# Singular, Op-Amp, and Nonunique Adjoint Circuits

R. Mukunda*, R. Rakshit+ and R. Newcomb*+

*Applied Mathematics Program
+Electrical Engineering Dèpartment
University of Maryland
College Park, Maryland 20742 USA

Abstract:

Adjoint circuits have proven extremely valuable in the computer aided evaluation of derivatives, especially for sensitivity and optimization of active circuits. Among the active circuits those incorporating op-amps are among the most valuable. But in the literature the adjoint of op-amp circuits does not seem to appear. Here we present the main result, the adjoint of an op-amp, which is another op-amp but reversed. Since the op-amp can be represented by singular circuit elements, the adjoints of norators and nullators are presented. Finally the singular sources of Oono are used to show that the adjoint circuit is not necessarily unique.

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## I. Introduction

Since the introduction of the concept of interreciprocity applied to electrical network by Bordewijk [1], the adjoint circuits have proven extremely valuable in the computer aided design [2] and network sensitivity [3][4]. A formulation of the adjoint network $N^a$ has also been developed in terms of the definition [5,p.7] of an n-port network N through its allowed pairs of voltage and current, $[v,i]\in N$ [6].

For a given n-port network N defined through $[v,i]\in N$, the adjoint network $N^a$ is defined [6] by $[v^a,i^a]\in N^a$ if and only if for all $[v,i]\in N$ and all t and $\tau$

$$[\tilde{v}(t),\tilde{i}(t)]\begin{bmatrix} i^a(\tau) \\ -v^a(\tau) \end{bmatrix}=0 \tag{1}$$

where ~denotes the transpose.

For time-invariant networks, in the frequency domain this can be written as

$$[\tilde{V}(p),\tilde{I}(p)]\begin{bmatrix} I^a(p) \\ -V^a(p) \end{bmatrix}=0 \tag{2}$$

If N is linear and characterized in the frequency domain by the general description

$$AV=BI \tag{3}$$

we can characterize $N^a$ by

$$[\tilde{V},\tilde{I}]\begin{bmatrix} \tilde{A} \\ -\tilde{B} \end{bmatrix} X=0 \tag{4}$$

where X is an arbitrary n-vector [6].

Comparing with equation (2)

$$I^a = \tilde{A}X$$
$$V^a = \tilde{B}X \tag{5}$$

## II. General description of the adjoint network.

By finding a common left multiple M of $\tilde{A}$ and $\tilde{B}$ for equation (5) following [7,pp.35-36] we have

$$[B^a, A^a] \begin{bmatrix} \tilde{A} \\ -\tilde{B} \end{bmatrix} = 0_{nxn} \tag{6a}$$

or $$M = B^a \tilde{A} = A^a \tilde{B} \tag{6b}$$

where $[B^a, A^a]$ has maximum possible rank n. Hence equation (5) gives

$$B^a I^a = B^a \tilde{A}X = MX = A^a \tilde{B}X = A^a V^a \tag{7}$$

which completely describes the adjoint network in view of the rank n condition satisfied by $[B^a, A^a]$.

## III. Adjoint Op-amp circuit.

An ideal op-amp, Fig. 1a), is the 2-port described by

$$AV = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = BI \tag{8}$$

To obtain the adjoint we can form using row and column operations

$$\begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} \tilde{A} & 0_n \\ -\tilde{B} & 0_n \end{bmatrix} = \begin{bmatrix} D & 0_n \\ 0_n & 0_n \end{bmatrix} \tag{9a}$$

with the W matrix non-singular from which we have

$$W_{21} \tilde{A} = W_{22} \tilde{B} = B^a \tilde{A} = A^a \tilde{B} \tag{9b}$$

$$\therefore \quad B^a = W_{21} \qquad A^a = W_{22} \tag{9c}$$

In the op-amp case

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10a}$$

$$\therefore \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = W_{21} = B^a \quad W_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = A^a \tag{10b}$$

Thus, for the adjoint op-amp network $N^a$ we have

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_1^a \\ V_2^a \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_1^a \\ I_2^a \end{bmatrix} \tag{11}$$

Thus, the adjoint of an op-amp has $V_2^a = I_2^a \equiv 0$ which is another op-amp but with reversed ports, as in Fig. 1b).

