It should be recalled that L. Solari was a Navy Officer and strictly cooperated in the Marconi work from 1899 to 1936; in 1939 he wrote "Storia della Radio" (History of Radio), where we found the above news.

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A Diagram for Stability, Gain and Bandwidth of a Linear One-Stage Tunnel Diode Amplifier*

For designing pulse or wide-band amplifiers as well as tuned amplifier circuits with tunnel diodes, it is useful to have a diagram relating diode and circuit parameters to stability, maximum obtainable gain, frequency of maximum gain and bandwidth.[1]

Fig. 1 shows the basic circuit. The magnitude V of the voltage gain can be calculated by conventional means. The following abbreviations are involved:

$$\Omega = \frac{\omega}{\omega_0}\,;\ \omega_0 = \frac{G}{C}\,;\ \alpha = G^2\cdot\frac{L}{C}\,;\ \beta = R_i\cdot G. \quad (1)$$

Using the circuit parameters α and β as coordinates, the curves of constant maximum magnitude V_m of voltage gain and related frequency Ω_m are plotted in Fig. 2. The stability conditions $\alpha<\beta<1$ prove to form a rectangular triangle within the α, β-plane and the curves of constant V_m and constant Ω_m which are plotted belong to stable circuits. Curves of constant 3-db bandwidth are shown in Fig. 3.

Within the stability range, the pairs of circuit parameters α, β, which belong to the low-pass wide-band amplifier type, lie at the left of the curve $\Omega_m=0$ in Fig. 2. They have their maximum gain V_m at zero-frequency where

$$V_m/_{\Omega=0} = \frac{1}{1-\beta}\,. \quad (2)$$

The 3-db bandwidth $\Delta\Omega$ is approximately given by ($\alpha\ll 1$)

$$\beta = \Delta\Omega\cdot\alpha + \frac{1}{1+\Delta\Omega}\,. \quad (3)$$

$\Delta\Omega$ equals the absolute bandwidth $\Delta\omega$ referred to the cutoff frequency ω_0 of the combination G parallel C.

Pairs of the circuit parameters α, β, which belong to the narrow-band amplifier type, lie in the vicinity of the dynamic stability boundary $\beta=\alpha$. Their 3-db bandwidth is approximately given by ($\Delta\Omega\ll 1$)

$$\Delta\Omega = \frac{\beta}{\alpha} - 1. \quad (4)$$

* Received February 25, 1963; revised manuscript received March 19, 1963.

[1] A more detailed evaluation will be published in *Archiv der elektrischen Uebertragung* (in German).

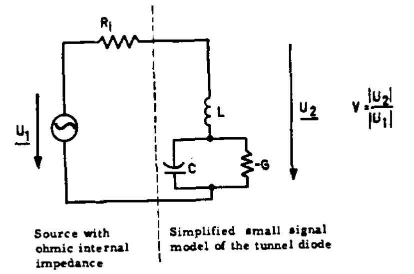

Fig. 1—Basic circuit for calculation of voltage gain.

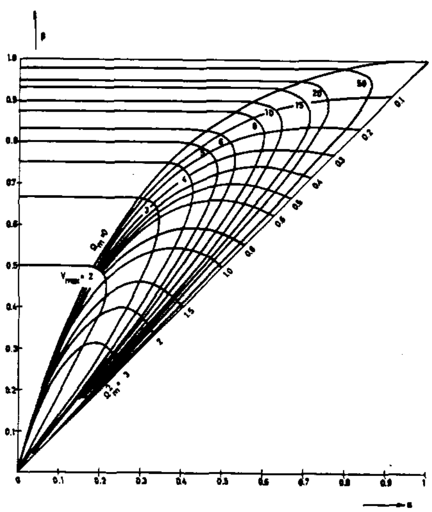

Fig. 2—Curves of constant maximum gain and constant frequency of maximum gain within the stability range.

