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SCATTERING MATRIX FOR N-SECTION IN-LINE MODEL OF SAW TRANSDUCERS

ABSTRACT

The scattering matrix of N-equal sections of an in-line model for a SAW device is calculated with the aid of chain and admittance matrices.

1. INTRODUCTION

In the past decade, due to successful applications to the design of different kinds of filters and delay lines, considerable attention has been given to the phenomena of surface acoustic waves (SAW) created on piezoelectric substrates. It has been shown that a circuit model representation can be the key for the development and design of SAW devices. Especially, a frequency domain equivalent circuit model representation [1,2,3] of a transducer has been found to be very successful [2] in characterizing the excitation and detection of surface waves. In these references the transfer characteristic and input admittance at the electric port of the interdigital transducer have been determined with the aid of chain and admittance matrices. Smith, et. al [3], and Gerard [4] have calculated reflection coefficients, but only at the synchronious frequency. Recently, Hribsek and Newcomb [5] have outlined the formalism for obtaining the scattering and transfer scattering matrices of a SAW device section as a function of frequency. While Debnath, Ajmera and Newcomb [6] have calculated the scattering matrix of N-equal sections of a crossed-field model for a SAW section.

The SAW devices offer special interest to system theorists since they rely upon piezoelectrically generated mechanical waves in realizing practical devices for which the theory is anything but complete. And, of course, critical to the operation are the input and output transducers which can be considered as 3-ports having one electrical port and two mechanical (acoustic) ports (one for forward and one for backward acoustic wave motion). These transducers can be characterized by what have been called "in-line" and "crossed-field" models for the basic sections. This paper is concerned with the scattering matrix for

N sections of a SAW transducer described by the in-line model.

The primary objective is to develop a general mathematical technique by using the known admittance matrix of a 3-port SAW transducer section. The starting point is the equivalent circuit of a basic one electrode section of the SAW transducer. This is then used to obtain the 3-port admittance matrix of a full (interdigital) transducer by first finding the matrix of one interdigital period and then applying a cascading formalism. The 3-port admittance matrix for an entire N-section transducer is found by connecting the N sections in cascade acoustically and in parallel electrically. Using these chain and admittance matrix results, the scattering matrix of N equal basic sections expressed in terms of the in-line model is determined.

2. ADMITTANCE MATRIX FOR A THREE-PORT N-SECTION NETWORK

The details of this section repeat those of reference [6] but are recorded here for completeness of the treatment.

We consider a 3-port section with two identical mechanical ports (of variables $F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$ = force applied, $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ = velocity of material) and one electrical port (of variables V_3 = voltage, I_3 = current applied to the interdigital electrodes), as shown in Fig. 1.

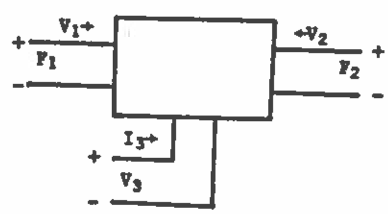

Figure 1. Block-diagram of a three-port Model

Then

$$\begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{11} & y_{13} \\ y_{13} & y_{13} & y_{33} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ V_3 \end{bmatrix} \tag{1}$$

is taken as the admittance matrix y where $y_{22} = y_{11}$ and $y_{23} = y_{13}$ by symmetry. y is symmetric by assumed reciprocity.

We now connect N of these sections in cascade at the mechanical ports and in parallel, with every other one reversed, at the electrical ports as shown in Fig. 2.

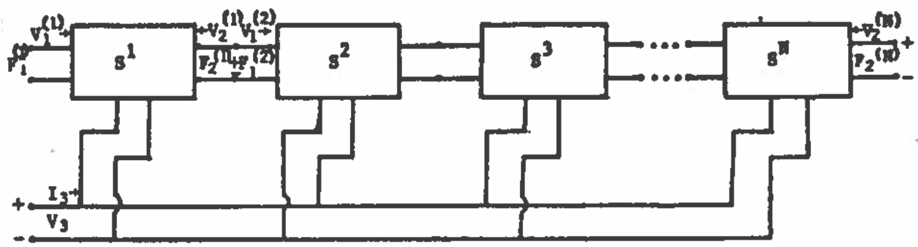

Figure 2. Cascaded one-electrode sections for a SAW transducer.

