,3], appears to have been the first to use the capacity of an arbitrary bounded region R in n-dimensions, doing this by having a potential of one on R and zero at infinity (using $-1nr$ at infinity for $n = 2$).

Wiener recognized that the capacity gave a precise measure of the importance of a region to the solution of problems such as that of Dirichlet which he was attempting to solve[3]. This has been taken over into the area of Fourier series[4], where the Fourier series is defined with respect to a given measure. It was then found that the sets of zero capacity, with respect to the measure, were the sets where convergent Fourier series could diverge (as for the Gibbs Phenomena)[5]. Besides this importance, sets of zero capacity became fascinating in their own right, since they may be different from sets of Lebesque measure zero, as was exhibited by the example of de La Vallee Poussin[6], where a positive measure is given on the Cantor set to give it nonzero capacity, although the Cantor set has zero length (that is, zero Lebesque measure). Indeed, this has led to general studies of Cantor sets and their capacities[7]. As a summary work, Choquet[8] gives a relatively complete treatment of the potential theory aspects of capacity from a mathematical point of view, while a good and more modern treatment using Schwartz distributions is given by Deny[9].

Because of the difficulty of characterizing sets of capacity zero, mathematicians were led to treat them as unions or intersections of sets on which the capacity

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min–max and limit expressions[10] which allow the mathematical theory of capacity to be directly tied to the theory of capacity used in computer networking as discussed below. Early mathematical works in this area are associated with Fekete's 1923 transfinite diameter[11], which was recognized by 1931 to be the capacitance constant of a set by Polya and Szego[12].

The connection between these mathematical concepts and the analogous application of potential theory inherent in robotic electrical components to the analogy of computer networking terminology for capacity is now outlined. In computer networking, Kleinrock[13, 14] defines capacity as

- the measure of average flow the computer system channel must exceed to prevent congestion,
- the maximum rate at which a computer system can perform work.

Therefore, the average rate of demand must be less than the capacity to prevent system congestion. Computer system components that can also be analysed for capacity include the processor storage[15, 16] and the terminal storage.

THEORETICAL ANALOGY

In the computer network, capacity is usually defined in relation to the expression for delay, as in Kleinrock[13]:

$$T = \frac{1}{\gamma}\sum_{i=1}^{n} \lambda_i T_i \qquad (1)$$

where

$\gamma = \Sigma_j \Sigma_k \gamma_{jk}$ π_{jk} is a message path originating at node j and terminating at node k
j, k: C_j for π_{jk}

μ_i = service rate
λ_i = interarrival rate
$T_i = 1/(\mu_i C_i - \lambda_i)$
C_i = capacity for network topology links $i = 1, \ldots, n$
$\lambda_i/\mu_i = \rho_i < 1$ (ρ_i = ith utilization factor) (2)
γ_{jk} = external input message rate with origin j and destination k.

This expression then represented in terms of capacity C_i, *is stated as*

$$C_i = \left(\frac{\lambda_i}{\mu_i}\right) + \left(\frac{1}{T_i\mu_i}\right) \qquad (3)$$

Equation (3) for link capacity in a computer network is then related to the three theoretical measures described above as characteristic of robotic neural component capacity theory in the following manner:

The Newtonian capacity satisfaction of

$$C(K_1 \cup K_2) + C(K_1 \cup K_2) \leq C(K_1) + C(K_2) \qquad (4)$$

relates the inner Newtonian capacity and the general nonadditivity property[17]

$$\mathrm{CAP}(\cup_n X_n) \leq \Sigma \mathrm{cap} X_n \qquad (5)$$

This theory can be observed when the individual computer network capacity equation (3) is substituted into equation (1). Also, the bilinear form of the Newtonian capacity is illustrated from Kelvin's Principle as given by Bleidtner[18]:

$$\mathrm{CAP}_a\omega = \begin{cases} 0 & \text{If } \phi_\omega^{a,1} = \phi \\ \dfrac{1}{a(u_\omega, u_\omega)} & \text{If } \phi_\omega^{a,1} \neq \phi \end{cases} \qquad (6)$$

for all open subsets $\omega \subset \Omega$, where

$a(\cdot,\cdot)$ = bilinear form for network circuit section considered
u_ω = pure a-potential
$\phi_\omega^{a,1}$ = real set that is a closed convex subset of a real Hilbert Space
ϕ = empty set

$$\mathrm{CAP}_a\omega = \begin{cases} +\infty & , \text{If } H_\omega = \phi \\ a(u_\omega, u_\omega), \text{ If } H_\omega \neq \phi \end{cases} \qquad (7)$$