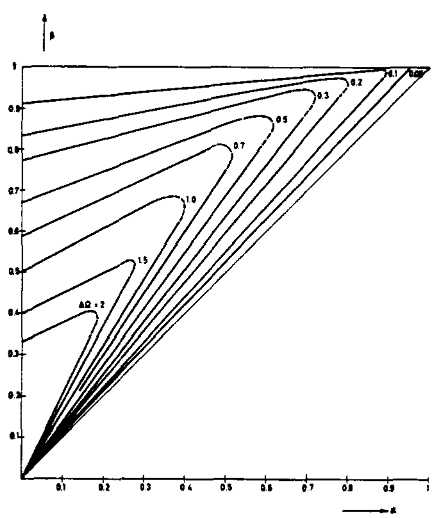

Fig. 3—Curves of constant bandwidth within the stability range.

The peak value of voltage gain amounts to

$$V_m = \frac{1}{\Delta\Omega\sqrt{1-\beta}} \quad (5)$$

at the frequency

$$\Omega_m^2 = \frac{1}{\beta} - 1 + \frac{\Delta\Omega}{\beta}(1-\beta). \quad (6)$$

The usefulness of the diagram is based on the fact that the boundaries of stability involved in the designing problem are always present as boundaries of the diagram. Thus, it can also answer the question of which amount of gain is obtainable if certain tolerances of the circuit parameters are given and, therefore, a certain distance from the stability boundaries is necessary.

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Distributional Impulse Response Theorems*

Here we call attention to some results of L. Schwartz which rigorously justify the use of the impulse response while pointing out the conditions of validity for its use. We adhere to Schwartz's notation[1] which we try to clarify to the extent possible in the short space available.

Consider a physical system which can be mathematically represented by a transformation $\mathcal{S}[\ ]$ which maps an input x, which is a real valued function of time t, into an output y, which is also a real valued function of t. Denoting the set of allowed inputs by X and the set of allowed outputs by Y, we then write

$$y = \mathcal{S}[x];\ x\in X,\ y\in Y. \quad (1)$$

Before giving the results we state the assumptions needed, which are as follows:

A-1: $\mathfrak{D}\subset X$ and $Y\subset\mathfrak{D}'$,

A-2: $\mathcal{S}[\ ]$ is linear,

A-3: $\mathcal{S}[\ ]$ is continuous.

A-1 says that the input space X contains the space $\mathfrak{D}$ of all infinitely (continuously) differentiable functions which are zero for t outside a finite interval (that is, of compact support), while the output space Y is included in the space $\mathfrak{D}'$ of distributions (the common functions, doublets, impulses etc.). A-2 means that for all real constants α and β and all x_1, $x_2\in X$

$$\mathcal{S}[\alpha x_1+\beta x_2] = \alpha\mathcal{S}[x_1]+\beta\mathcal{S}[x_2]. \quad (2)$$

The interpretation of A-3 is that if $\{x_n\}$ is a convergent sequence of inputs in $\mathfrak{D}$ then

$$\mathcal{S}\left[\lim_{n\to\infty} x_n\right] = \lim_{n\to\infty}\mathcal{S}[x_n]. \quad (3)$$

Actually the precise notions of convergence, which rest on topological considerations[2,3] are somewhat complicated because the topologies in $\mathfrak{D}$ and $\mathfrak{D}'$ cannot be described in simple terms.

The most general of Schwartz's results is the following theorem.[4] For this one defines a distributional kernel to be any distribution in two real variables.[3]

* Received February 18, 1963; revised manuscript received February 28, 1963. This work was carried out under the sponsorship of the Air Force Office of Scientific Research, Grant No. AF-AFOSR-62-349.

[1] L. Schwartz, "Théorie des Distributions," Hermann Publishing Co., Paris, France, vols. I, II; 1957, 1959. See especially vol. II, p. 169.

[2] *Ibid.*, vol. I, pp. 68–74.

[3] L. Schwartz, "Théorie des Noyaux," *Proc. Internat'l. Congress of Mathematics*, Cambridge, Mass., pp. 220–230; 1950. See especially p. 221.