Thus for the i^{th} section we have

$$\begin{bmatrix} V_1^{(i)} \\ V_2^{(i)} \\ I_3^{(i)} \end{bmatrix} = \begin{bmatrix} y_{11}^{(i)} & y_{12}^{(i)} & y_{13}^{(i)} \\ y_{12}^{(i)} & y_{11}^{(i)} & y_{13}^{(i)} \\ y_{13}^{(i)} & y_{13}^{(i)} & y_{33}^{(i)} \end{bmatrix} \begin{bmatrix} F_1^{(i)} \\ F_2^{(i)} \\ V_3^{(i)} \end{bmatrix} , \tag{2}$$

while the interconnection laws are (for mechanical ports, and $i > 1$)

$$\left.\begin{aligned} F_1^{(i)} &= F_2^{(i-1)} \\ V_1^{(i)} &= -V_2^{(i-1)} \end{aligned}\right\} \tag{3}$$

where

$$\left.\begin{aligned} F_1^{(1)} &= F_{in} = F_{\text{left boundary}} , \quad V_1^{(1)} = V_{in} \\ F_2^{(N)} &= F_{out} = F_{\text{right boundary}} , \quad V_2^{(N)} = V_{out} . \end{aligned}\right\} \tag{4}$$

Also for electrical ports with $i \geq 1$, the interconnection laws are

$$\begin{aligned} V_3^{(i)} &= (-1)^{i-1} V_3 \\ I_3 &= \sum_{i=1}^{N} (-1)^{i-1} I_3^{(i)} \end{aligned} \tag{5}$$

In order to find the interconnected network 3-port description we convert to a reverse - chain matrix type of description. From equation (2) it follows that

$$\begin{bmatrix} F_2^{(i)} \\ -V_2^{(i)} \end{bmatrix} = \psi_R^{(i)} \begin{bmatrix} F_1^{(i)} \\ V_1^{(i)} \end{bmatrix} + A^{(i)} V_3 \tag{6}$$

where

$$\psi_R^{(i)} = -\frac{1}{y_{12}^{(i)}}\begin{bmatrix} y_{11}^{(i)} & -1 \\ (y_{12}^{(i)})^2-(y_{11}^{(i)})^2 & y_{11}^{(i)} \end{bmatrix} , \tag{7}$$

$$A^{(i)} = \frac{(-1)^i\, y_{13}^{(i)}}{y_{12}^{(i)}}\begin{bmatrix} 1 \\ y_{12}^{(i)} - y_{11}^{(i)} \end{bmatrix} \tag{8}$$

and

$$I_3^{(i)} = -\frac{y_{13}^{(i)}}{y_{12}^{(i)}}[y_{11}^{(i)} - y_{12}^{(i)}, 1]\begin{bmatrix} F_1^{(i)} \\ V_1^{(i)} \end{bmatrix} + \frac{(-1)^{i-1}}{y_{12}^{(i)}}[y_{33}^{(i)}y_{12}^{(i)} - (y_{13}^{(i)})^2]V_3 , \tag{9}$$

Equation (6) is the key for this analysis. Using the mechanical interconnections, we obtain

$$\begin{bmatrix} F_2^{(1)} \\ -V_2^{(1)} \end{bmatrix} = \psi_R^{(1)}\begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + A^{(1)}V_3 \quad , \tag{10}$$

$$\begin{bmatrix} F_2^{(2)} \\ -V_2^{(2)} \end{bmatrix} = \psi_R^{(2)}\begin{bmatrix} F_1^{(2)} \\ V_1^{(2)} \end{bmatrix} + A^{(2)}V_3 \tag{11}$$

or

$$\begin{bmatrix} F_2^{(2)} \\ -V_2^{(2)} \end{bmatrix} = \psi_R^{(2)}\,\psi_R^{(1)}\begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + (\psi_R^{(2)}\,A^{(1)} + A^{(2)})\,V_3 \tag{12}$$

