SIMULATION OF SURFACE ACOUSTIC WAVE RESONATOR

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ABSTRACT

The phenomenon of surface acoustic waves (SAW) generated on a piezoelectric substrate has proven useful in the design of various types of filters and delay lines. A preliminary investigation of the modeling of complex acoustic resonant structures is presented. The components of the resonant structures under investigation include several transducer sections and reflective gratings. As a first step, Mason equivalent circuits are adopted toward modeling of these devices. The results of a simulation of a simple SAW resonator are presented to demonstrate the feasibility of the Mason modeling approach.

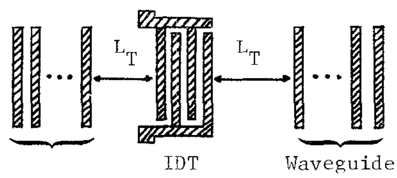

Figure 1b. SAW Resonator Detail of Construction

I. Introduction

In the past few years, there has been considerable interest in the surface acoustic wave (SAW) resonators. The SAW resonator is a relatively new device which can be used for frequency control and filter synthesis in the VHF and UHF frequency ranges. The model of SAW resonator used in the present paper consists of two gratings, which act as distributed reflectors to form an acoustic cavity. An interdigital transducer placed between the reflectors is used to couple energy into and out of the cavity. Ash [1] proposed the use of the gratings for the design of resonators, which were later studied by several authors [2-6].

A SAW resonator is essentially a cavity where an oscillating wave is reflected upon itself many times. Once the cavity is formed it is coupled to a transmitting and receiving electrical network. Surface acoustic wave reflection gratings have the special property that they are efficient and, at the same time, frequency selective. This latter feature allows such devices to operate in the 30 - 3000 MHz.

The model of a typical SAW resonator is shown in Figure 1. When a RF voltage is applied to an interdigital transducer (IDT),

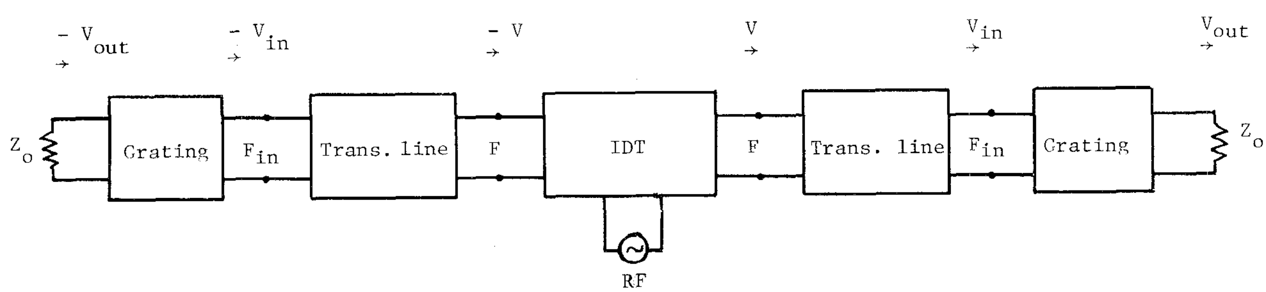

Figure 1a. Model of the SAW Resonator

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the surface acoustic waves are generated and propagate away from the IDT in both directions going to the grating reflectors; this sets up a large standing wave within the cavity.

A matrix representation of the IDT and the gratings will be employed in this paper. The matrices representing the grating along with a section of acoustic transmission line are cascaded with the two acoustic ports of a three-port representation for the IDT. From this, the electrical admittance of the resonator is obtained. The mathematical formulation is discussed in Section II. Finally, the electrical admittance and the phase of the SAW resonator are plotted as a function of frequency.

II. MATHEMATICAL FORMULATION

A. Transmission Matrix of an Interdigital Transducer (IDT)

The use of so-called "crossed-field" and "in-line" models for IDT's have been firmly established [7]. These models have the advantage of ease of implementation on a digital computer and were found to predict transducer performance accurately in a variety of applications. In the present analysis, the IDT will be characterized by the crossed-field model, because of its simpler structure.

The Mason equivalent circuit for a crossed-field model for an active section of a piezoelectric material is shown in Figure 2 [8]. Where

$$\theta_o = \frac{L_o}{V_o} \cdot \omega ,$$

is the acoustic transit angle of the substrate.

