THE SEMISTATE DESCRIPTION FOR CASCADE AND FEEDBACK CONNECTIONS

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ABSTRACT

The canonical semistate equations are obtained for cascade and feedback interconnections of nonlinear, time-varying systems. It is shown that coupling always enters in a linear, constant, off-diagonal manner.

INTRODUCTION

Previously we have introduced the semistate [1] and shown that it leads to useful characterization for nonlinear and time-variable systems [2]. In particular the canonical form

$$G\dot{x} + \mathcal{B}(x,t) = \mathcal{A}u \quad , \quad x=\text{semistate} \qquad (1a)$$

$$y = \mathcal{C}x \quad , \quad u=\text{input}, y=\text{output} \qquad (1b)$$

has been introduced where $G, \mathcal{A}$, and $\mathcal{C}$ are constant linear operators which are generally singular. By the use of some equivalences and the choice of the semistate as tree branch voltages and link branch currents it has been shown [2] that this canonical form holds for almost all electrical networks with $\mathcal{B}(.,.)$ a single valued function (even when the system exhibits hysteresis [3]). Similar results undoubtedly hold for other classes of systems, especially where graphs can be introduced [4]. Since large scale systems are most often to be thought of as interconnections of subsystems [5], it is of interest to obtain the canonical semistate equations for various interconnections. Here we do this for the cascade and feedback interconnections. Since in the network case all connections can be considered special cases of the cascade one [6, pp. 63-64], the results really hold for all network connections such as the series, parallel, and hybrid ones, while extensions to other connections, as the canonical subsystem measurement structure [7, p. 41], should be possible.

*The material of the second author is based in part upon work supported by the US National Science Foundation under Grant ECS-81-05507.

THE CASCADE CONNECTION

We consider as given two uncoupled systems described in the canonical form of equations (1) and indexed by subscripts $i=1,2$. The <u>cascade systems connection</u> is defined by

$$u=u_1, \; y_1=u_2, \; y=y_2 \qquad (2a)$$

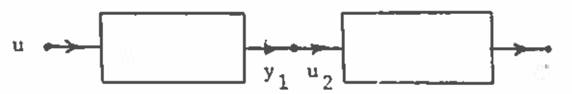

Figure 1
Cascade Systems Connection

Since the systems are otherwise uncoupled, we can naturally define the system semistate as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (2b)$$

from which we can recover x_1 and x_2 through projection operators P_1 and P_2

$$x_1=P_1x=[I_1,0]x \quad , \quad x_2=[0,I_2]x \qquad (3)$$

with I_i appropriate sized identities. Thus, the system descriptions can be rewritten as

$$G_1\dot{x}_1 + \mathcal{B}_1(x_1,t) = \mathcal{A}_1u_1 \qquad (4a)$$

$$= G_1P_1\dot{x} + \mathcal{B}_1(P_1x,t) = \mathcal{A}_1u \qquad (4b)$$

$$y_1=u_2=\mathcal{C}_1x_1 = \mathcal{C}_1P_1x \qquad (4c)$$

$$G_2P_2\dot{x} + \mathcal{B}_2(P_2x,t) = \mathcal{A}_2u_2 \qquad (4d)$$

$$= \mathcal{A}_2\mathcal{C}_1P_1x \qquad (4e)$$

$$y=y_2=\mathcal{C}_2P_2x \qquad (4f)$$

or

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}\dot{x} + \begin{bmatrix} 0 & 0 \\ -\mathcal{A}_2\mathcal{C}_1 & 0 \end{bmatrix}x + \begin{bmatrix} \mathcal{B}_1(P_1x,t) \\ \mathcal{B}_2(P_2x,t) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_1 \\ 0 \end{bmatrix}u \qquad (5a)$$

$$y = [0, \mathcal{C}_2]\, x \qquad (5b)$$

Equations (5) are canonical semistate equations for the cascade connection. They show the very interesting result that, even for nonlinear time-variable systems, the coupling is linear and constant and occurs "off diagonal" [that is, in the (2,1) submatrix position of the linear multiplier of x].

It should be commented that for cascade connections of electrical circuits the currents in y_1 may need to be multiplied by minus one to achieve $y_1=u$, as is recognized in chain matrix theory.

