suitable as a prototype (subject to the presence of physical symmetry in the case of the elliptic type), a variety of frequency responses is available for the matched structure.

ACKNOWLEDGMENT

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Letters to the Editor

An *RC* Op Amp Chaos Generator

R. W. NEWCOMB AND S. SATHYAN

Abstract*—An *RC* op amp chaos generator is presented with experimental results showing the practicality and convenience of the system. The circuit is achieved by inserting dynamics on a nondynamic semistate variable of a second-order bent hysteresis Lienard system, the idea stemming from observations of Shinriki *et al.

I. INTRODUCTION

Electronically generated chaotic signals would give means of monitoring and controlling effects that can occur in biological systems without actually testing, and possibly damaging, the biological system itself, as for example the effects of drugs on neuromuscular junctions in living organisms [1]. In attempting to obtain a practically useful electronic chaos generator, particularly for the construction of stochastic resistors for such biomedical simulations, we have looked at a number of differential equation systems which give what is presently classified as chaos [2]. Probably the most well known and popular of these differential chaotic systems is that of Lorenz [3, p. 135]. But, although the Lorenz system has been simulated in *RC* op amp multiplier electronic form [4], it uses an unduly large number of components, especially the inconvenient multipliers. As one alternative we noticed that the nonlinear *R* linear *LC* circuit of Shinriki *et al.*, [5], called a modified Van der Pol oscillator, was observed to give "random" responses among the three modes of oscillation cited. By redoing the theory to be based upon *RC* active circuits, bent hysteresis [6], and a theory of Lienard systems incorporating bent hysteresis [7], we have been able to obtain a class of chaos generators which are quite simply realized in *RC* op amp form. Here we present the main circuit and typical experimental results with a more thorough design theory reserved for a later treatment.

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II. THE IDEA AND THE CIRCUIT

The basic idea is to begin with a second order Lienard system whose nonlinearity is bent hysteresis [7]. This Lienard system is then modified by increasing the degree to three through the introduction of first-order dynamics on an internal semistate variable of the bent hysteresis. The modification is so made that each cycle of the Lienard oscillator brings the hysteresis to a different point of its loop thus giving chaotic types of responses.

Fig. 1 shows the circuit in which the system of op amps O_1, O_2, and O_3 form a second-order Lienard system with bent hysteresis nonlinearity, the latter being formed in the feedback loop of op amps O_4, O_5, O_6, and O_3. For convenience the basic portion of the bent hysteresis used is identically that presented in [6, p. 481]. However, dynamics is inserted into the bent hysteresis via the capacitor C_0, which for convenience was chosen equal to C_1. Fig. 2(a) shows the bent hysteresis as measured in place, the input voltage being inserted at point e with lead $a-e$ opened and the output measured at point d with lead $b-c$ opened. Control on the slope of the bent hysteresis, which can be used to change the characteristics of the chaos, is through the variable resistors R_{01} and R_{02}; in essence these control the shape of three stable limit cycles of the Lienard system when C_0 is absent [7]. When C_0 is present the system has a unique trajectory in three-dimensional state space, but, when projected upon the two-dimensional x_1-x_2 plane, corresponding to a state space for the $C_0=0$ case, the system trajectories appear to jump between the above-mentioned limit cycles at random points. Fig. 2(b) shows this phenomena by presenting an x_1-x_2 plane oscilloscope trace of a trajectory. For comparison Fig. 2(a) and (b) are presented on the same scales (1 V/div) and, as seen, the trajectory in Fig. 2(b) fills portions of the plane. Any of the voltages measured at points $a-g$ of Fig. 1 will appear chaotic; Fig. 2(c) and (d) show two samples of x_2 measured at point b on a Tektronix 7834 storage scope used for all the measurements.

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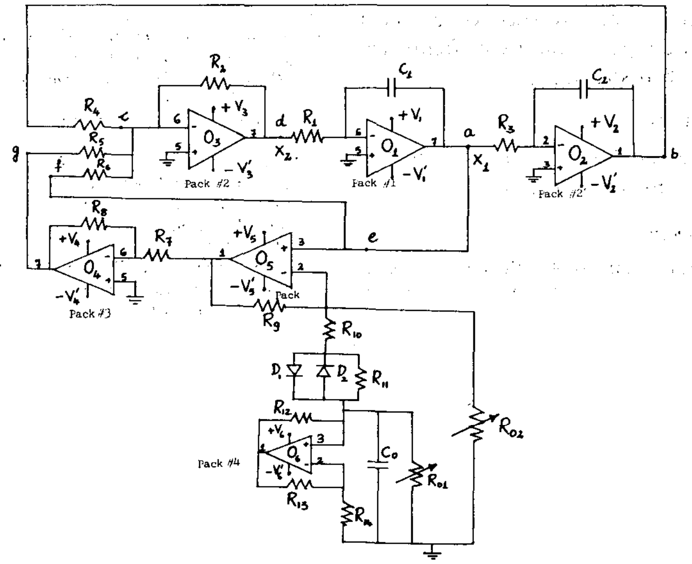
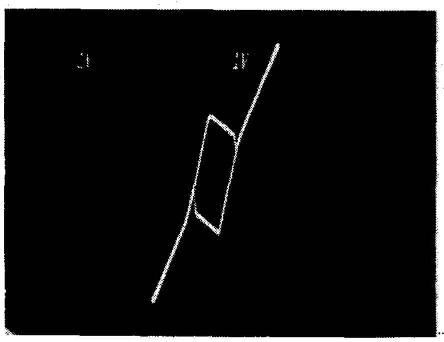


