Porous Media," Ind.
ugh Porous Media,"
il Volume Averaging
ol. 3, pp. 333-340
raged Equations of
lows," ANL-80-51,
and E. I. H. Lin,
n-Phase Component
-77-96, NUREG/CR-
ih, "COMMIX-1A: A
ogram for Thermal
tems," ANL-82-25,
Formulation for
Liquid Metal Fast
draulics, Nuclear
n Continua, Chap.
." AIChE J., Vol.
amics Calculation
197-213 (1971).
Hemisphere Publ.
d Thermal Mixing
" Electric Power
d Thermal Mixing
Electric Power
Sha, J. H. Kim,
the Cold Leg of
, pp. 1057-1058

Proceedings Int. AMSE Symposium "Modelling & Simulation", Bermuda, March 1-3, 1983, p. 181-187

A SEMISTATE MODEL FOR EQUAL SLOPE HYSTERESIS

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Abstract:

Semistate equations are presented which will model any sloping hysteresis at DC where the two branches of the hysteresis have equal slope. The model uses step function nonlinearities which are of operational amplifier type and the hysteresis parameters are determined in terms of semistate coefficients.

I. Introduction

Sloping hysteresis is hysteresis with two linear branches while semistate theory is a generalization of state variable theory which allows the characterization of multivalued operators through single valued ones. Sloping hysteresis has recently been introduced in the framework of design of solitary pulse waves [1] while semistate theory has been shown to advantageously model hysteresis [2]. Here the two concepts are merged to develop a semistate model of sloping hysteresis for the most useful case where both hysteresis branches have the same slope.

Semistate equations of nonlinear systems can be put in the canonical form [3]

$$\mathcal{A}\dot{x} + \mathcal{B}(x,t) = \mathcal{D}u \quad (1a)$$

$$y = \mathcal{F}x \quad (1b)$$

where $\mathcal{A}, \mathcal{D}$, and $\mathcal{F}$ are constant generally singular matrices and x, u, and y are the semistate, input, and output vectors, respectively. Because $\mathcal{A}$ is singular those

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semistate equations can model systems in terms of all variables of interest thus allowing multivalued operators, as those of hysteresis, to be described by single-valued ones. In the case of hysteresis interest is usually in the DC characteristic in which $\dot{x} = 0$ is set in (1a). Hysteresis results when multiple solutions for the semistate x are obtained from $\mathcal{B}(x,t) = \mathcal{D}u$. Often [2] these solutions can be looked upon as a moving load line, parameterized by the input u, intersecting a nonlinear characteristic. In this paper we show that this philosophy can be used to model sloping hysteresis where for convenience we assume that both hysteresis branches have equal slope. Figures 1 & 2 d) (and their negatives) show the nature of the sloping hysteresis modeled by this theory using step function (op-amp type) nonlinear characteristics.

II. The Model

From as yet unpublished results stemming from experimentation on op-amp circuits we are led to consider the semistate equations

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 0 & -1 & 0 \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} x + \begin{bmatrix} 0 \\ K(x_1) \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \qquad \text{(a2)}$$

$$y = [0,\ 1,\ 1]x \qquad \text{(2b)}$$

Here we will take

$$b_{21} < 0,\ b_{23} > 0,\ \ b_{31} > 0,\ \ b_{23} - b_{33} > 0 \qquad \text{(3a)}$$

and $K(\cdot)$ to be a step function op-amp characteristic of one of the two forms

$$K_+(x_1) = \begin{cases} V_+ & \text{for } x_1 > 0 \\ -V_- & \text{for } x_1 < 0 \end{cases} \qquad \text{(3b)}$$

or

$$K_-(x_1) = -K_+(x_1) \qquad \text{(3c)}$$

Setting the second and third semistate equations equal, as they both equal the input u, we can solve for x_3. Using this result and the third semistate equation gives

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$$-(b_{23}-b_{33})x_3 = K(x_1) - (b_{31}-b_{21})x_1 - b_3 + (b_{22} - b_{32})x_2 \qquad \text{(4a)}$$

$$-(b_{23}-b_{33})x_3 = \frac{(b_{23} - b_{33})}{b_{33}} \left[b_{31}\, x_1 + b_3 - u + b_{32}\, x_2\right] \qquad \text{(4b)}$$

