On Parasitics in the Semistate Formulation

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Abstract:

An example, the first degree Hazony filter section, is used to show that semistate equations become regularized by parasitics in a manner different than postualted in the iterature.

I. Introduction

The inclusion of parasitics unnecessarily complicates the design process for some systems, as for example those containing hysteresis. However, in some such cases the practical operation of the finally designed system depends critically upon the presence and effects of parasitics, as for example in determining single-valued hysteresis systems. In the case of semistate described systems, available literature implies that the parasitics lead to a nonsingular coefficient matrix on the semistate derivative (see for example [1, p.112]). Here we show by example that such need not be the case.

II. Canonical Equation Framework

Given a semistate describable system it has the canonical description [2]

$$\mathcal{A}\dot{x} + \mathcal{B}(x,t) = \mathcal{D}u \qquad (1a)$$

$$y = \mathcal{H}x$$

where the $\mathcal{A}$, $\mathcal{D}$, and $\mathcal{H}$ are constant matrices, $\mathcal{A}$ being nxn for n-vector semistates x; u and y are inputs and outputs, respectively, and · denotes differentiation in time t.

To tie in with previously published material [1], [3], we bring the coefficient of $\dot{x}$ to be the identity on the nonsingular part of $\mathcal{A}$ and zero on the singular portion, as others have done in the linear case [4, p.33][5]. That is, by an equivalence transformation, of nonsingular matrices P and Q, we write

$$P\mathcal{A}Q = 1_m \dotplus 0_{n-m} \qquad (2a)$$

$$Q^{-1}x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{matrix}\text{dimension } x_1 = m \\ \text{dimension } x_2 = n-m\end{matrix} \qquad (2b)$$

where 1_m is the mxm identity, m = rank $\mathcal{A}$, $\dotplus$ denotes the direct sum, 0_{n-m} is the (n-m)x(n-m) zero marix. Then (1) is rewritten as

$$\dot{x}_1 + \mathcal{B}_1(x_1,x_2,t) = \mathcal{D}_1 u \qquad (3a)$$

$$\mathcal{B}_2(x_1,x_2,t) = \mathcal{D}_2 u \qquad (3b)$$

$$y = \mathcal{H}_1 x_1 + \mathcal{H}_2 x_2 \qquad (3c)$$

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In [1, p.112] it is postulated that parasitics add to the left of (3b) the term $\varepsilon\dot{x}_2$ where ε is a small parameter. In the following examples we show that such need not be the case.

III. An Example

To show the effect on the form of the equations it is initially sufficient to consider a linear example, for which we choose the loaded degree-one Hazony section of Fig. 1a), this section playing an important role in active filter synthesis [6, p. 158]. For the circuit graph of Fig. 1b) we choose the tree as branches 1&2 in which case the technique of [2] gives the canonical semistate equations as (G = 1/R, g = gyration conductance)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C & -C & 0 \end{bmatrix}\dot{x} + \begin{bmatrix} 0 & g & 1 \\ -g & G & -1 \\ 0 & 0 & -1 \end{bmatrix}x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}u \qquad (4a)$$

$$y = [0,\ 1,\ 0]x \qquad (4b)$$

where

$$u = i_g,\ y = v_2,\ x = \begin{bmatrix} v_1 \\ v_2 \\ i_3 \end{bmatrix} \qquad (5)$$

Using

$$P = \begin{bmatrix} 0 & 0 & 1/C \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (6a)$$

$$Q^{-1}x = \begin{bmatrix} v_1 - v_2 \\ v_2 \\ i_3 \end{bmatrix}; \ x_1 = v_1 - v_2,\ x_2 = \begin{bmatrix} v_2 \\ i_3 \end{bmatrix} \qquad (6b)$$

gives for (3)

$$\dot{x}_1 + [0,\ 0,\ -1/C]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [0]u \qquad (7a)$$

$$\begin{bmatrix} -g & G-g & -1 \\ 0 & g & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \qquad (7b)$$

$$y = [0]x_1 + [1,0]x_2$$

It is observed that if G = 0, that is, an open circuit load, then x_2 has a singular coefficient matrix in (7b) and, hence, can not be eliminated from (7a) to obtain state-variable equations.