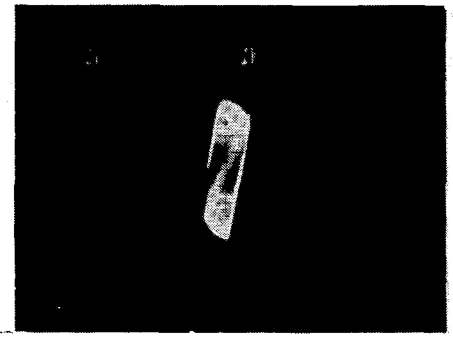
Fig. 1. Chaos generator circuit using op amps MC 1458, p-i-n numbers as shown:

$$\text{bias voltages} \begin{cases} +V_1 = +V_2 = +V_3 = +V_4 = +V_5 = 9\text{ V} \\ -V_1' = -V_2' = -V_3' = -V_4' = -V_5' = -9\text{ V} \\ +V_6 = 2{\cdot}5\text{ V},\ -V_6' = -4{\cdot}6\text{ V} \end{cases}$$

$R_1 = R_3 = R_7 = R_8 = 100$ kΩ; $R_2 = 330$ kΩ; $R_4 = 220$ kΩ; $R_5 = R_6 = 150$ kΩ; $R_9 = 20$ kΩ; $R_{10} = 3$ kΩ; $R_{11} = 56$ kΩ; $R_{12} = R_{13} = R_{14} = 10$ kΩ; $R_{01} =$ set at 22 kΩ; $R_{02} =$ set at 100 kΩ; $C_0 = C_1 = 0{\cdot}01$ μF; $C_2 = 0{\cdot}001$ μF; $D_1 = D_2 =$ 2N4123 n-p-n transistors connected as diodes.



(a)



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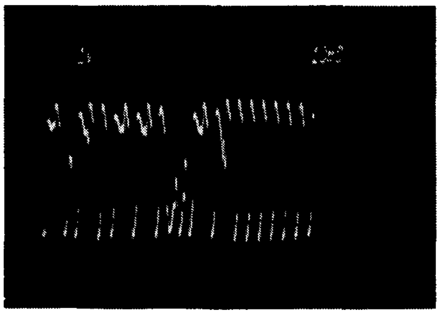
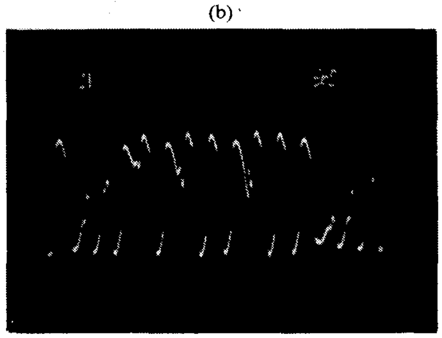


(c)



(d)

Fig. 2. Typical oscilloscope traces. Horizontal: 1 V/div. Vertical: 1 V/div. (a) Bent hysteresis used (measured at 10 Hz). (b) $x_1 - x_2$ plane trajectory. (c) and (d) Portions of x_2 signal.

Once the nature of operation of the system is realized various types of chaotic responses can be obtained by various changes in the design. For example, greater or lesser excursions between maximal maxima and minimal maxima can be obtained by displacing the limit cycles of the second-order Lienard system, this in turn being achieved by restructuring the bent hysteresis.

III. Conclusions

Chaotic signals with controllable properties can be practically realized by the *RC* active chaos generator that is herein presented, the idea stemming from observations of Shinriki *et al.* The system is structurally stable in the sense that small changes in element values do not destroy the chaos though the effects on the actual nature of the chaos need more study.

