

LINEARIZING TRANSFORMATIONS ON QUADRATIC SYSTEMS

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ABSTRACT

In this abstract we summarize the ideas which will be more fully developed in the presentation and later prepared for publication.

Previously it has been shown [1] that all unforced polynomial systems can be described within a nonassociative algebra by the canonic quadratic system  $\dot{x}=x.x$ . Indeed many reasonable, even forced systems can be approximated by such a quadratic system.

Adapting some ideas of Takata [2] we here outline the technique to show that this canonical system can be further transformed into a linear system. This can be done by using n-dimensional Hermite polynomials [3] to transform the quadratic equation  $\dot{x}=x.x$  of an n-dimensional nonassociative algebra into the linear equation  $\dot{h}=a.h$  in a new but possibly infinite dimensional nonassociative algebra. In so doing we give an indexing scheme [4] which makes it possible to write the equation in this form  $\dot{h}=a.h$ . For example the system

$\dot{x}_1=(x_2)^2$ ,  $\dot{x}_2=-x_1x_2$  is written as  $\dot{x}=x.x$  in the algebra where basis functions  $u_1$  and  $u_2$  satisfy

$$\begin{bmatrix} u_1u_1 & u_1u_2 \\ u_2u_1 & u_2u_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2}u_2 \\ -\frac{1}{2}u_2 & u_1 \end{bmatrix}$$

and then an expansion in 2-dimensional Hermite polynomials yields the infinite set.

$$\begin{aligned} \dot{H}_{m_1, m_2}(x) &= m_1(m_2+1)H_{m_1-1, m_2}(x) \\ &- m_2(m_2-1)H_{m_1+1, m_2-2}(x) + m_1H_{m_1-1, m_2+2}(x) \\ &- m_2H_{m_1+1, m_2}(x) \end{aligned}$$

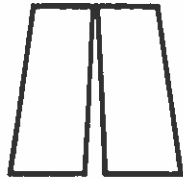
where  $H_{m_i m_j} = H_{m_i}(x_i)H_{m_j}(x_j)$

The equation  $\dot{h}=ah$  is obtained by using the indexing scheme  $H_{m_1 m_2} \rightarrow h_k$  with  $k = \frac{1}{2}[(m_1+m_2)^2 + 3m_1+m_2]$ .

This new system is completely equivalent to the original quadratic system and with this indexing allows the new system to be rewritten in a new nonassociative algebra as  $\dot{h}=a.h$  thus linearizing the original quadratic differential equation.

#### REFERENCES

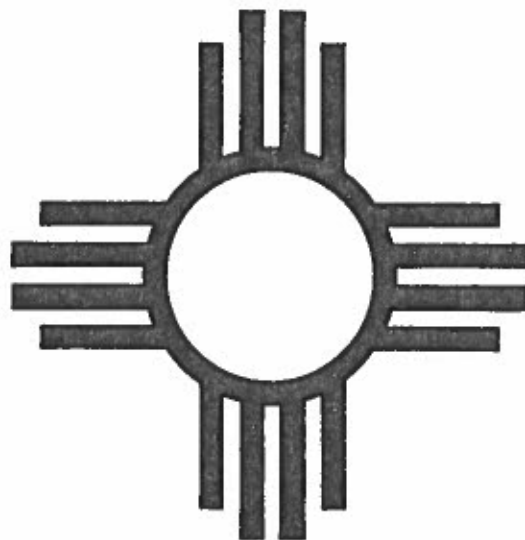
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