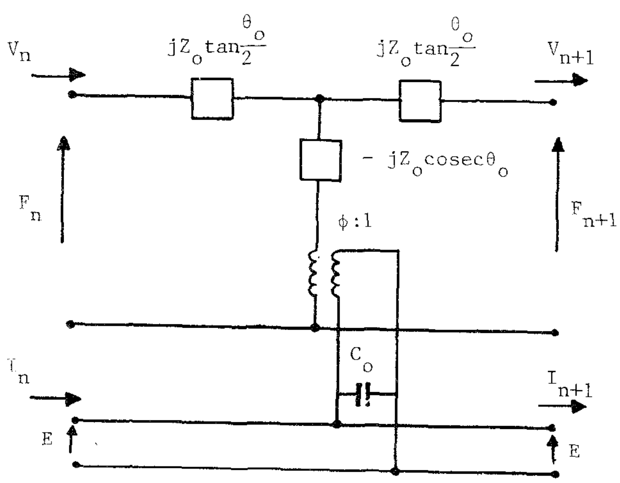

Figure 2. Crossed-field Model for Active Section.

$\phi = \left(\frac{K^2 C_o Z_o V_o}{L_o}\right)^{1/2}$, is the transformer turns ratio.

$\omega = 2\pi f$, f = frequency.

Z_o = acoustic characteristic impedance.

K^2 = piezoelectric coupling coefficient.

C_o = capacitance for the nth section.

V_o = velocity of the acoustic propagation.

L_o = length of the nth section.

The standard network analysis leads to the following transmission matrix equation for this four-port device.

$$\begin{bmatrix} F_n \\ V_n \\ E \\ I_n \end{bmatrix} = \begin{bmatrix} \cos\theta_o & jZ_o\sin\theta_o & (1-\cos\theta_o)\phi & 0 \\ \frac{j}{Z_o}\sin\theta_o & \cos\theta_o & -\frac{j\phi}{Z_o}\sin\theta_o & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{j\phi}{Z_o}\sin\theta_o & (1-\cos\theta_o)\phi & j(\omega C_o + \frac{\phi^2\sin\theta_o}{Z_o}) & 1 \end{bmatrix} \times \begin{bmatrix} F_{n+1} \\ V_{n+1} \\ E \\ I_{n+1} \end{bmatrix} \quad (1)$$

The equivalent circuit for an inactive section of a piezoelectric substrate results by taking $\phi = C_o = 0$ in Figure 2 and in Equation (1); such a section will be called a "transmission line model".

The crossed-field and the transmission line model are cascaded as a pair to design a complete transducer. Thus, the matrix equation (1) with appropriate ϕ's and C_o's yields upon multiplication the transmission matrix for each complete section of the IDT. The transmission matrix of the whole transducer can, therefore, be written in the following form.

$$\begin{bmatrix} F_{in} \\ V_{in} \\ E \\ I_{in} \end{bmatrix} = \left(\prod_{i=1}^{k} T_i \right) T_M \begin{bmatrix} F_{out} \\ V_{out} \\ E \\ I_{out} \end{bmatrix} \quad (2)$$

where T_i represents the transmission matrix for the ith section, and k denotes the total number of sections in the transducer. T_M is the transmission matrix for the inactive section of the substrate.

B. Metal Strip Grating

The grating consists of a large number of uniformly spaced regions with adjacent regions having differing mechanical impedances. When a surface wave is incident on a grating, each region boundary exhibits a small reflection due to the discontinuity in impedance associated with it.

The transmission line model discussed previously is employed to model the acoustic response of a metal strip grating. But in this case the electrical port is omitted.

The acoustic transmission characteristics of the device can therefore be expressed as

$$\begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} = \left(\prod_{i=1}^{N} G_i \right) G_M \begin{bmatrix} F_{out} \\ V_{out} \end{bmatrix} \quad (3)$$

Here G_i is the transmission matrix of the ith section of the grating. Each section of the grating is made up of a metalized and a non-metalized region. Thus the transmission matrix for the ith section is given by

$$G_i = \begin{bmatrix} \cos\theta_1 & jZ_1\sin\theta_1 \\ \frac{j}{Z_1}\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} \cos\theta_o & jZ_o\sin\theta_o \\ \frac{j}{Z_o}\sin\theta_o & \cos\theta_o \end{bmatrix} \quad (4)$$

where Z_o and θ_o are Z_1 and θ_1 for the metalized region.

Equation (4) may also be written in the form

$$G_i = \begin{bmatrix} \cos(\theta_o+\theta_1) & jZ_o\sin(\theta_o+\theta_1) \\ \frac{j}{Z_1}\sin(\theta_o+\theta_1) & \cos(\theta_o+\theta_1) \end{bmatrix} \quad (5)$$

In deriving Equation (5), the ratio $\frac{Z_1}{Z_o}$ is assumed approximately equal to unity, as it will be clear later from Equation (26).