H_ω = a real Hilbert space over open subsets $\omega \subset \Omega$, and so

$$\mathrm{cap}_a\omega = \mathrm{cap}_a\omega \qquad (8)$$

Again, equation (3) is of this bilinear form, because:

$$C_i = \frac{1}{a_i} + \rho_i, \text{ where } a_i = \mu_i T_i \qquad (9)$$

and $\rho_i < 1$ for stability in an infinite storage queueing system; i.e. as ρ_i increases from 0 to 1 the system load increases, and therefore stability decreases.

Hausdorff measure dimension[17]

This analogy is stated owing to the property of the Hausdorff measure being characterized by an increasing function h(t) which decreases on shrinking sets, t representing the diameter of the set[4, 10, 19]. In a computer network, the function $h(t)$ may be taken as the scaled waiting-time density function $f_y(y)$ where $y = 1/t$ represents the service minus interarrival time. For the Lindley integral equation solution of the M/M/1 system[13], $f_y(y) = \lambda(1-\rho)e^{-\mu C(1-s)y}$ where μC = service rate = (messages/bits) (bit/s) and λ = interarrival rate, ρ = utilization factor, these all being for a single channel.

For output channels and finite storage approximations we take, for the ith channel,

$$h_i(t) = \left.\frac{f_y(y)_i}{\lambda_i(1-\rho_i)}\right|_{y=1/t} = (e^{-(1-\rho_i)}(\mu_i/t))^{C_i}$$

a formula which further illustrates this analogy. The exponent in the $h(t)$ function yields the Hausdorff dimension[4] and this exponent most often agrees with the potential theory capacity[20]. By our choice of $h(t)$ for the computer network, we see that computer networking capacity also agrees with the Hausdorff dimension. For inactive output channels and finite storage approximations, other formulae could be developed which also illustrate this anology.

Analytic capacity[10, 17]

Analytic capacity is defined[17] in a plane region R by $\max_{g \in G} b_1$ where G is a family of holomorphic functions g(z) in D, where D is the unbounded component of the complement of R. (Here $|g(z)| < 1$ and $g(z) = (b_1/z) + (b_2/z^2) + \ldots$ around infinity.) Therefore, the research by Polya and Szego[21] is consistent with this definition, because the capacity for a plane is shown to be $P(M)$, where M is an arbitrary plane bounded closed point set and $P(M) = d(M) = k(M)$, where k = capacity and d = transfinite diameter. Since $P = \lim_n P_n$ and $P = d$ was proven by Fekete in Reference 11,

$$P_n = \min_{p_i} \max_{p \in M} \sqrt[n]{|pp_1|\,|pp_2|\cdots|pp_n|} \tag{10}$$

where p_i = fixed points in M varied for the maximum
n = number of points considered
$|pp_i|$ = distance between p and p_i

Since the result of equation (10) can be applied to a two-dimensional surface in three-dimensional space for the robotic neural components, this can define capacity for the surfaces of neural components realized by three-dimensional VLSI. Also, by taking the derivative of T_i with respect to C_i, and equating the result to zero, after a LaGrangian expression has been formed, a mix–max[14, 22] expression similar to that in equation (10) of Fekete in formal capacity theory can be found. The computer networking definition and derivation of the analogy in equation (10) requires considering m groups of delays for n capacities, grouped in groups $G_1, \ldots, G_m$, where $C_o = C_{o1} + C_{o2} + , \ldots , C_{om}$; here C_{oi} is the total capacity of group $i (i = 1 \ldots m)$.

$$T_k(z_k) = \sum_{i \in G_k} \lambda_i \left[\frac{1}{[\mu_i C_{oi} - \lambda_i]}\right], k = 1, \ldots, m \tag{11}$$

where γ is as in equation (1)