In general for N Sections

$$\begin{bmatrix} F_{out} \\ -V_{out} \end{bmatrix} = \begin{bmatrix} F_2^{(N)} \\ -V_2^{(N)} \end{bmatrix} = \psi_R^{(N)}\begin{bmatrix} F_1^{(N)} \\ -V_1^{(N)} \end{bmatrix} + A^{(N)}V_3$$

$$= \psi_R^{(N)}\, \psi_R^{(N-1)} \cdots\cdots \psi_R^{(2)} \psi_R^{(1)} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + (A^{(N)} + \psi_R^{(N)}\, A^{(N-1)} + \cdots \psi_R^{(N)} \psi_R^{(N-1)} \cdots$$

$$\cdots \psi_R^{(3)}\, A^{(2)} + \psi_R^{(N)}\, \psi_R^{(N-1)} \cdots\cdots \psi_R^{(3)}\, \psi_R^{(2)}\, A^{(1)}) \tag{13}$$

Thus for the entire cascaded system

$$\begin{bmatrix} F_{out} \\ -V_{out} \end{bmatrix} = P_R \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + A\, V_3 \tag{14}$$

where

$$P_R = \prod_{i=1}^{N}{}_R\, \psi_R^{(i)} \tag{15}$$

$$A = \sum_{i=1}^{N} \left(\prod_{k=i}^{N-1}\Big|_R\, \psi_R^{(k+1)} \right) A^{(i)} \tag{16}$$

The notation $\prod|_R$ means multiply lower indexed terms on the right and $\prod_{k=N}^{N-1}$ $\equiv$ identity. If all sections are identical then it follows from (7), (15) and (16) that

$$P_R \equiv (\psi_R^{(1)})^N = \frac{(-1)^N}{y_{12}^{\;N}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^{\;2} - y_{11}^{\;2} & y_{11} \end{bmatrix}^N \tag{17}$$

and

$$A = \sum_{i=1}^{N} (\psi_R^{(1)})^{N-i} (-1)^{i-1} A^{(1)} = \sum_{k=0}^{N-1} (\psi_R^{(1)})^k (-1)^{N-k} \frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} \tag{18}$$

where $N-i \equiv k$. Continuing with the identical section case we have for each section

$$\psi_R = -\frac{1}{y_{12}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^{\;2} - y_{11}^{\;2} & y_{11} \end{bmatrix}, \quad A = -\frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} \tag{19}$$

so that

$$\psi_R^2 = \frac{1}{y_{12}^{\;2}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^{\;2} - y_{11}^{\;2} & y_{11} \end{bmatrix} \begin{bmatrix} y_{11} & -1 \\ y_{12}^{\;2} - y_{11}^{\;2} & y_{11} \end{bmatrix} \tag{20}$$

or

$$\psi_R^{\,2} = \frac{1}{y_{12}^{\,2}}\begin{bmatrix} 2y_{11}^{\,2} - y_{12}^{\,2} & -2y_{11} \\ 2y_{11}(y_{12}^{\,2} - y_{11}^{\,2}) & 2y_{11}^{\,2} - y_{12}^{\,2} \end{bmatrix} \tag{21}$$

Thus it leads to the recursion relation

$$\psi_R^{\,2} = -\psi_R^{\,0} - \frac{2y_{11}}{y_{12}}\,\psi_R^{\,1}\,, \tag{22}$$

Where

$$\psi_R^{\,0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{23}$$

Relation (22) can be used to reduce the higher powers of ψ_R to the first.

We next proceed to find the admittance matrix of the overall 3-port network. Explicit expansion of (14) yields

$$\begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} -P_{R_{12}}^{-1} P_{R_{11}} & P_{R_{12}}^{-1} & -P_{R_{12}}^{-1} A_{11} \\ -P_{R_{12}} + P_{R_{22}} P_{R_{12}}^{-1} P_{R_{11}} & -P_{R_{22}} P_{R_{12}}^{-1} & P_{R_{22}} P_{R_{12}}^{-1} A_{11} - A_{21} \end{bmatrix} \begin{bmatrix} F_{in} \\ F_{out} \\ V_3 \end{bmatrix} \tag{24}$$

Where $P_{R_{11}}$, $P_{R_{12}}$, $P_{R_{22}}$ etc. represent the corresponding elements of the final matrix P_R for the entire cascaded network system.

