Introduction and Open Problems on SAW's

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Prologue: "O Mother Poland - Whose lips would dare
To boast that they will find that word so rare
And lift the gravestone from the hearts of men
And unlock eyes that brim with tears again."

[0, p.284]

Abstract

A quick overview of the SAW field is presented along with a list of open problems suitable for profitable investigation.

# I. SAW Importance

The surface acoustic wave (SAW) field is undergoing rapid changes because of the versatility of SAW applications and advances in technology allowing for device realizations suitable to the applications. Specifically SAW devices can be economically used as TV IF filters, tapped delay lines and conventional band-pass filters while they appear suitable for other applications, such as radar pulse-compression filters, high frequency oscillators, and spread spectrum correlators. Presently they operate economically in the 20MHz to 600MHz range and offer in many cases a 10:1 size advantage to comparable LC filters along with characteristics fixed by integrated circuit (IC) process steps. They do have some problems, such as relatively large insertion loss, the necessity of specialized equipment for construction, and relatively large temperature coefficients for low loss materials.

<sup>\*</sup>Talk prepared for ECCTD 80, Warsaw, September 2-5, 1980.

Presently, designs are almost exclusively done via computer-aided design (CAD), which can mean high initial design costs [1].

Because of their increasing importance and because their theories are not as yet extensively developed, SAW's offer considerable challenge to the field of circuit theory. To understand this challenge the basics of SAW device operation will first be reviewed.

#### 11. The Basic SAW Device

Figure 1 shows the basic SAW device. This consists of a piezo-electric substrate (such as Quartz =  $\mathrm{SiO}_2$  or Lithium Niobate =  $\mathrm{Li\,NbO}_3$ ) which is set into mechanical displacement by the electric field between the fingers of an input interdigital transducer (IDT). This motion sends a displacement wave in the positive and negative x directions, the one in the negative direction being absorbed to prevent reflections. The positively traveling wave is detected by the output transducer which obtains an electric field converted through the piezoelectric effect of the mechanical motion. By limiting to surface waves, signal injection and detection can be limited to the surface allowing for constructions through integrated circuit processing techniques.

In order to understand the open problems it is necessary to understand the nature of presently used mathematical formulations. For this a brief summary of the key techniques are next presented.

The following ideas somewhat follow the development of the excellent book by Sapriel [2, p. 38]. Let  $u_{m}$  be the displacement along the  $x_{m}$  axis (m = 1,2,3), then the components of the strain tensor are

$$S_{mn} = \frac{1}{2} \left( \frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m} \right) = S_{nm}$$
 (1)

while the resultant stress tensor (internal forces/area) components are denoted  $T_{mn} = T_{nm}$ . Using the convention of summing on repeated indices, the equations of displacement motion are

$$\frac{\partial T_{mn}}{\partial x_n} = \rho \frac{\partial^2 u_m}{\partial t^2} \quad ; \quad \rho = \text{mass density}$$
 (2)

subject to the piezoelectric conditions

$$T_{mn} = c_{mnpq} S_{pq} - e_{pmn} E_{p}$$
 (3a)

$$D_{m} = e_{mpq} S_{pq} + \varepsilon_{mp} E_{p}$$
 (3b)

where  $\mathbf{E}_k$  and  $\mathbf{D}_m$  are components of the electric field and the electric displacement field; the stiffness tensor  $\{\mathbf{c}_{mnpq} = \mathbf{c}_{nmqp}\}$ , the piezoelectric tensor  $\{\mathbf{e}_{mpq} = \mathbf{e}_{mqp}\}$ , and the permittivity tensor  $\{\mathbf{e}_{mp} = \mathbf{c}_{mp} = \mathbf{c}_{mp}\}$  diagonal for piezoelectric materials) are tabulated for various materials of interest [3, pp. 357-382] (these are given in coordinates that may need to be transformed to conform with those of Fig. 1). There are also Maxwell's equations for which a "quasi-static" assumption ( of slow time variations) is made to get  $\mathbf{E} = -\nabla \mathbf{u}_4$ ,  $\mathbf{u}_4 = \mathbf{potential}$  function, and considering first the electrode free surface a zero free charge assumption is made, giving  $\mathbf{V} \cdot \mathbf{\hat{D}} = \mathbf{0}$ ; in component form

