LINEARIZING TRANSFORMATIONS ON QUADRATIC SYSTEMS

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ABSTRACT

In this abstract we summarize the ideas which will be more fully developed in the presentation and later prepared for publication.

Previously it has been shown [1] that all unforced polynomic systems can be described within a nonassociative algebra by the canonic quadratic system $\dot{x}=x.x$. Indeed many reasonable, even forced systems can be approximated by such a quadratic system. Adapting some ideas of Takata [2] we here outline the technique to show that this canonical system can be further transformed into a linear system. This can be done by using n-dimensional Hermite polynomials [3] to transform the quadratic equation $\dot{x}=x.x$ of an n-dimensional nonassociative algebra into the linear equation $\dot{h}=a.h$ in a new but possibly infinite dimensional nonassociative algebra. In so doing we give an indexing scheme [4] which makes it possible to write the equation in this form $\dot{h}=a.h$. For example the system $\dot{x}_1=(x_2)^2$, $\dot{x}_2=-x_1x_2$ is written as $\dot{x}=x.x$ in the algebra where basis functions u_1 and u_2 satisfy

$$\begin{bmatrix} u_1u_1 & u_1u_2 \\ u_2u_1 & u_2u_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2}u_2 \\ -\frac{1}{2}u_2 & u_1 \end{bmatrix}$$

and then an expansion in 2-dimensional Hermite polynomials yields the infinite set.

$$\dot{H}_{m_1,m_2}(x)=m_1(m_2+1)H_{m_1-1,m_2}(x)$$
$$-m_2(m_2-1)H_{m_1+1,m_2-2}(x)+m_1H_{m_1-1,m_2+2}(x)$$
$$-m_2H_{m_1+1,m_2}(x)$$

where $H_{m_im_j}=H_{m_i}(x_i)H_{m_j}(x_j)$

The equation $\dot{h}=ah$ is obtained by using the indexing scheme $H_{m_1m_2}\rightarrow h_k$ with $k=\frac{1}{2}[(m_1+m_2)^2+3m_1+m_2]$.

This new system is completely equivalent to the original quadratic system and with this indexing allows the new system to be rewritten in a new nonassociative algebra as $\dot{h}=a.h$ thus linearizing the original quadratic differential equation.

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24th MIDWEST SYMPOSIUM ON CIRCUITS AND SYSTEMS

June 29-30, 1981

Edited by
S. Karni

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