Following the use of induction, an explicit evaluation of the matrix elements in (24) gives the following results for all N = 1, 2, 3, ...

$$Y_{13} = y_{13} \tag{25}$$

$$Y_{23} = (-1)^{N-1}\, y_{13} \tag{26}$$

Y_{ij} are the matrix elements of the final admittance matrix Y for the N cascaded sections, and y_{ij} are the matrix elements for the single electrode section. It remains to calculate I_3 to complete the admittance matrix. We have from (5)

$$I_3 = \sum_{i=1}^{N}(-1)^{i-1} I_3^{(i)} = \sum_{i=1}^{N}(-1)^{i-1} y_{13}^{(i)}\,[F_1^{(i)} + F_2^{(i)}] + \sum_{i=1}^{N} y_{33}^{(i)} V_3 \tag{27}$$

where

$$F_1^{(i)} = F_2^{(i-1)}\,, \tag{28}$$

$$\begin{bmatrix} F_2^{(i)} \\ -V_2^{(i)} \end{bmatrix} = \prod_{j=1}^{i} \psi_R^{(j)} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + \sum_{j=1}^{i}\left(\prod_{k=j}^{i-1} \psi_R^{(k+1)}\right) A^{(j)}\, V_3\,. \tag{29}$$

and

$$\begin{bmatrix} F_1^{(i)} \\ -V_2^{(i-1)} \end{bmatrix} = \prod_{j=1}^{i-1} \ldots$$

for which the (1,1) ...
substituted.

For identical sec...

$$\begin{bmatrix} F_2^{(i)} \\ -V_2^{(i)} \end{bmatrix} = \frac{(-1)^{i-}\ldots}{y_{12}^{\,i-1}} \ldots$$

$$+ \{ \sum_{k=0}^{i-2} (\psi_R^{(1)})^{k} \ldots$$

$$\begin{bmatrix} F_1^{(i)} \\ -V_2^{(i-1)} \end{bmatrix} = \frac{(-1)^{i}\ldots}{y_{12}^{\,i-}\ldots}$$

$$+ \sum_{k=0}^{i-2} (\psi_R^{(1)})^{k} \ldots$$

After some algebric ...
$[F_1^{(i)} + F_2^{(i)}]$. O...
(27) we get

$$I_3 = \sum_{i=1}^{N} y_{33} V_3 \ldots$$

$$-(y_{12}-y_{11})[y_{12}+y_{11}- \ldots$$

$$+ (y_{12}-y_{11})(\psi_R^{(1)})^{i-1} \ldots$$

From this result fo...
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$$\vdots$$

$$\begin{bmatrix} F_1^{(i)} \\ -V_2^{(i-1)} \end{bmatrix} = \prod_{j=1}^{i-1} \phi_R^{(j)} \begin{bmatrix} V_{in} \\ V_{in} \end{bmatrix} + \sum_{j=1}^{i-1} \left(\prod_{k=j}^{i-2} \phi_R^{(k+1)} \right) A^{(j)} \; V_3 \qquad (30)$$

for which the (1,1) entries are to be summed and the expression for V_{in} substituted.

For identical sections, it turns out that

$$\begin{bmatrix} F_2^{(i)} \\ -V_2^{(i)} \end{bmatrix} = \frac{(-1)^{i-1}}{y_{12}^{\,i-1}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix}^{i-1} \frac{(-1)}{y_{12}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix}$$

$$+ \left\{ \sum_{k=0}^{i-2} (\phi_R^{(1)})^k (-1)^{i-k} + (\phi_R^{(1)})^{i-1} (-1)^{i-(i-1)} \right\} \frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} V_3 \; . \qquad (31)$$

$$\begin{bmatrix} F_1^{(i)} \\ -V_2^{(i-1)} \end{bmatrix} = \frac{(-1)^{i-1}}{y_{12}^{\,i-1}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix}^{i-1} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix}$$

$$+ \sum_{k=0}^{i-2} (\phi_R^{(1)})^k (-1)^{i-1-k} \frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} V_3 \; . \qquad (32)$$

After some algebric manipulation of (31) and (32), we find an expression for $[F_1^{(i)} + F_2^{(i)}]$. On substitution of this expression for $[F_1^{(i)} + F_2^{(i)}]$ into (27) we get

$$I_3 = \sum_{i=1}^{N} y_{33} V_3 + \sum_{i=1}^{N} (-1)^{i-1} \frac{y_{13}}{y_{12}} \Big[\{ (y_{12} - y_{11} - P_{R_{12}}^{-1} P_{R_{11}}) ({\phi_R^{(1)}}^{i-1})_{11}$$

$$- (y_{12} - y_{11}) [y_{12} + y_{11} - (-P_{R_{12}}^{-1} P_{R_{11}})] ({\phi_R^{(1)}}^{i-1})_{12} \} F_{in} + \{ ({\phi_R^{(1)}}^{i-1})_{11}$$

$$+ (y_{12} - y_{11}) ({\phi_R^{(1)}}^{i-1})_{12} \} P_{R_{12}}^{-1} F_{out} \Big] \qquad (33)$$