$z_k = \{C_{oi} \mid C_{oi}$ representing a delay of $G_k\}$

Then, for the m groupings, the objective min–max expression becomes

$$\min_{C_{oi}} \max_{G_j} [T_1(z_1), \ldots, T_m(z_m)] = \min_{x_1, \ldots, x_m} \{\min_{z_1} \min_{z_2}, \ldots, \min_{z_m} \{\max_{G_j} [T_1(z_1, \ldots, T_m(z_m)\}\}\} \tag{12}$$

where $0 \le x_k \le 1$ is given by $x_k = \dfrac{C_o}{\sum_{i=1}^{k} C_{oi}} \ge 0$

and $\sum_{i=k+1}^{m} C_{oi} = (1 - x_k) C_o$,

and for fixed x_i with $\sum_{i=1}^{m} x_i = 1$. The minimization of $T_1, \ldots, T_m$ is the standard capacity assignment problem and,

$$A_k = \gamma^{-1}\left(\sum_{i \in G_k} \lambda_i^{1/2}\right)^2, \quad B_k = \sum_{i \in G_k} \lambda_i \mu_i^{-1} \tag{13}$$

is also a convex function. Expanding the A_k, B_k and defining C_{ok}, the solution for the LaGrange multiplier α, a convex function of capacity weighting becomes

$$\frac{A_1}{x_1 C_{o1} - B_1} = \frac{A_2}{x_2 C_{o2} - B_2} = \ldots = \frac{A_{m-1}}{x_{m-1} C_{o,\,m-1} - B_{m-1}} = \frac{A_m}{x_m C_{om} - B_m} = \alpha$$

$$x_i = \left(\frac{A_i}{\alpha} + B_i\right)\frac{1}{C_{oi}}$$

$$\frac{1}{\alpha} = \left(1 - \sum_{i=1}^{m} B_i\right)\left(\sum_{i=1}^{m} \frac{A_i}{C_{oi}}\right)^{-1} \tag{14}$$

ROBOTIC NEURAL SYSTEM MEASURES AND CAPACITY

Impulse coding concepts

Information in the robotic neural system is contained in the action-potential-like pulses being processed. Therefore, we first define some terms associated with these concepts for which it will be useful to refer to Figure 1.

Impulse

We call the information-carrying pulses of a robotic neural system impulses. These are action potential types of pulses that can be idealized to mathematical impulses for our purposes.

Time slot

A time slot is the time interval during which impulses are observed. Usually this is taken to be long enough that impulse frequency, impulse bursts and impulse burst frequencies can be monitored and measured.

Impulse burst

This is the sequence of pulses that comprises the pulse train, where the pulses occur as a group when the system is active. Time-slot 2 of Figure 1 indicates an impulse burst.

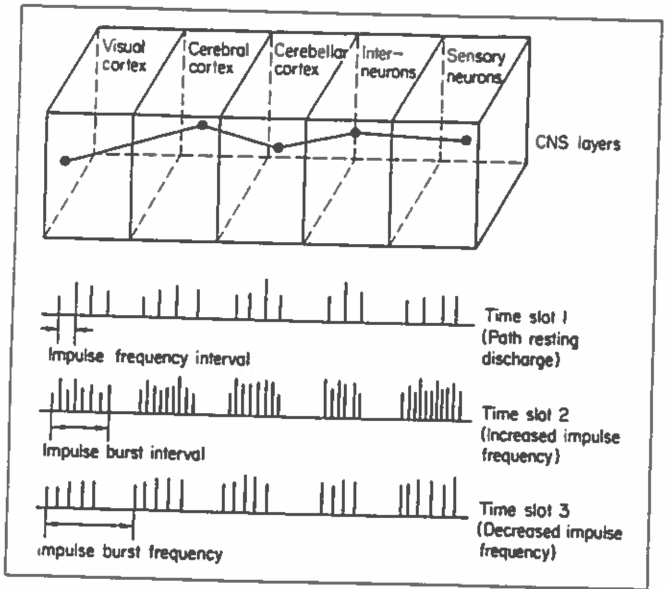

Figure 1. Coding of all-or-none response path in robotic neural system hardware and software

Impulse frequency

This is the inverse of the impulse frequency interval, the interval being the time between pulses in an impulse burst. This time is indicated in time slot 1 of Figure 1. It is assumed that the frequency is constant during a burst and that there is an upper impulse frequency, f_u, at which pulses can occur.

