STATE VARIABLE EQUATIONS FOR A NONLINEAR DELAY CIRCUIT MODEL OF IMPATT DIODES

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SUMMARY

I. INTRODUCTION

IMPATT is an acronym for IMPact Ionization Avalanche Transit Time. Using these two properties, IMPATT diodes produce negative resistance at millimeter wave frequencies. In the following sections, we present the state variable equations for a first-order nonlinear circuit model of IMPATT diodes under assumptions made in the literature. This allows one to perform a large signal analysis of the device using a digital computer. For convenience we attach a list of symbols in the appendix.

II. A FIRST-ORDER NONLINEAR CIRCUIT MODEL FOR IMPATT DIODES

A first-order nonlinear circuit model for IMPATT diodes given by Gannett and Chua [1] is shown in Fig. 1 (see end of paper). To avoid duplication, the description of assumptions made concerning the diode and the derivation of the circuit model will be omitted here. However we mention that the equivalent circuit of Fig. 1 uses the primary assumptions of (i) equal ionization rates for holes and electrons (ii) negligible carrier diffusion effects and (iii) an assumed p-i-n doping profile in the avalanche region. The circuit model consists of an avalanche region modeled using a nonlinear inductor, a nonlinear resistor, and a nonlinear voltage source, these accounting for the nonlinearities in the physical mechanism taking place within the device. The dynamic current source I_e models the transit time delay as the injected carriers pass through the low-field drift region. C_a and C_d are simply the geometrical capacitances of the avalanche and drift region respectively.

III. STATE VARIABLE EQUATIONS

In this section, we shall give the state variable equations for the first-order nonlinear circuit model of Fig. 1. This allows one to perform a time domain analysis of IMPATT diode circuits on a digital computer. The input is chosen as a terminally applied current I and the output is taken as the voltage V across the input terminals. We choose (I_a, V_a, I_e) as the state variables and (I_o, V_o) as the bias parameters. From Fig. 1 we have for the three dynamic equations

$$\frac{dI_e}{dt} = \frac{1}{\tau d} \left(I_a(t) - I_a(t-\tau_d) \right) \tag{1}$$

$$\frac{dV_a}{dt} = \frac{1}{Ca} \left(I - I_o - I_a \right) \tag{2}$$

$$\frac{dI_a}{dt} = \frac{\alpha'(E_c)}{k_e \tau_e} \left(V_a I_a + I_o V_a \right) - \frac{\alpha''(E_c)}{2Wk_e \tau_e} \left(I_a V_a^2 + I_o V_a^2 \right) - \frac{I_s}{k_e \tau_e} \frac{I_a}{I_o} \tag{3}$$

and for the output equation

$$V = V_o + V_a + \tau_d \frac{I_e}{C_1} \qquad (4)$$

Equations (1), (2), (3) and (4) represent the state variable equations for the first-order non-linear circuit model of Fig. 1. Since (1) has a delay term and (3) is nonlinear in I_a and V_a, the state variable equations are clearly nonlinear delay-differential equations.

IV. COMPUTER-AIDED DESIGN (CAD) PROGRAM

The Euler method has been employed to solve this set of state variable equations on a digital computer. Of course, the success of any computer simulation program depends on the choice of model parameters. In carrying out an example we have used the following model parameters calculated by Gannett and Chua [1, p. 305] for a GaAs Read diode as described by Schroeder and Haddad [2, p. 175, Fig. 26, diode 3]

$C_a = 46.0\ \mathrm{pF/cm^2}$

$C_d = 3.46\ \mathrm{pF/cm^2}$

$I_o = 500\ \mathrm{A/cm^2}$

$I_s = 10^{-4}\ \mathrm{A/cm^2}$

$W = 0.21\ \mu\mathrm{m}$

$W_d = 2.79\ \mu\mathrm{m}$

$V_o = 56\mathrm{V}$

$\tau_d = 3.49 \cdot 10^{-11}\mathrm{s}$

$k_e \tau_e / \alpha'(E_c) = 2.94.10^{-12}\ \mathrm{V.s}$

$I_s / \alpha'(E_c) = 3.36.10^{-4}\ \mathrm{A.V}$

$\alpha''(E_c)/(2W\alpha'(E_c)) = -\ 6.73.10^{-3}\mathrm{V}^{-1}$

The input I was chosen as a sinusoidal current source of frequency 5 GHz.

V. DISCUSSIONS AND CONCLUSIONS

The state space equations of IMPATT diodes have been presented. These allow for circuit analysis of nonlinear circuits, which include IMPATT diodes. Advanced numerical techniques can be employed to solve these equations to perform a time domain analysis on a digital computer. But, since the differential equations are really delay-differential equations [3] for which the delay dependence is expected to be more important when the restrictive assumptions are relaxed, techniques and properties of the more general delay-differential equations

$$\dot{x}(t) = f(x(t), x(t-\tau)) \qquad (5)$$

are under investigation. The computer results can be successfully used to design IMPATT diodes for optimum device performance. The technique also presents an efficient method for extracting model parameters for devices with specific geometries. However, we feel that the important contribution of the technique described here will be in the analysis and design of systems incorporating IMPATT devices.