And since we are interested here solely in the DC characteristics, we take $\dot{x} = 0$ from which the first of the semistate equations gives

$$x_2 \equiv 0 \qquad \text{(4c)}$$

In such a case (4b) serves as a load line, with parameter u, on the characteristic curve of (4a). For $K = K_+$ and b_{33} 0, Fig. 1 shows the situation; in Fig. 1a) eq. (4a) is sketched while in Fig. 1b) eq. (4b) is shown. Starting with a large enough negative u, as u increases the load line of Fig. 1b) sweeps to the right across the characteristic of Fig. 1a) as shown in Fig. 1c); at $u = u_a$ there is one intersection, at $u = u_b = u_f$ there are two intersections until $u = u_c = u_e$ is reached followed by one intersection at $u = u_d$. Plotting the value of $y = x_3$ at the intersections versus u giving the intersection yields the sloping hysteresis as shown in Fig. 1d). By eliminating x_1 from (the linear portions of) (4a) & (4b) we find

$$y = x_3 = \frac{b_{31} - b_{21}}{b_{31}b_{23} - b_{21}b_{33}} \left\{ u + \left[\frac{b_{21}b_3 - b_{31}K(\cdot)}{b_{31} - b_{21}}\right]\right\} \qquad \text{(5)}$$

where $K(\cdot)$ takes the values $\pm V_+$, $\mp V_-$, depending upon which point on the curve and which K ($=K_+$ or K_-) is under consideration. Using (5) we can give the (u,y) coordinates of points on the hysteresis curve, for example, at point c (from Fig. 1c).

$$-(b_{23}-b_{33})y_c = -V_- - b_3 \Rightarrow y_c = \frac{b_3 + V_-}{b_{23}-b_{33}} \qquad \text{(6a)}$$

with (5) giving

$$u_c = \frac{b_{23}b_3 + b_{33}\, V_-}{b_{23} - b_{33}} \qquad \text{(6b)}$$

By symmetry, replacing $-V_-$ by $+ V_+$,

$$y_f = \frac{b_3 - V_+}{b_{23} - b_{33}}\ , \qquad u_f = \frac{b_{23}b_3 - b_{33}V_+}{b_{23} - b_{33}} \tag{6c}$$

Consequently, the hysteresis is centered at (set $V_+ = V_- = 0$ in (6a&b))

$$(u_o, y_o) = \frac{b_3}{b_{23} - b_{33}}(b_{23}, 1) \tag{6d}$$

and has (from (5))

$$\text{slope} = \frac{b_{31} - b_{21}}{b_{31}b_{23} - b_{21}b_{33}} \tag{6e}$$

with

$$\text{width} = u_c - u_f = \frac{b_{33}}{b_{23} - b_{33}}(V_+ + V_-) \tag{6f}$$

The vertical distance between the two overlapping portions of the hysteresis is found by setting u = 0 and subtracting the lower value of the curve from the upper one as

$$\text{height} = \frac{b_{31}}{b_{31}b_{23} - b_{21}b_{33}}[V_+ + V_-] \tag{6g}$$

Summarizing, we see via equations (6) that we can choose the parameters, subject to the constraints of (3a), such that arbitrary curves of the shape of Fig. 1d) can be obtained when $K = K_+$ and $b_{33} > 0$.

In the event that $K = K_-$ and $b_{33} < 0$ the results are somewhat similar, except that some constraint has to be placed upon the slopes of the load line and characteristic curve to insure hysteresis. As seen from Fig. 2, the requirement is $[(b_{23} - b_{33})/(-b_{33})]b_{31} > b_{31} - b_{21}$ or

$$b_{23}b_{31} > b_{21}b_{33} \tag{7}$$

Equation (5) still remains valid and the properties of the resulting hysteresis, Fig. 2d), can be determined as at equations (6). The center, slope and height equations are identically as before while the width equation gets a negative multiplier.

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References

[1]. R. W. Newcomb an
Manuscript prepa

[2]. G. Kiruthi, H. Y
Semistate Equati
and Systems, Tol

[3]. R. W. Newcomb, "
Circuits," IEEE
January 1981, pp

Again, any curve of the form of Fig. 2d) can be obtained when $K = K_-$ and $b_{33} < 0$.

Similar arguments show that only a single-valued curve results (i.e., no hysteresis) when $K = K_-$ and $b_{33} > 0$ while a negative slope hysteresis (left portion above the right) is obtainable when $K = K_+$ and $b_{33} < 0$ subject to a reversal of inequality (7). However, the negatives of both of the hystereses in Figs. 1d) & 2d) can be obtained by multiplying the output semistate equation, (2b), by -1.