The produce matrix $(G_i)^N$ for N identical periodic sections of a grating is obtained by using the similarity transformation for diagonalization of the matrix G_i [9]. The result is

$$(G_i)^N = \begin{bmatrix} \cos\theta & -j\sqrt{Z_oZ_1}\sin\theta \\ -\frac{j}{\sqrt{Z_oZ_1}}\sin\theta & \cos\theta \end{bmatrix} \quad (6)$$

$$\text{where } \theta = N(\theta_o+\theta_1) \quad (7)$$

Equations (1), (5) and (6) can be used to find the transmission matrix for the overall grating.

C. The SAW Resonator

The complete structure of the SAW resonator was shown in Figure 1. When the acoustic transmission line is combined with the grating, as in Figure 1, the input and output parameters are related by the following transmission matrix equation.

$$\begin{bmatrix} F \\ V \end{bmatrix} = \begin{bmatrix} \cos\theta_T & jZ_o\sin\theta_T \\ \frac{j}{Z_o}\sin\theta_T & \cos\theta_T \end{bmatrix} \begin{bmatrix} \cos\theta & -j\sqrt{Z_oZ_1}\sin\theta \\ -\frac{j}{\sqrt{Z_oZ_1}}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta_1 & jZ_1\sin\theta_1 \\ \frac{j}{Z_1}\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} F_{out} \\ V_{out} \end{bmatrix} \quad (8)$$

or it can be rewritten in the form

$$\begin{bmatrix} F \\ V \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} F_{out} \\ V_{out} \end{bmatrix} \quad (9)$$

where

$$A = \cos\theta\ (\cos\theta_1\ \cos\theta_T - \frac{Z_o}{Z_1}\sin\theta_1\ \sin\theta_T) +$$

$$\sqrt{\frac{Z_o}{Z_1}}\ \sin\theta\ \sin(\theta_1+\theta_T) \qquad (10)$$

$$B = jZ_o\left[\cos\theta\ (\cos\theta_1\ \sin\theta_T + \frac{Z_1}{Z_o}\sin\theta_1\ \cos\theta_T)\right.$$

$$\left. - \sqrt{\frac{Z_1}{Z_o}}\ \sin\theta\ \cos(\theta_1+\theta_T)\right] \qquad (11)$$

$$C = \frac{j}{Z_o}\left[\cos\theta\ (\cos\theta_1\ \sin\theta_T + \frac{Z_o}{Z_1}\sin\theta_1\ \cos\theta_T)\right.$$

$$\left. - \sqrt{\frac{Z_o}{Z_1}}\ \sin\theta\ \cos(\theta_1+\theta_T)\right] \qquad (12)$$

$$D = \cos\theta\ (\cos\theta_1\ \cos\theta_T - \frac{Z_1}{Z_o}\sin\theta_1\ \sin\theta_T) +$$

$$\sqrt{\frac{Z_1}{Z_o}}\ \sin\theta\ \sin(\theta_1+\theta_T) \qquad (13)$$

and θ_T is the acoustic transit angle of the transmission line defined by

$$\theta_T = \frac{L_T}{V_o}\cdot\omega$$

L_T denotes the length of the acoustic transmission line.

Again, since the output of the circuit is terminated by the impedance Z_o, we have

$$F_{out} = Z_o\ V_{out} \qquad (14)$$

From equations (9) and (14), we get

$$F = (Z_oA + B)\ V_{out} \qquad (15)$$

$$V = (Z_oC + D)\ V_{out} \qquad (16)$$

Equations (15) and (16) can now be used to obtain the acoustic impedance Z_A as seen by the transducer, and is given by

$$Z_A = \frac{F}{V} = \frac{Z_oA + B}{Z_oC + D} \qquad (17)$$

The overall transmission matrix of the transducer, defined by (2), is of the form

$$\begin{bmatrix} F \\ -V \\ E \\ I_{in} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & 0 \\ T_{21} & T_{22} & T_{23} & 0 \\ 0 & 0 & 1 & 0 \\ T_{41} & T_{42} & T_{43} & 1 \end{bmatrix} \begin{bmatrix} F \\ V \\ E \\ I_{out} \end{bmatrix} \qquad (18)$$

Explicit expansion of Equation (18) yields

$$F = T_{11}\ F + T_{12}\ V + T_{13}\ E \qquad (19)$$

$$-V = T_{21}\ F + T_{22}\ V + T_{23}\ E \qquad (20)$$

$$I = T_{41}\ F + T_{42}\ V + T_{43}\ E \qquad (21)$$

Utilizing Equations (17), (19) and (20), the values of F and V can be written in terms of E by the following relations

$$F = \frac{T_{13}}{1 - T_{11} - \frac{T_{12}}{Z_A}}\ E \qquad (22)$$

$$V = -\frac{T_{23}}{1+T_{21}Z_A+T_{22}}\ E \qquad (23)$$

Finally, substitution of Equations (22) and (23) into (21) yields the value of I given by

$$I = \frac{T_{41}\ T_{13}}{1 - T_{11} - \frac{T_{12}}{Z_A}}\ E - \frac{T_{42}\ T_{23}}{1+T_{21}Z_A+T_{22}}\ E + T_{43}E \qquad (24)$$