[4] *Ibid.*, p. 223.

Theorem 1: If $\mathcal{S}[\;]$ satisfies A-1, A-2 and A-3, then there exists a unique distributional kernel $K(t, \tau)$ such that for $x \in \mathfrak{D}$,

$$y(t) = \int_{-\infty}^{\infty} K(t, \tau)x(\tau)d\tau = \mathcal{S}[x]. \quad (4)$$

Here the integral is symbolically written for the precisely defined scalar product[5] $y = K \cdot x$ which is actually the integral of (4) when K is an integrable function. $K(t, \xi)$ physically represents the response to an impulse at time ξ, $x(\tau) = \delta(\tau - \xi)$. Since $\mathfrak{D}$ is dense[6] in $\mathfrak{D}'$, (4) can be used for any $x \in X$ for which a sequence $\{x_n\}$ converges to $x \in \mathfrak{D}'$, $x_n \in \mathfrak{D}$, such that the right of (3) exists.

Another assumption of interest is

A-4: $\mathcal{S}[\;]$ is time invariant.

In other words, every pair $[y, x]$ for which (1) holds and for every real finite $T \geq 0$, there is an $x_0 \in X$ with $y_0 = \mathcal{S}[x_0]$ such that for all t

$$x(t) = x_0(t + T) \quad (5a)$$

$$y(t) = y_0(t + T). \quad (5b)$$

Time invariance in conjunction with linearity, and continuity implies that $\mathcal{S}[\;]$ commutes with differentiation and we can state the following.[7]

Theorem 2: If $\mathcal{S}[\;]$ satisfies A-1, A-2, A-3 and A-4, then for $x \in \mathfrak{D}$, $\mathcal{S}[\;]$ is a convolution operator. That is there exists a $k \in \mathfrak{D}'$ such that $y = k * x$ which means that (4) is

$$y(t) = \int_{-\infty}^{\infty} k(t - \tau)x(\tau)d\tau = \mathcal{S}[x]. \quad (6)$$

Since (6) is a special case of (4) the same extension to $x \notin \mathfrak{D}$ applies, in fact (6) always holds for $x = \delta =$ unit impulse.[8] Note, however, that since the convolution cannot be defined for an arbitrary pair of distributions, neither $k * x$ of (6) nor $K \cdot x$ of (4) may have meaning for some allowed inputs x, $x \in X$. For example, if $k(t - \tau) = u(t - \tau)$ and $x(\tau) = u(-\tau)$, then the integral of (6) diverges; here u is the unit step function.

Of course for physical systems the input and output spaces to be considered, as well as α and β in (2), are real, but sometimes complex ones are met in theoretical studies (when letting $x(t) = \exp[-pt]$ in (6) to get the Laplace transform, for instance). The above hold for real or complex spaces, but linearity shows that we only need to consider real spaces at the start. For multivariable systems we merely replace x and y by vectors and K by a matrix, which is again unique. If the inputs are such that they are all zero for $t < c =$ fixed constant, then A-1 is violated. An alternate assumption is

A-1′: $\mathfrak{D}_{(c,+\infty)} \subset X$ and $Y \subset \mathfrak{D}'$

where $\mathfrak{D}_{(c,+\infty)}$ is the same as $\mathfrak{D}$ except that the functions vanish for $t < c$.[9] Replacing $\mathfrak{D}$ by $\mathfrak{D}_{(c,+\infty)}$ where needed [at (3), (4) and (6)] and observing Schwartz's proofs, A-1 can be replaced by A-1′ in Theorems 1 and 2; $T \geq 0$ was used at (5) to get Theorem 2 using A-1′. If the system is antecedal[10] (sometimes called "causal") then, distributionally,

$$K(t, \tau) = 0 \text{ for } \tau > t \quad (7)$$

and if it is also time invariant, (6) can be used[11] for every $x \in \mathfrak{D}_+'$, that is, for every distribution which is zero until a finite time.