Impulse burst frequency

This is the inverse of the time between initiation of adjacent impulse bursts, as shown in time slot 3 of Figure 1. Thus, this is the rate at which impulse bursts travel along specific neural links (such as dendrites and axons).

Table 1 lists other specific computer networking and neural network analogies which are of interest in relating the robotic neural system to computer networking.

Capacity

In the robotic neural system, there are two notions of capacity, one, C_I, measures the capacity to handle individual impulses and the other, C_B, measures the capacity to handle impulse bursts. Different neural components will require different evaluations. However, C_I is expressed as the maximum frequency of impulses that the component can process. Similarly, C_B is the maximum impulse burst frequency that the component can process. Inherent in these capacity measures for most components is the basic idea of weights and amplitude[23] attached to the impulses indicating their effectiveness and routing coding information. In general, the amplitude of input impulses will determine, subject to weighting, which of the neurons in a signal path will fire (that is, give an impulse output due to the impulses present at input dendrite buttons).

Consider a robotic neural pathway made of neural lines and junctions, where at the junctions the outputs are weighted sums of the inputs. In terms of frequencies, if f_j is the frequency on line j and weight w_{ij} is the weight of f_j yielding the ith junction output, then we describe an n_i input junction by

$$V_i = \sum_j^{n_i} w_{ij} \cdot f_j \tag{15a}$$

where the firing frequency of the kth neuron is

$$f_k = F(V_k)$$

and

$$F(V_i) = \begin{cases} V_i & \text{if } F_k > 0 \quad (15b) \\ 0 & \text{if } F_k \leq 0 \quad (15c) \end{cases}$$

That is, we assume that the output frequency is linearly related to the input frequencies (one of which may be the output when feedback occurs). Since there are excitatory and inhibitory inputs, we further break the weights into positive and negative portions[24].

$$w_{ij} = \begin{cases} w_{ij}^{+} \text{ if } w_{ij} \geq 0 \text{ (excitatory)} \\ -w_{ij}^{-} \text{ if } w_{ij} < 0 \text{ (inhibitory)} \end{cases} \tag{16}$$

By appropriate numbering, we will take the first m_i of these to be non-negative. Since not all real numbers are allowed as weights, the values of $| w_{ij} |$ are restricted to lie in some weight set W. The envelope of these amplitudes will be defined mathematically by the quantization measure[25] which determines the interval between action potentials by sampling the interarrival and service rates of action potentials at path neurons.

We can now define the capacity of the neural pathway as the smallest maximum frequency that can be transmitted through the pathway. That is,

$$C = \min_i \max_{f_j} F(V_i) \tag{17}$$

In other words, we maximize each f_i by choice of the f_j and then look for the smallest f_i in the pathway. As per the above, we can use the impulse frequencies or the impulse burst frequencies; hence we again have at least two types of capacity for the neural pathway. In fact, there are many more than two types of neural pathway capacities, since for a given usage it may be that one component is limited by its upper impulse frequency and another component by the upper impulse burst frequency. Since the f_j in the definition become maximized, they can be replaced by the component capacities and the max term dropped, in which case the total path capacity C is expressed in terms of the component capacities C_i (where C_i is C_{I_i} or C_{B_i}). Thus