From this result for $N = 1, 2, 3$ the expected reciprocity results, $Y_{31} = Y_{13}$ and $Y_{32} = Y_{23}$, can be easily verified. By induction it follows that the reciprocity relations are valid for an arbitrary N.

Therefore, the final simplified form of the admittance matrix Y for N equal section case is given by

$$Y = \begin{bmatrix} -P_{R_{12}}^{-1} P_{R_{11}} & P_{R_{12}}^{-1} & y_{13} \\ P_{R_{12}}^{-1} & -P_{R_{12}}^{-1} P_{R_{11}} & (-1)^{N-1} y_{13} \\ y_{13} & (-1)^{N-1} y_{13} & N y_{33} \end{bmatrix} \quad (34)$$

This is the most general form of the admittance matrix for the "N cascaded" network shown in Fig. 2. The combined network structure consequently may be defined by a single section as shown in Fig. 3.

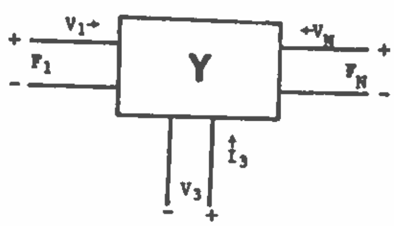

Figure 3. Combined Network Structure

3. EQUIVALENT CIRCUIT OF THE IN-LINE MODEL AND THE ADMITTANCE MATRIX

The development of the interdigital transducer transfer function begins with the one dimensional in-line model for a single electrode section; this is shown for the sinusoidal steady state in Fig. 4. The model of a single electrode section has three ports, corresponding to the two symmetric acoustic ports and the electric port. The stresses and particle velocities at the acoustic ports are represented by equivalent voltages and currents, respectively, to which they are numerically equal. A convenient method for describing the relationship between these currents and voltages, $V_j = y_{jk}F_k$, is in terms of the admittance matrix as for equation (1). The y_{jk}'s for the n^{th} section are given by [5]

$$y_{11} = \frac{1}{jZ_o} \frac{(\cos\theta_n - \gamma \sin\theta_n)}{\sin\theta_n + 2\gamma(\cos\theta_n - 1)} = y_{22} \quad (35)$$

$$y_{12} = \frac{j}{Z_o} \frac{(1 - \gamma \sin\theta_n)}{\sin\theta_n + 2\gamma(\cos\theta_n - 1)} = y_{21} \quad (36)$$

$$y_{13} = \frac{j\phi}{Z_o} \frac{(\cos\theta_n - 1)}{\sin\theta_n + 2\gamma(\cos\theta_n - 1)} = y_{23} = y_{32} = y_{31} \quad (37)$$

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$$y_{33} = j\omega C_o \left[1 + \frac{2K^2}{\theta_n} \cdot \frac{1 - \cos\theta_n}{\sin\theta_n + 2\gamma(\cos\theta_n - 1)} \right] \qquad (38)$$

where

$\theta_n = \frac{\ell_n}{V} \cdot 2\pi f$, is the acoustic transit angle of the n^{th} section; f = frequency.

$\phi = \left(\frac{K^2 C_o Z_o V}{\ell_n}\right)^{1/2}$ is the transformer turns ratio.