In summary, Theorems 1 and 2 state what most electrical engineers already intuitively know. However, the assumptions for their validity are made precise by Schwartz's theorems. In particular it appears that A-3 has been somewhat overlooked by engineers, except perhaps in some network studies.[12] Of course (1) is not the most general model for a system, and, if a "state" can be defined, A-2 essentially assumes that "initially" the system is in the zero state (that is initially "at rest").

Acknowledgment

The valuable suggestions of Prof. C. Desoer for the improvement of the readability of this communication are gratefully acknowledged.

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[5] *Ibid.*, p. 220.
[6] Schwartz, *op. cit.*, vol. I, p. 75.
[7] Schwartz, *op. cit.*, vol. II, p. 18.
[8] *Ibid.*, p. 11.
[9] *Ibid.*, p. 28.

[10] M. Bunge, "Causality, chance, and law," *Am. Scientist*, vol. 49, pp. 432–448; December, 1961. See especially p. 438.
[11] Schwartz, *op. cit.*, vol. II, p. 29.
[12] D. Youla, L. Castriota and H. Carlin, "Bounded Real Scattering Matrices and the Foundations of Linear Network Theory," IRE Trans. on Circuit Theory, vol. CT-6, pp. 102–124; March, 1959. See especially p. 113, postulate 4.

Comments Concerning "On a New Security Problem"*

Mr. McAuley[1]

In his communication, Mr. Geffe states:

1) ". . . In other words, arguing on probabilistic grounds, it is almost certain that the alien culture would be some centuries, millennia, or eons ahead of us in the development of physical science."
2) "It follows therefore that any military contact between the communicants would find us in a position of complete helplessness."

I disagree. If alien cultures do exist, it is highly improbable that they are at the same level of technological development that we are. But must they be ahead of us? Why can't they be centuries, millennia, or eons *behind* us?

If the alien cultures are behind us in the development and application of the physical sciences:

1) We will probably be incapable of discovering the existence of these cultures for some time and
2) We, not they, will be in a dominating position.

The problem will then be to prevent humans from exploiting and destroying these alien cultures.

And if the alien cultures are more advanced than we are? We will not necessarily be in a position of complete military helplessness. Rome was far ahead of the barbarian peoples in the development and application of the physical sciences, yet Rome fell to the barbarians.

If the alien cultures are very far ahead of us in the physical sciences, they probably already know of our existence, our capabilities, and our limitations. There would be little point in refusing to engage in interstellar communications with them if this is the case.

In summary, if other cultures do exist, we should seize every opportunity to communicate with them. We have everything to gain and little to lose.

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Mr. Lott:[2]

In reply to Mr. Geffe,[1] may I point out that he is too late, far too late, with his comment (6) with regard to preventing irresponsible attempts at interstellar communication with alien cultures. For several decades there has been an increasing number of broadcast and television programs on shorter and shorter wavelengths at higher and higher powers of types which readily penetrate the ionized layers surrounding the earth, and there is no doubt in my mind whatsoever that if the alien cultures exist with the level of technicological advancement suggested by Mr. Geffe, the deduction and analysis of the vast amount of information which we pour out into the universe daily will already be an accomplished fact about which we are able to do absolutely nothing at this stage.

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Author's Reply[3]

McAuley's first argument is based on a misunderstanding of my point. In my letter I considered, not those alien cultures which merely exist, but those which would be in communication with us. In view of the age of our galaxy, this fixes their level of technological development as being equal to ours with a tolerance of $^{-10^9}_{+10^9}$ years, approximately. Cultures at a lower level are irrelevant to the subject of my letter. I admit that there is no security reason not to contact lower cultures, and I would like to know how to do so.

* Received December 5, 1962.
[1] P. R. Geffe, Proc. IRE (*Correspondence*), vol. 50, p. 2126; October, 1962.

[2] Received December 10, 1962.
[3] Received January 8, 1963.