IV. Nullator-Norator Adjoints.

The nullator is a one-port described by [5,p.13].

$$v \equiv i \equiv 0 \tag{12}$$

Equation (2) shows that the adjoint of a nullator is the one-port described by

$$v^a = \text{arbitrary}, \qquad i^a = \text{arbitrary} \tag{13}$$

Such a one-port is the norator [5,p.13]. Thus these two "singular" one-ports are adjoints of each other. Since the op-amp can be considered as a nullator at the input and a norator at the output (sometimes called a nullor [5,p.20]), we have another check on the op-amp adjoint. Along the same lines the norator being the adjoint of the nullator can be checked by finding the adjoint of the circuit realization of the nullator, Fig. 2a) [5,p.13], in which the gyrator gets reversed to yield the adjoint (following standard rules [6]) which is then the circuit realization of the norator of Fig. 2b).

## V. Nonunique Adjoint Circuits

Oono [8,p.484] has given a one-port defined as

$$v=E=\text{constant}, \quad i=J=\text{constant} \tag{14}$$

Figure 3a) shows the circuit, which we may call a singular source. Equation (2) gives for the adjoint, if $J\neq 0$,

$$E\, I^a - JV^a = 0 \text{ or } V^a = \frac{E}{J} I^a \tag{15}$$

Thus, the adjoint of the singular source is a resistor of resistance $R^a=E/J$. Now a resistor of resistance R has as an adjoint another resistor of resistance R. Consequently, we see that a resistor has at least two possible adjoints, one being a singular source and the other being another resistor. By the same reasoning, the adjoint of the adjoint need not be the original circuit (for the singular source adjoint is a resistor, so the adjoint of the adjoint of the singular source can be a resistor rather than the original singular source).

It should be noted that eq. (15) also describes singular sources, one being

$$V^a=kE, \quad I^a=kJ, \quad k=\text{arbitrary constant} \tag{16}$$

Figure 3b) shows two possible adjoints of the singular source.

## VI. Conclusions-Discussion

Here we have given the adjoint of the op-amp, this being practically useful for CAD of active circuits. Since the op-amp can be considered as a realization of the nullor the study has led us to obtain the adjoints of singular circuit elements. In so doing we have seen that when singular sources are allowed the adjoint will not be unique, as even the resistor has multiple adjoints (uncountably many in fact). However, although independent sources must be present to drive circuits, independent (and singular) sources are nonlinear and hence the nonuniqueness we have noted here has really occurred due to nonlinearities. Thus it seems worth a consideration of adjoint circuits for nonlinear networks.

## VII. References


[1] J.L. Bordewijk, "Interreciprocity Applied to Electrical Network," Applied Science Research, V. B6, 1957, pp. 1-74.

[2] S.W. Director and R.A. Rohrer, "On the Design of Resistance n-Port Networks by Digital Computer", IEEE Transactions on Circuit Theory, Vol. CT-16, No. 3, August 1969, pp. 337-346.

[3] S.W. Director and R.A. Rohrer, "The Generalized Adjoint Network and Network Sensitivity," IEEE Transactions on Circuit Theory, Vol. CT-16, No. 3, August 1969, pp. 318-323.

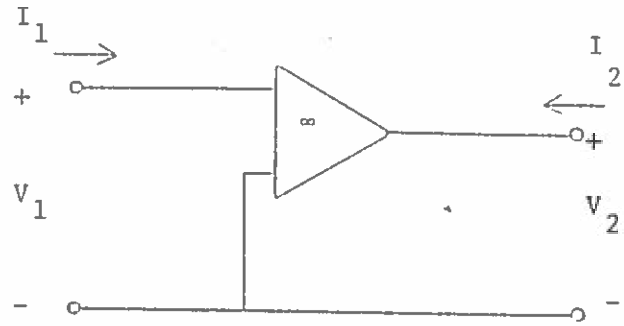
[4] G.C. Temes and J.W. LaPatra, "Introduction to Circuit Synthesis and Design," McGraw-Hill Book Co., New York, 1977; see pages 379-458.

[5] R.W. Newcomb, "Linear Multiport Synthesis," McGraw-Hill Book Co., New York, 1966.

[6] R.W. Newcomb and Q.M. On, "Time-Invariant Adjoint Networks: Generalized Description Formulation", Proceedings of the 1978 IEEE International Symposium on Circuits and Systems, New York, May 17-19, 1978, pp. 432-433.

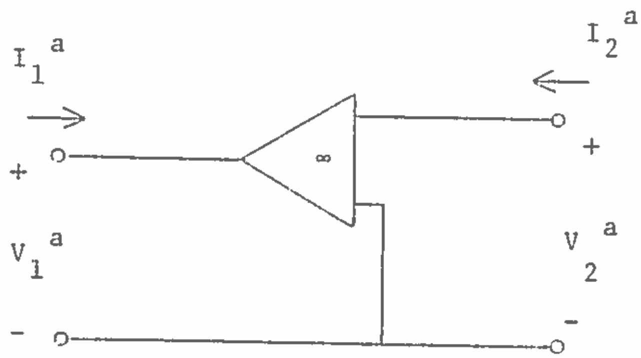
[7] C.C. McDuffee, "The Theory of Matrices", Chelsea Publishing Co., New York, 1956.