THE FEEDBACK CONNECTION

The feedback systems connection is illustrated in Fig. 2 and defined by

$$u_1=u+y_2\ ,\quad y=y_1=u_2 \qquad (6)$$

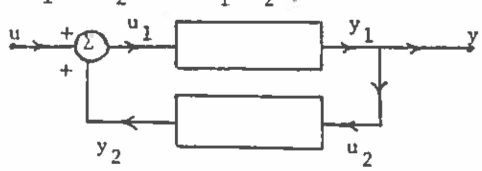

Figure 2
Feedback Systems Connection

Again assuming the subsystems are uncoupled prior to interconnection, we can define the system semistate as for the cascade connection, that is, via (2b). Then we have, again using the projection operators of (3),

$$\mathcal{L}_1 P_1 \dot{x} + \mathcal{A}_1(P_1x,t) = \mathcal{B}_1(y_2+u) \qquad (7a)$$

$$= \mathcal{B}_1\mathcal{C}_2 P_2 x + \mathcal{B}_1 u \qquad (7b)$$

$$y=y_1 = \mathcal{C}_1 P_1 x \qquad (7c)$$

$$\mathcal{L}_2 P_2 \dot{x} + \mathcal{A}_2(P_2x,t) = \mathcal{B}_2 u_2 \qquad (7d)$$

$$= \mathcal{B}_2\mathcal{C}_1 P_1 x \qquad (7e)$$

Thus, canonical semistate equations are

$$\begin{bmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_2 \end{bmatrix} \dot{x} + \begin{bmatrix} 0 & -\mathcal{B}_1\mathcal{C}_2 \\ -\mathcal{B}_2\mathcal{C}_1 & 0 \end{bmatrix} x + \begin{bmatrix} \mathcal{A}_1(P_1x,t) \\ \mathcal{A}_2(P_2x,t) \end{bmatrix} = \begin{bmatrix} \mathcal{B}_1 \\ 0 \end{bmatrix} u \qquad (8a)$$

$$y = [\mathcal{C}_1, 0]x \qquad (8b)$$

Again the coupling enters only linearly (and in time-invariant) form in the new $\mathcal{A}(.,.)$

$$\mathcal{A}(x,t) = \begin{bmatrix} 0 & -\mathcal{B}_1\mathcal{C}_2 \\ -\mathcal{B}_2\mathcal{C}_1 & 0 \end{bmatrix} x + \begin{bmatrix} \mathcal{A}_1(P_1x,t) \\ \mathcal{A}_2(P_2x,t) \end{bmatrix} \qquad (9)$$

The coupling is still "off diagonal" but is in the feedback, (1,2), terms as well as in the feedforward, (2,1), terms, as is to be expected.

DISCUSSION

Semistate equations offer some advantages as system descriptions due to the nature of their canonical form. Among these advantages are that multivalued systems operators, for example, hysteresis, can be characterized by single-valued ones. As shown here, they also allow important interconnections to be described through linear constant coupling terms. For the computer aided analysis of large scale nonlinear systems this should prove of importance.

REFERENCES

[1]. B. Dziurla and R. Newcomb, "The Drazin Inverse and Semi-State Equations," Proceedings of the International Symposium on Mathematical Theory of Networks and Systems, Delft, July 1979, pp. 283-289.

[2]. R.W. Newcomb, "The Semistate Description of Nonlinear Time-Variable Circuits", IEEE Transactions on Circuits and Systems, Vol. CAS-28, No. 1, January 1981, pp. 62-71.

[3]. G. Kiruthi, H. Yazdani, and R.W. Newcomb, "A Hysteresis Circuit Seen Through Semi-State Equations", Proceedings of the 23rd Midwest Circuit Theory Symposium, Toledo, August 1980, pp. 279-282.

[4]. F.T. Boesch, Editor, "Large-Scale Networks: Theory and Design," IEEE Press, New York, 1976.

[5]. D.D. Siljak, "Large-Scale Dynamic Systems, Stability and Structure," North Holland, New York, 1978.

[6]. R.W. Newcomb, "Linear Multiport Synthesis", McGraw-Hill Book Co., New York, 1966.

[7]. E.C. Tacker and E.W. Sanders, "Decentralized Structure for State Estimation in Large Scale Systems," Large Scale Systems Theory and Applications, Vol. 1, No.1, February 1980, pp. 39-49.

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1982 IEEE INTERNATIONAL LARGE SCALE SYSTEMS SYMPOSIUM

The Cavalier Resort

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82CH1741-8