Discussion

As shown above, by appriopriate choices of the b's any sloping hysteresis with equal slopes of the two branches can be modeled at DC by the semistate equations (2), possibly with a sign reversal in the output equation. Since it appears that these semistate equations can be realized by RC-op-amp circuits, the theory should be of use in practical simulation studies.

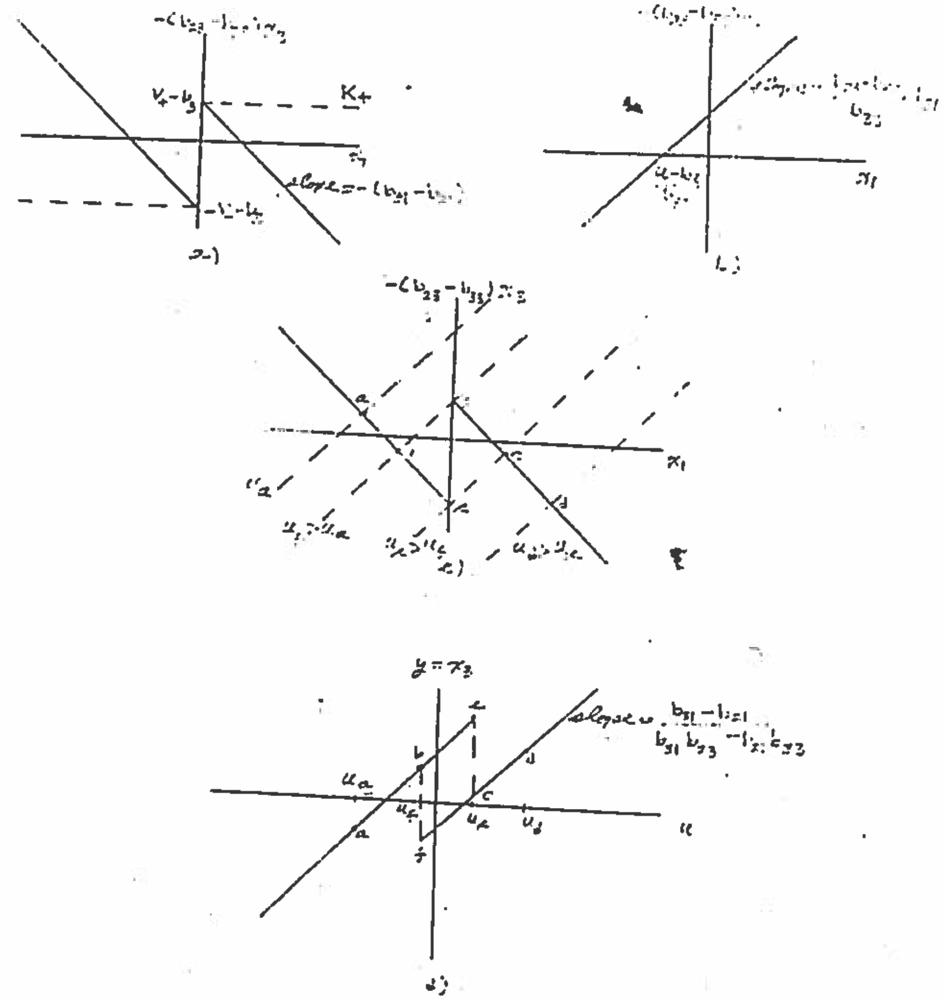
Our study here has been primarily of the DC behavior since that is the reqime in which the hysteresis is primarily formulated. However, the semistate equations (2) give a dynamic model which should be of use when sloping hysteresis is used in non-static situations. In the dynamic regime, as shown by eqs. (4), the load line and nonlinear characteristic both shift. Practically this must occur such that there is a single valued traversal (as fixed by b_{22} & b_{32}) of the DC hysteresis curve at a rate fixed by the time constant a_{11}. Certainly the dynamics of this hysteresis is interesting in its own right and merits further study.

References

[1]. R. W. Newcomb and R. C. Ajmera, "A Hysteretic RC Line for Solitary Pulses," Manuscript prepared for publication.

[2]. G. Kiruthi, H. Yazdani, and R. W. Newcomb, "A Hysteresis Circuit Seen through Semistate Equations," Proceedings of the 23rd Midwest Symposium on circuits and Systems, Toledo, August 1980, pp. 279 - 283.

[3]. R. W. Newcomb, "The Semistate Description of Nonlinear and Time-Variable Circuits," <u>IEEE Transactions on Circuits and Systems</u>, Vol. CAS-28, No. 1, January 1981, pp. 62 - 71.