[8] Y. Oono, "Formal Realizability of Linear Networks", Proceedings of the Symposium on Active Networks and Feedback Systems, Vol. 10, New York, April 19-21, 1960, Polytechnic (Institute of Brooklyn) Press, pp. 475-486.




$V_1 = I_1 \equiv 0$

a)
op-amp

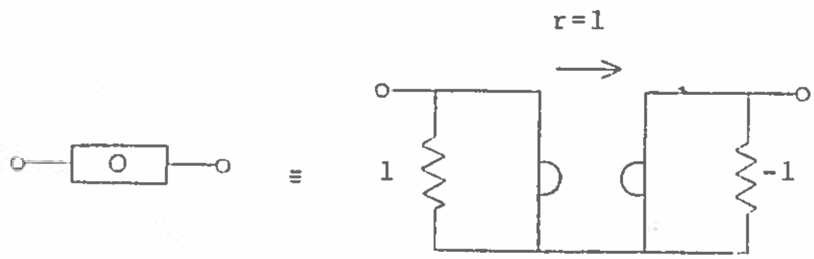



$V_2^a = I_2^a = 0$

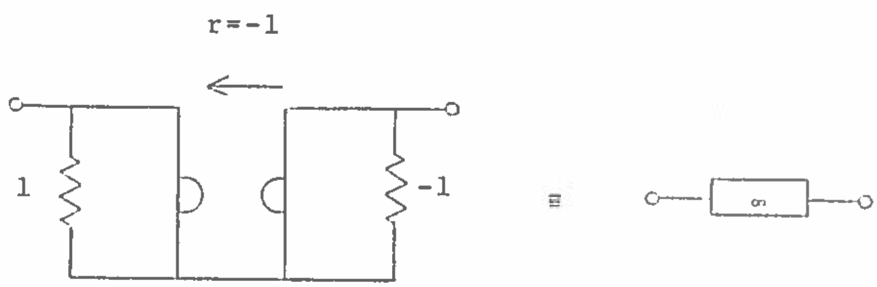
b)
Adjoint of op-amp

Figure 1

Op-Amp and Its Adjoint



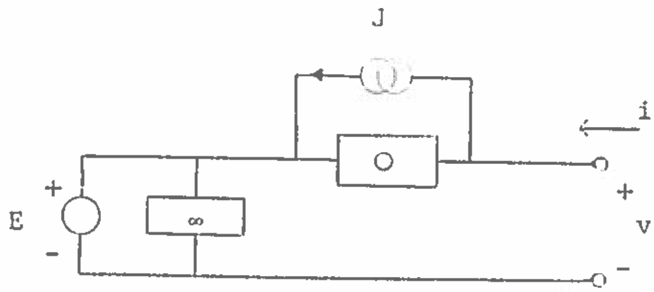

a)
Nullator Circuit



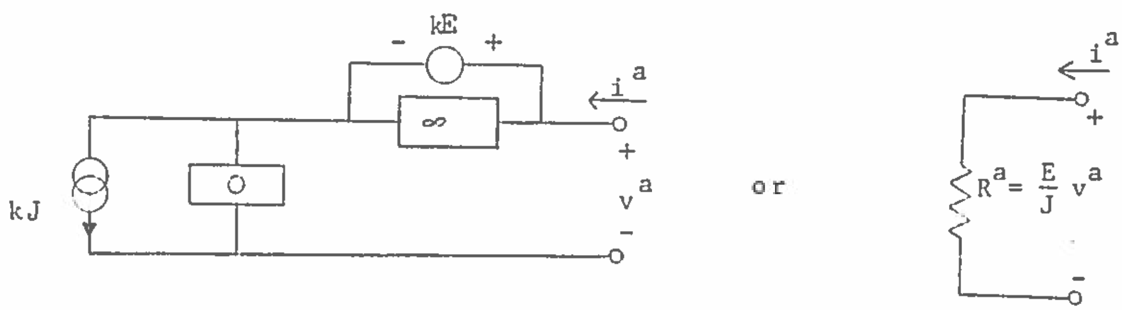

b)
Adjoint Circuit of Nullator=Norator

Figure 2

"Singular" Circuit Adjoints




a)
Singular Source




b)
Two Singular Source Adjoints

Figure 3

Circuits with Nonunique Adjoints

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