$$E_{m} = -\frac{\partial u_{4}}{\partial x_{m}} \tag{4a}$$

$$\frac{\partial D}{\partial x_{m}} = 0 \tag{4b}$$

Taking the position vector as

$$\vec{r} = x_{m} \vec{a}_{m}$$
 ,  $\vec{a}_{m} = unit \ vector$  (5a)

and the propagation vector as

$$\vec{\beta} = \beta_{\mathbf{m}} \vec{a}_{\mathbf{m}} \tag{5b}$$

we assume plane wave solutions at frequency  $\omega$ 

$$u_{m}(\vec{r},t) = U_{m} e^{j\omega(t - \frac{\vec{\beta} \cdot \vec{r}}{\omega v_{s}})}, \quad j = \sqrt{-1}$$

$$m \in \{1,2,3,4\}$$
(6)

This gives a surface wave travelling in the  $\mathbf{x}_1$  direction with velocity  $v_s$ , acoustic when  $v_s << c = velocity of light, if <math>\vec{\beta} = \beta_1 \vec{a}_1 + j \sigma_3 \vec{a}_3$ ,  $\beta_1 > 0$ ,  $\sigma_3 < 0$ 

$$\vec{\beta} = \beta_1 \vec{a}_1 + j \sigma_3 \vec{a}_3 , \quad \beta_1 > 0, \quad \sigma_3 < 0$$
 (7)

The term  $\sigma_3 < 0$  meaning an attenuation in the  $x_3 > 0$  direction occurs through the term  $e^{\sigma_3 x_3/v_s}$  giving propagation on the surface.  $\vec{\beta}$  of the form of (7) defines the  $x_1-x_3$  plane as the "sagital plane". To obtain this SAW propagation the displacement of (6) must satisfy (3) and (4); direct substitution leads to the 4-vector matrix equation:

$$(A - [\rho v_s 1_3 + 0]) U = 0$$
 (8a)

where  $l_3 = 3x3$  identity,  $\dot{+} = \text{direct sum}$ ,  $U = [U_m]$ ,  $A = [A_{mn}]$  (having  $m, n \in \{1, 2, 3, 4\}$ ) with, for  $m, n, p, q \in \{1, 2, 3\}$ 

$$A_{mn} = c_{mnpq} \beta_p \beta_q \tag{8b}$$

$$A_{m4} = A_{4m} = e_{mpq} \beta_p \beta_q$$
 (8c)

$$A_{44} = -\varepsilon_{pq}\beta_{p}\beta_{q}$$
 (8d)

In order for there to be a nonzero solution U to (8a)

$$\det \{A - \{\rho v_s 1_3 + 0\}\} = 0$$
 (9)

which is seen, by looking at the four diagonal terms of (8b) - (8d), which are each of degree two in  $\beta_3$ , to be of degree 8 in  $\beta_3$ . For suitable  $v_s$  imaginary  $\beta_3 = j\sigma_3$  with  $\sigma_3 < 0$  result, as is verified for example by experimentally observed SAW's. Such SAW's must be subjected to further conditions at the boundary  $x_3 = 0$ , these being the continuity of the potential function,  $u_4$ , the continuity of the normal component of  $\vec{D}$ ,

$$D_{3 \text{ internal}} = (e_{3pq}S_{pq} - c_{3p}\frac{\partial u_4}{\partial x_p})| = -c_0\frac{\partial u_4}{\partial x_3}| = D_3 \text{ external} \quad (10a)$$

$$x_3 = 0_+ \qquad x_3 = 0_-$$

and the vanishing of body forces

$$T_{3n} = 0$$
 ,  $n \in \{1, 2, 3\}$  (10b)  
 $x_3 = 0$ 

Equation (9) fixes ratios of the  $\mathbf{U}_{\mathbf{m}}$  while (10) gives their actual values.

At the transducer electrodes  $\nabla \cdot \vec{D} = Q = \text{charge need not be zero}$  and excitations can occur. By considering mirror electrodes fictitiously placed under the substrate, a rather lengthy development following the longitudinal mode theory of quartz of Mason [4, pp. 195-205] yields a "cross-field" model (the electric field, being in the  $x_3$  [Mason's X] direction, crosses the  $x_1$  [Mason's Y] direction of [longitudinal] propagation).