REFERENCES

[1] J.W. Gannett and L.O. Chua, "A nonlinear circuit model for IMPATT diodes," IEEE Trans. Circuits and Systems, Vol. CAS-25, pp. 299-308, May 1978.

[2] W.E. Schroeder and G.I. Haddad, "Nonlinear properties of IMPATT devices," Proc. IEEE, Vol. 61, pp. 153-182, Feb. 1973.

[3] L.E. El'sgol'ts, "Introduction to the Theory of Differential Equations with Deviating Arguments," Holden-Day, San Francisco, 1966.

APPENDIX-NOMENCLATURE

A	Cross-sectional area
E_c	Static electric field in the p-i-n avalanche region
I	Terminal current
I_A	Spatially averaged particle current in the avalanche region
I_a	$\triangleq I_A - I_o$ (see I_o below)
I_o	Terminal current under static conditions
I_{ns}	Electron current incident on the avalanche region
I_{ps}	Hole current incident on the avalanche region
I_s	$\triangleq I_{ns} + I_{ps}$
v	Carrier velocity
V	Terminal voltage
V_a	Variation of the avalanche region voltage from its static value
V_d	Variation of the drift region voltage from its static value
V_o	Terminal voltage under static conditions
W	Width of p-i-n avalanche region
W_d	Width of the drift region
α	Ionization coefficient
α'	$\triangleq d\alpha(E)/dE$
α	$\triangleq d^2\alpha(E)/dE^2$
E	Dielectric permittivity of the semiconductor

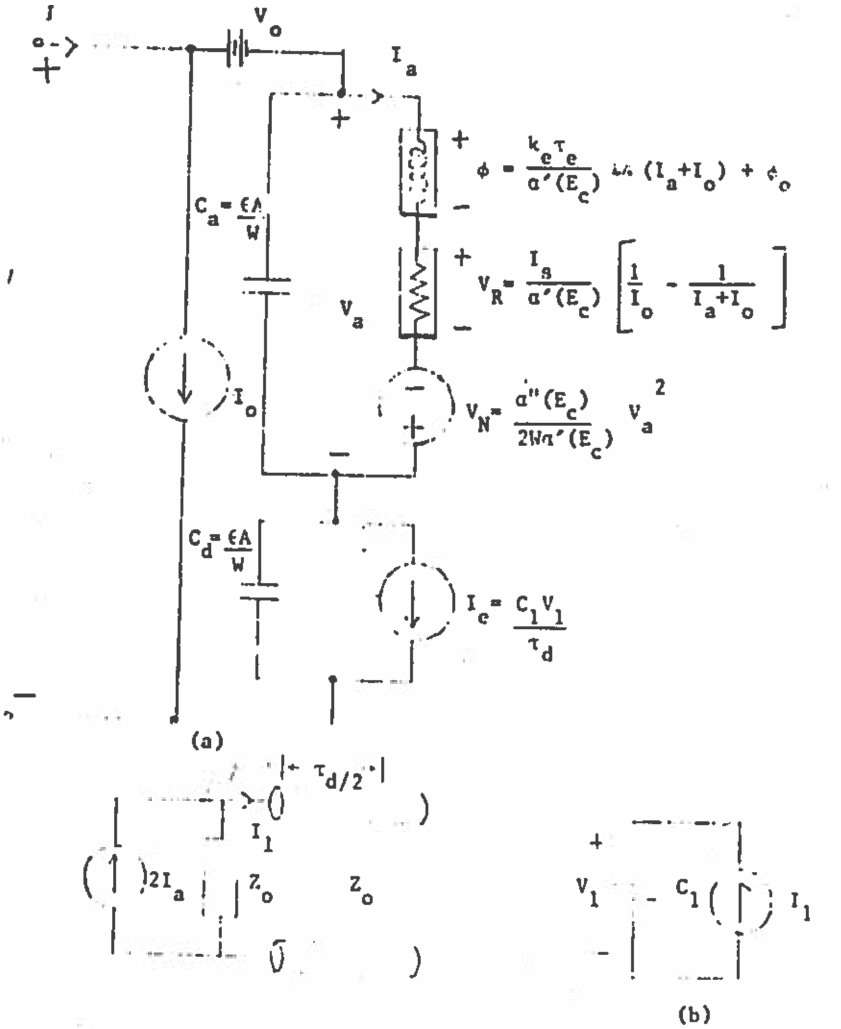

Fig. 1.(a)A first-order nonlinear circuit model for an SDR (single drift region) IMPATT diode $k_e\tau_e = W(v_p+v_n)/(6v_pv_n)$ and $\tau_d = W_d/v$, where v is the velocity of the majority carriers in the drift region. (b) An open-circuited transmission line of impedance Z_o and delay $\tau_{d/2}$ used to model I_e. The impedance Z_o and the capacitance C_1 can be any convenient values.

INTERNATIONAL SYMPOSIUM ON MICROWAVES AND COMMUNICATION

29-30, DECEMBER, 1981

PROCEEDINGS

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