Although the characterization of parasitics for even this simple system is a difficult problem in itself, let us assume that the device is to be realized by a fully integrated circuit and operated at low frequencies. Then inductive parasitics can be ignored since capacitive and resistive effects will dominate, leading to the circuit of Fig. 2. In Fig. 2, C_1 & R_1 are Norton equivalent parasitic input capacitance and resistance for the gyrator, R_2 is the similar output resistance, C_2 combines the similar output capacitance with the parasitic capacitance of the load while R_3 is the parasitic resistance of the capacitor C. The same graph may be used as for Fig. 1 resulting in

$$\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ C & -C & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} G_1 & g & 1 \\ -g & G + G_2 & -1 \\ G_3 & -G_3 & -1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \quad (8a)$$

$$y = [0,\ 1,\ 0]x \quad (8b)$$

where (5) still holds to define u, x, and y. The use of P and Q of (6a) brings these to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & C_2 & 0 \\ C_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} G_3/C & 0 & -1/C \\ -g & G - g + G_2 & -1 \\ G_1 & g + G_1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u \quad (9a)$$

$$y = [0,\ 1,\ 0,] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (9b)$$

It is observed that (9a) is not of the form of (7a, b) with $\epsilon\dot{x}_2$ added to the left of (7b). Indeed the $\mathcal{A}$ matrix in (9a) is of a different form and too small a rank. Nevertheless, the third component of x, i_3, can be eliminated in (9) to reduce (9) to state-variable equations, even in the case, G = 0, in which no such reduction is possible when parasitics are not considered. That is, the parasitics considered in Fig. 2 are adequate to regularize the system but their insertion does not regularize the equations in the manner postulated in the literature.

IV. Discussion

The example shown here illustrates that parasitics come into play in the semistate equations in a manner not previously foreseen. Nevertheless semistate theory readily handles parasitics. handles parasitics. Beyond that, semistate theory shows how parasitics can be inserted in design to guarantee unique solutions; this is by inserting parasitics until a $\mathcal{B}_2$ is obtained at (3b) that can be uniquely solved for x_2. As such, we are led to answer the questions of regularization for RLC circuits raised by Smale [7, p. 209] since any of his circuits can be placed in the framework of the canonical semistate equations of (1).

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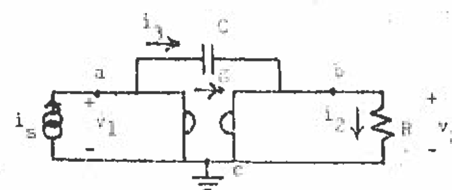
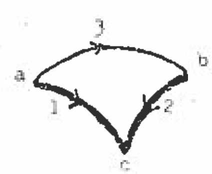


a) Loaded Hazony Section



b) Graph

Figure 1

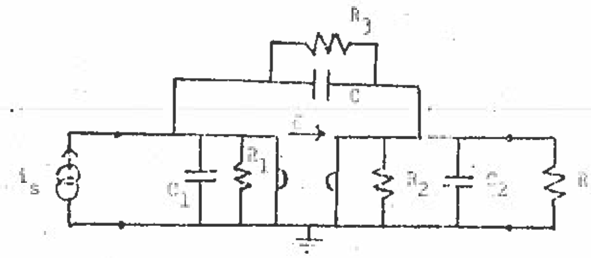


Figure 2

Hazony Section with

Relevant Parasitics

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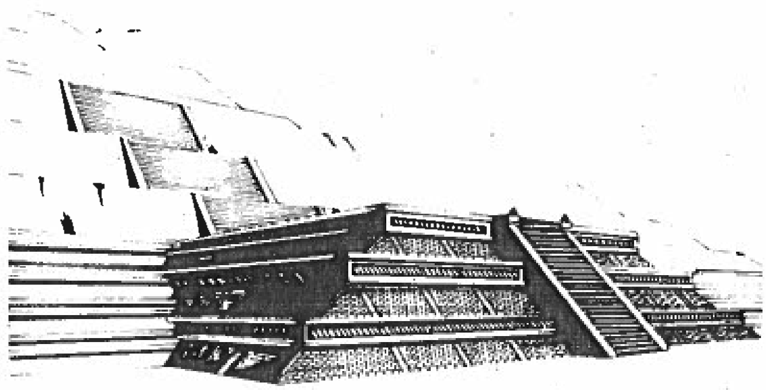

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