1. Sloping Hysteresis for $K = K_+$, $b_{33} > 0$

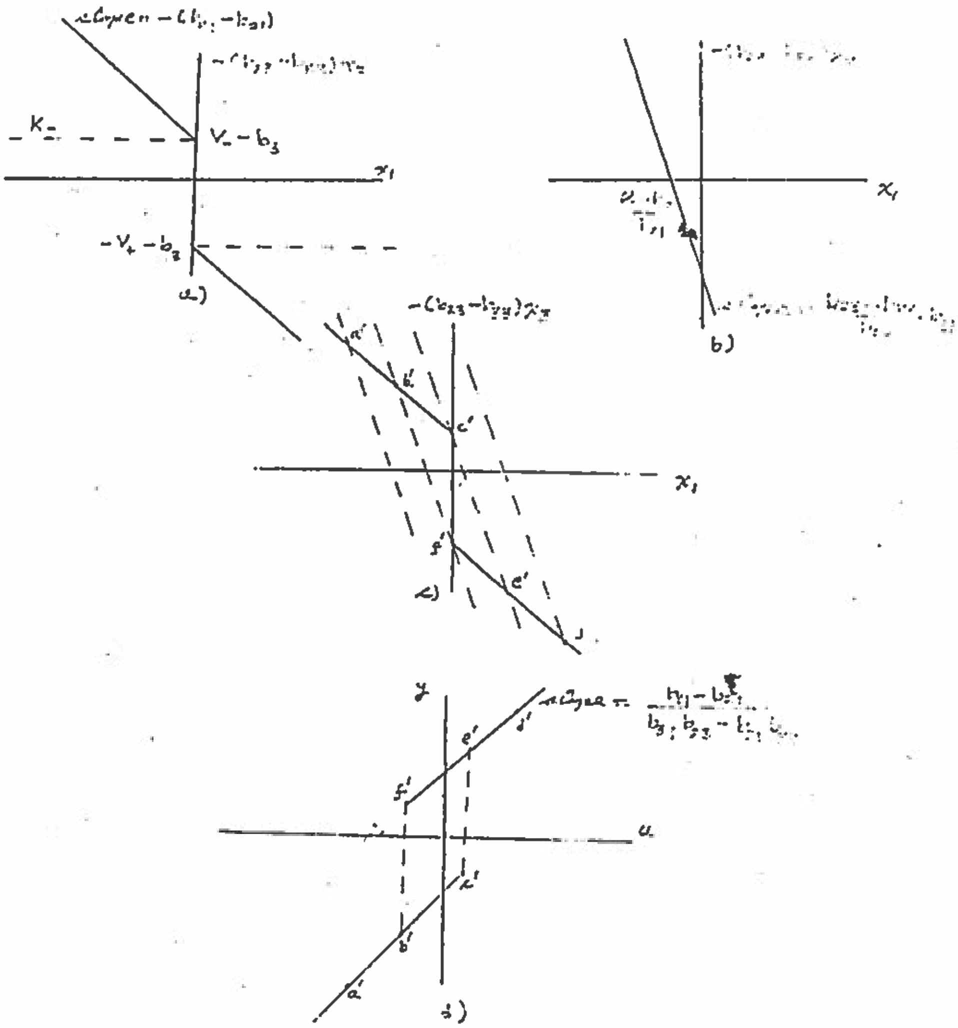
   a). Nonlinear Characteristic

   b). Load-Line, u = parameter

   c). Intersections of Nonlinear Characteristic and Load-Line for Various u

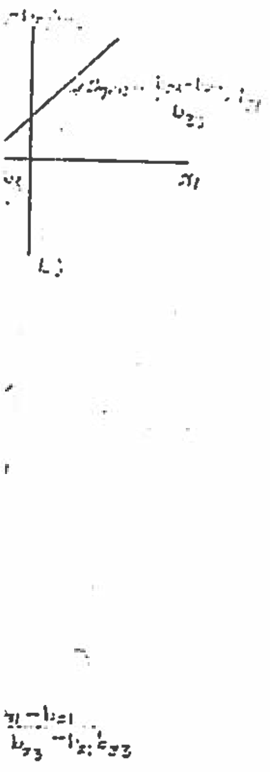
Fig. 1

2. Sloping Hyst



2. Sloping Hysteresis for $K = K_-$, $b_{33} < 0$

Fig. 2



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