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Designing Active *RC* Biquads with Improved Performance

ROLF SCHAUMANN

Abstract—**It is shown that multiamplifier-active *RC* biquads of significantly better performance than that of known filters can be obtained by using composite amplifiers whose frequency dependence is tailored to the specific filter requirements rather than being designed for the usual criteria, such as maximum bandwidth or minimum phase shift.**

It is generally recognized that the best two-amplifier active *RC* biquads show a better performance with lower sensitivities than single amplifier biquads [1]. Specifically, the best two-amplifier resonator, the circuit based on a general impedance converter (GIC) [2], has passive Q-sensitivities of magnitude $\leqslant 1$, a gain–sensitivity product GS^Q of value $2Q$, and its deviations of pole-frequency ω_0 and quality factor Q due to the amplifiers' gain-bandwidth product ω_t are given by [3]

$$\frac{\Delta\omega_0}{\omega_0} \triangleq \delta_1 \simeq -\frac{H^2}{2(H-1)}\left(\frac{\omega_0}{\omega_t}\right)$$

$$\frac{Q}{Q_m} \simeq 1+\left(1-2Q\frac{2-H}{H}\right)\delta_1+2\left(3-\frac{4}{H}\right)Q\delta_1^2$$

where Q is the designed and Q_m the measured quality factor, and H is the gain constant. In comparison, the best single-amplifier biquads (SAB's), e.g., the Deliyannis–Friend circuit [4], have passive Q-sensitivities of the order of $2Q/r$ and a gain–sensitivity product of the approximate value (for $r>3$) rQ where r (a resistor ratio, Fig. 1) is a parameter to be chosen for an acceptable tradeoff between active and passive sensitivities ($r \simeq 6$) [5]. The deviations of ω_0 and Q due to finite ω_t are expressed as

$$\Delta\omega_0/\omega_0 \simeq -GS^Q(\omega_0/\omega_t)/(2Q)$$

$$Q/Q_m \simeq 1-GS^Q(\omega_0/\omega_t)(Q^{-1}-\omega_0/\omega_t).$$

Of course, both circuits, as do all good active *RC* filters, have passive ω_0-sensitivities at their theoretical minimum.

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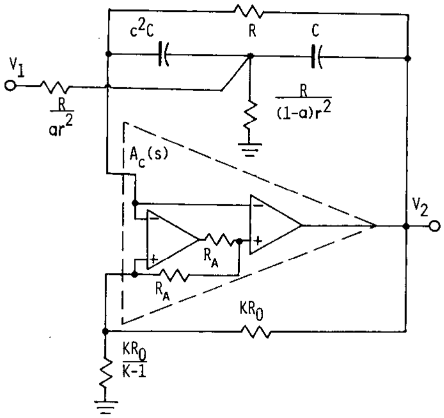

Fig. 1. Deliyannis–Friend SAB using a composite amplifier.

Comparing sensitivities and ω_0- and Q-deviations in the SAB and the GIC filters, one finds the latter circuit indeed to be better, notably in its passive Q-sensitivities, although the improvement is by no means striking. The question arises, therefore, whether the expense associated with the additional amplifier in the GIC-circuit cannot be used more advantageously in order to arrive at a "much better" two-amplifier filter.

The circuit in Fig. 1 suggests one possibility of designing a far superior two-amplifier biquad. The reader will recognize in Fig. 1 a normal Deliyannis–Friend resonator with the single op amp replaced by a "suitable" composite amplifier, in this example taken from those proposed in [6], specifically the unit C20A-4. Here, suitable means that the amplifier's frequency response is not optimized for criteria such as widest bandwidth [7], [8], but rather that it must be tailored to the specific requirements of the filter. For the "composite-amplifier biquad" (CAB) being discussed, the necessary model for $A_c(s)$ can be shown to be

$$A_c(s) = (n_2 s^2 + n_1 s + n_0)/s^2.$$

Then, the resulting CAB has passive ω_0- and Q-sensitivities of value 0.5 and $\simeq 2Q/r$, respectively, and $GS^Q \equiv A_c(0)S^Q_{A_c(0)} = rQ$, just as the regular SAB; however, the deviations due to finite ω_t are now

$$\Delta\omega_0/\omega_0 \simeq GS^Q(\omega_0/\omega_t)^2/Q^2$$

$$Q/Q_m \simeq 1-2GS^Q(\omega_0/\omega_t)^2$$

and, due to the reduced effect of GS^Q, r can be chosen to equal approximately 12 to 15 or larger, depending on Q. With these values of r, the passive sensitivities are of the same order as those of the GIC circuit, as are the deviations of Q due to ω_t, but as should be evident from the above equations, the more important ω_0-errors are significantly smaller (by about 2–3 orders of magnitude)! Table I illustrates the improvement for a bandpass with $Q=20$ and $f_0=10$ kHz, assuming $f_t=1.5$ MHz. The only price paid for the improvement is a larger element (resistor) spread.

The theory was verified by a number of experiments; representative measured results are given in Table II for a CAB, SAB, and GIC bandpass filter, built with 741-type dual opamps with the relatively small value $f_t=690$ kHz, and designed for (1) $f_0=8414$ Hz, $Q=31.8$, and (2) $f_0=18.02$ kHz, $Q=20$. Varying f_t (by changing amplifiers) in $560 \leqslant f_t/\text{kHz} \leqslant 1050$ produced in bandpass (1) *no* measurable change of f_0 in the CAB, but changes of 1.26 percent in the GIC and 3.60 percent in the SAB; simi-

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