The electrical admittance Y_E of the resonator structure in Figure 1, can therefore be written in the following form.

$$Y_E = \frac{I}{E} = \frac{T_{41}\ T_{13}}{1 - T_{11} - \frac{T_{12}}{Z_A}} - \frac{T_{42}\ T_{23}}{1+T_{21}Z_A+T_{22}} + T_{43} \qquad (25)$$

The Equation (25) has been utilized to obtain the resonator response.

The acoustic resonator being considered is shown in Figure 1b. Since the gratings are not dispersive, the results of Equation (25) may be applied to obtain an efficient computer simulation of the resonator. To further simplify the calculations, both gratings are assumed to be identical with the transducer centered in the acoustic cavity.

Since the grating considered in this analysis consists of metal strips, the impedance mismatch will have two different components; one is due to the mechanical loading of the substrate and the other component is due to the shorting of the local electric field. It can be shown [10] that

$$\frac{Z_1}{Z_o} = 1 + \omega HM + E_o \qquad (26)$$

where H denotes the metal thickness in meters, M is known as mechanical mismatch constant, which primarily depends upon the substrate material and metalization. ω represents the design centre frequency of the device in radian/sec, and E_o is an electrical mismatch constant depending upon the finger width-gap width ratio and the effective surface wave coupling coefficient. For equal finger and gap width [10]

$$E_o \approx 0.37\ K^2$$

Another parameter of common interest is the velocity mismatch defined by [10]

$$\frac{V_1}{V_o} = 1 + \omega\ \theta_L\ H - \frac{1}{2} K^2$$

Here θ_L is the velocity change coefficient from the mechanical loading.

III. PHYSICAL PARAMETERS

Using the above results, a simulation was made for which the substrate material was chosen to be Lithium niobate ($LiNbO_3$) along with the aluminum metalization. The following list shows the numerical values of all physical parameters used in the simulation.

The number of IDT sections = 9
The number of sections in each grating = 200

$L_1 = L_o = 0.6866 \times 10^{-3}$ in.

$L_T = 0.274$ in.

$V_o = 1.37 \times 10^5$ in/s

$C_o = 0.635 \times 10^{-11}$ farads/in

$K^2 = 0.045$

$\theta_L = -\ 0.123 \times 10^{-4}$

$H = 2 \times 10^{-7}$ m

$M = 3.04 \times 10^{-5}$

where L_1 denotes the length of the nth section under metalization.

IV. RESULTS

The preliminary responses for Real Y_E and phase obtained at the center frequency of 50 MHz are shown in Figure 3 and Figure 4.

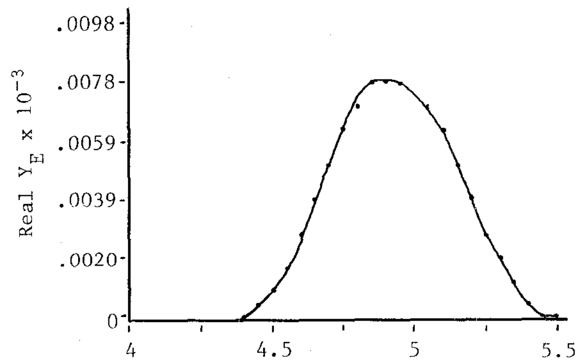

Figure 3

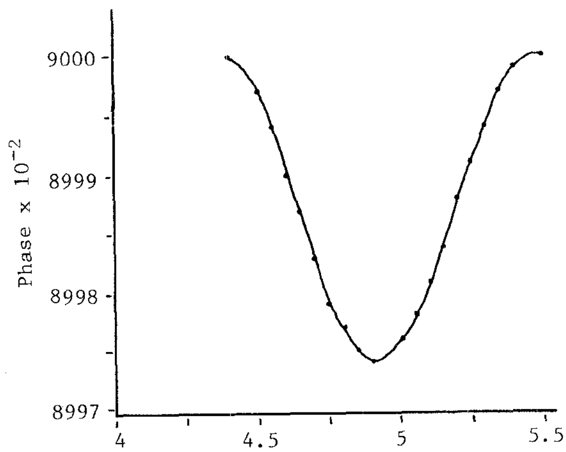

Figure 4

V. CONCLUSION

The preceding discussion shows that the Mason equivalent circuit can be used to model the SAW resonator as just simulated. Very efficient computer programs can be developed to predict the response of complicated interconnections of transducer and gratings. These programs can also be used to varify filter designs and procedures.

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