Table 1. **Computer networking and neural networking analogies**

	Computer networking components	Robotic neural networking analogies
	Computer nodes (buffer storage + front end (routing processor))	Soma (cell body) (buffer function + routing function)
	Input communications channels	Dendrite trees
	Output communications channels	Axons
	Communications processor ports	Buttons
Terminology	Parameter definitions	Analogies
Service time probability	Service/time message	Threshold time/potential
Density function parameters	Messages served/time interval t	Potentials achieving threshold/time interval t
Interarrival rate probability density function parameters	Interarrival time/message	Interarrival time/potential
	Messages arriving/time interval t	Potentials achieving the threshold/time slot t
Marginal overflow probability density function	Marginal message overflow/time interval t	Marginal potential overflow/time interval t
$\alpha_1 + \alpha_2$ = complexity of buffer storage at t^{th} time slot (interval)	α_1 = number of messages leaving buffer α_2 = number of messages entering buffer	α_1 = decrease in soma storage (potentials) α_2 = increase in soma storage (potentials)
Mean waiting time	Delay in buffer of each computer node	Delay in buffer function of each soma
Terminology	Computer networking definitions	Neural networking definitions
Packet switching network (PSN)	The set of communicating computers (nodes) connected with physical circuits, which carry bit strings using the store and forward concept	The neural network comprising a functional area of the nervous system
Packet	A sequence of bits (string) exchanged between nodes of a particular packet switched network	The quanta of information or burst of action potentials occurring over a discrete period of time from individual neurons. These pulses indicate by their *encoding* (combination of amplitudes and pulse time intervals) their routing destination
Message	A bit string comprised of packets exchanged between a PSN and an external device (a computer or another PSN)	A collection of bursts of impulses. The series of quanta of information from all neurons in one CNS functional area, i.e. the cerebellum, which are input to the dendrite tree branch buttons of a neuron in another functional area
Gateway	A node at the entrance to a PSN capable of sending/receiving from another PSN	Neurons at areas of the body referred to as 'pressure points' where there are many synaptic arrivals (action potentials) at the dendrite trees from gateway neurons of other functional nervous system areas
Internetwork	An abstract PSN structure resulting from the juxtaposition of several PSNs	Neural networks along path structures between human nervous system functional areas
Congestion	Stoppage of message flow because of system utilization exceeding 100%, total lockup of messages and therefore instability. A congested state is the point where flow exceeds capacity of the links and storage	The constant firing of neuron circuit action potentials resulting in *no* interval between bursts[27]

$$C = \min_{i} \sum_{j=1}^{n_i} w_{ij} C_j \quad (18)$$

If the w_{ij} are free to be chosen, as in a robot design, then they could be chosen, via say the LaGrange multiplier method mentioned above, to maximize further C, of course subject to the constraint $w_{ij} \varepsilon W$. And while in many cases the f_j and w_{ij} may be deterministic, in some cases they are probabilistic, in which case in the definition of capacity C we would place an expected value before the parentheses.

The next section applies these ideas to the layered neural pathway of Figure 1, in which no feedback is present.

Neural path example

Referring to Figure 1, and the cubic software structures representing sections of the central nervous system as described in Niznik and Newcomb[26], a neural path is set out where the capacity measurement is significant; it is described for strong and weak stimulus resting discharge and for all-or-none response paths. This robotic neural path is related to the computer networking capacity link assignment via LaGrange multiplier optimization following the neural optimization theory plan from Klopf[24]. The parameter being maximized is μ_2, a measure of transmission and synaptic frequencies defined as ($E[\cdot]$ is expected value of)

$$\mu_2(i) = E[\sum_{j=1}^{n_i} w_{ij}(t) C_j(t)] = E[\alpha_i(t) - \beta_i(t)] \quad (19)$$

where $\alpha_i(t) = \sum_{j=1}^{m_i} w_{ij}^{+}(t) C_j(t)$ = amount of excitation at i

$\beta_i(t) = \sum_{j=m+1}^{n_i} w_{ij}^{-}(t) C_j(t)$ = amount of inhibition at i

where j = the range of excitation
w_{ij} = synaptic transmittance for the jth input
$C_j(t)$ has been defined for equation (18)
$[m_i + 1 \leq j \leq n_i]$ = range of numbered inhibitory synapses.

Therefore,

$$\max_{W}(\mu_2(i)) = \max_{W} \{E[\alpha_i(t) - \beta_i(t)]\} \quad (20)$$

giving

$$\max_{W}(\mu_2(i)) = \max_{W}\{E[\alpha_i(t)]\} - \min\{E[\beta_i(t)]\} \quad (21)$$

We look for the worst possible case, the minimum of this expression, in order to use optimally the neural pathway. Therefore, in terms of the capacity of the robotic neural system, we look at the worst possible routing. On this pathway, the difference between the excitation and inhibition pulses is minimized by minimizing the maximum of the excitation and maximizing the minimum of the inhibition. The result gives the best frequency transmission obtainable in the worst possible situation.

CONCLUSIONS

The direct analogies between computer networking capacity and robotic neural capacity have been presented in terms of three historical circuit capacity definitions: Newtonian capacity, Hausdorff measure and analytic capacity. Specific computer networking terminology and analogies to robotic neural networks were also discussed. Finally, the measurement of the capacity of a robotic neural system path hardware and software example was illustrated.

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