$\gamma = \frac{K^2}{\theta_n}$

$C_s = \frac{C_o}{\phi^2}$, C_o = capacitance of the n^{th} section

$\omega = 2\pi f$

Z_o = acoustic characteristic impedance

K^2 = piezo-electric coupling coefficient

V = velocity of acoustic propagation

ℓ_n = lenght of n^{th} section

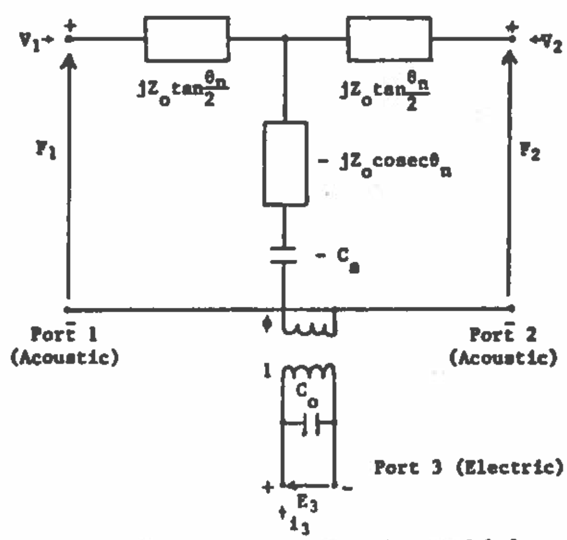

Figure 4. Equivalent circuit of the in-line model for one-electrode section.

4. SCATTERING MATRIX FOR N-SECTION IN-LINE MODEL

It is well known [6,7] that the scattering matrix of a network characterized by its admittance matrix Y is

$$\underset{\sim}{S} = \underset{\sim}{\mathbb{I}}_3 - 2\underset{\sim}{Y}\,(\underset{\sim}{\mathbb{I}}_3 + \underset{\sim}{Y})^{-1} \tag{39}$$

where $\underset{\sim}{\mathbb{I}}_3$ is the 3x3 identity matrix. This equation (39) will be used to find the scattering matrix for the N-section in-line model.

Application of a unitary transformation for diagonalization of $\phi_R^{(1)}$ given by (7) gives the product matrix P_R for the N-section in-line model as

$$P_R = (\phi_R^{(1)})^N = \sin N\delta \begin{bmatrix} \cot N\delta & -jZ_o \sin\delta(1-\Psi\sin\theta)(\sin\theta)^{-1} \\ -\dfrac{1}{Z_o}\dfrac{\sin\theta}{\sin\delta(1-\Psi\sin\theta)} & \cot N\delta \end{bmatrix} \tag{40}$$

where δ is given by the expression

$$\sin\delta = (\sin\theta + 2\Psi\cos\theta - 2\Psi)^{\frac{1}{2}}\,(1-\Psi\sin\theta)^{-1}\,(\sin\theta)^{\frac{1}{2}}. \tag{41}$$

Using (34), (37), (38) and (40), the admittance matrix Y for the N-section in-line model is obtained in the final form

$$Y = \begin{bmatrix} -\dfrac{1}{Z_o}\dfrac{\cot N\delta}{(1-2\Psi\tan\frac{\theta}{2})^{\frac{1}{2}}} & \dfrac{1}{Z_o}\dfrac{\operatorname{cosec} N\delta}{(1-2\Psi\tan\frac{\theta}{2})^{\frac{1}{2}}} & -\dfrac{1}{Z_o}\phi\dfrac{\tan\frac{\theta}{2}}{(1-2\Psi\tan\frac{\theta}{2})} \\ \dfrac{1}{Z_o}\dfrac{\operatorname{cosec} N\delta}{(1-2\Psi\tan\frac{\theta}{2})^{\frac{1}{2}}} & -\dfrac{1}{Z_o}\dfrac{\cot N\delta}{(1-2\Psi\tan\frac{\theta}{2})^{\frac{1}{2}}} & (-1)^N\dfrac{1}{Z_o}\phi\dfrac{\tan\frac{\theta}{2}}{(1-2\Psi\tan\frac{\theta}{2})} \\ -\dfrac{1}{Z_o}\phi\dfrac{\tan\frac{\theta}{2}}{(1-2\Psi\tan\frac{\theta}{2})} & (-1)^N\dfrac{1}{Z_o}\phi\dfrac{\tan\frac{\theta}{2}}{(1-2\Psi\tan\frac{\theta}{2})} & j\omega C_T(1+\dfrac{2K^2}{\theta}\dfrac{\tan\frac{\theta}{2}}{(1-2\Psi\tan\frac{\theta}{2})} \end{bmatrix} \tag{42}$$

where $C_T = NC_o$.

Finally, direct substitution of Y from (42) into (39) yields the explicit form of the scattering matrix for the N-section in-line model. A straightforward but lengthy calculations [6] gives the explicit and final form of the scattering matrix elements in terms of known parameters as

$$S_{11} = S_{22} = \frac{1}{|M|}\Big[(1-\frac{1}{\eta^2 Z_o^2}) + j\,\Big\{\omega C_T(1-\frac{1}{\eta^2 Z_o^2})$$
$$.(1+\frac{2K^2}{\eta^2\theta}\tan\frac{\theta}{2}) + \frac{2\phi^2}{\eta^5 Z_o^3}\tan^2(\frac{\theta}{2})(\cot N\delta + (-1)^{N+1}\operatorname{cosec} N\delta)\Big\}\Big] \tag{43}$$

$$S_{12} = S_{21} = \frac{2}{|M|}\left[\left\{\frac{\omega C_T}{\eta Z_o}\left(1+\frac{2K^2}{\eta^2\theta}\tan\frac{\theta}{2}\right)\operatorname{cosec} N\delta + (-1)^N\frac{\phi^2}{\eta^4 Z_o^{\,2}}\tan^2\left(\frac{\theta}{2}\right)\right\} - \frac{1}{\eta Z_o}\operatorname{cosec} N\delta\right] \tag{44}$$

$$S_{13} = S_{31} = \frac{2}{|M|}\left[\frac{\phi}{\eta^3 Z_o^{\,2}}\tan\frac{\theta}{2}\left\{\cot N\delta + (-1)^{N+1}\operatorname{cosec} N\delta\right\} + j\,\frac{\phi}{\eta^2 Z_o}\tan\frac{\theta}{2}\right] \tag{45}$$

$$S_{23} = S_{32} = (-1)^{N-1}\,S_{13} \tag{46}$$

and

$$S_{33} = \frac{1}{|M|}\left[\left\{\left(1+\frac{1}{\eta^2 Z_o^{\,2}}\right) - \frac{2\phi^2}{\eta^4 Z_o^{\,2}}\tan^2\left(\frac{\theta}{2}\right) - \frac{2\omega C_T}{\eta Z_o}\cot N\delta\left(1+\frac{2K^2}{\eta^2\theta}\tan\frac{\theta}{2}\right)\right\} + j\left\{\frac{2\phi^2}{\eta^5 Z_o^{\,3}}\tan^2\left(\frac{\theta}{2}\right)\left(\cot N\delta + (-1)^{N+1}\operatorname{cosec} N\delta\right) - \omega C_T\left(1+\frac{1}{\eta^2 Z_o^{\,2}}\right)\left(1+\frac{2K^2}{\eta^2\theta}\tan\frac{\theta}{2}\right) - \frac{2}{\eta Z_o}\cot N\delta\right\}\right] \tag{47}$$

where

$$|M| = \left[\left\{\left(1+\frac{1}{\eta^2 Z_o^{\,2}}\right) + \frac{2\phi^2}{\eta^4 Z_o^{\,2}}\tan^2\left(\frac{\theta}{2}\right) + \frac{2\omega C_T}{\eta Z_o}\cot N\delta\left(1+\frac{2K^2}{\eta^2\theta}\tan\frac{\theta}{2}\right)\right\} + j\left\{\omega C_T\left(1+\frac{1}{\eta^2 Z_o^{\,2}}\right)\left(1+\frac{2K^2}{\eta^2\theta}\tan\frac{\theta}{2}\right) - \frac{2}{\eta Z_o}\cot N\delta - \frac{2\phi^2}{\eta^5 Z_o^{\,3}}\tan^2\left(\frac{\theta}{2}\right)\left(\cot N\delta + (-1)^{N+1}\operatorname{cosec} N\delta\right)\right\}\right] \tag{48}$$

and

$$\eta = \left(1 - 2\Psi\tan\frac{\theta}{2}\right)^{1/2}. \tag{49}$$

Therefore, the scattering matrix for the N-section in-line model reflects the circuits reciprocity in its symmetric form. Although these expressions are rather bulky, they should be useful for computer aided analysis.

5. CONCLUSION

Here we have given, for the sinusoidal steady state behavior, explicit formulas for the scattering matrix entries of a SAW interdigital transducer based upon the in-line model of a one-electrode section, these formulas appearing in equations (43) - (49). In the form presented, they are suitable for computer aided design of SAW transducers while their existence paves the way for more extensive design formalisms.

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