But actually a considerable component of the electric field is in the direction of propagation and, hence, one conceives of a set of fictitious plates in the  $x_2$ - $x_3$  planes placed at the electrode centers to give an  $x_1$  directed electric field yielding, again by Mason's thickness mode theory [4, pp. 399-404], an "in-line" model. "Superosing" the two yields [5] the reasonably accurate equivalent circuit

model of a transducer section shown in Fig. 2 (where  $\alpha=0$  gives the cross-field model and  $\alpha=1$  the in-line one). In Fig. 2,  $V_3$  and  $I_3$  are the voltage and current at the transducer electrodes of a section while  $F_1$ ,  $v_1$ , and  $F_2$ ,  $v_2$  are the force-velocity pairs at the input and output mechanical portions of a section. The circuit of Fig. 2 is then seen to give a model for non-electroded sections by setting  $I_3=0$  (yielding simply the transmission line). Given the parameters of the equivalent circuit of Fig. 2, found for example by matching signal responses or through reduction of eqs. (1)-(3), computer-aided design of SAW systems can take place.

#### III. Some Observations

In order to further prepare for understanding of the SAW open problems we make some observations on the basic SAW device.

- 1. The SAW phenomena rests upon the transmission and processing of waves. Consequently, its description might best be given through scattering matrices (in the linear case).
- 2. Half of the waves launched by the input transducer are sent to the left in Fig. 1, hence lost for use. Further, waves are actually launched into the bulk of the substrate material (called "bulk waves") causing further loss. Since the substrate is relatively thin, it behaves in a linear manner only for very small signals.
- 3. Equations (3) represent linear behavior, as does the equivalent circuit model of Fig. 2. The cross-field model for Fig. 2 is derived assuming a (uniform) field between the top and bottom of the substrate and, hence, describes bulk, rather than surface, phenomena.

#### IV. Some Areas for SAW Research

Based upon the previous comments and what they can lead us to, and especially the observations just made, we can give some specific open problems toward which research could be profitably directed.

# 1. Linear and General Theory

## a) Scattering matrix developments

Since SAW's are apparently best described through scattering matrices, a continued development of scattering matrix theories for them should be vigorously pursued [6][7][8]. In particular synthesis techniques in terms of the circuit model of Fig. 2 appear possible following [9][10][11] with the development of these along CAD lines needed.

#### b) Development of more exact models

The model of Fig. 2 follows upon assumptions which are unrealistic in many situations, such as that of quasi-stationess. Relaxation of these assumptions appear possible (for example, the complete set of Maxwell's could be used). Further, the more solid stress-strain theory of Murnaghan could be used [12].

Consideration of the mathematics of eqs. (3) shows that uniform 7-port blocks (one port being electrical, six mechanical) of material could be most precisely used with interconnections appropriately made to incorporate boundary conditions; such a development appears straightforward.

#### 2. Improved basic devices

By placing output transducers, connected electrically in parallel, on both sides (right and left) of the input transducer the total launched wave can be detected [13, p. 170], other configurations are

worth developing for improved operation, such as for obtaining excitation to avoid bulk waves. For example, the use of implanted electrodes, even deep surface electrodes, properly spaced to avoid blockage of the wave should be considered. The challenge seems to be in the spacing.

# 3. Nonlinearities

- a) Use of third order strain energy.
- [14] gives the complete set of relationships for all crystal class. The constants for important materials do not seem to be completely available, and a search for which classes give the most useful nonlinear (square-law) effects would prove invaluable. The square-law algebra [15] is directly applicable for developing a general theory.
- b) Possibility of parametric amplification.

This is actually mentioned in [16, p. 12] and may rest on means of avoiding bulk waves, as mentioned in problem 2 above. But it should be investigated in detail.

c) Mixing, harmonic, and subharmonic generation.

The first two topics are outlined in the literature [17]
[18] [19] for squaring systems from which a design theory should be possible. Subharmonics seem quite possible but not yet considered.

Magnetic field SAW's.

Development of possibility of magnetic field SAW's.

[20, p. 48] gives the 90 magnetic point groups, an investigation of which should lead to a nice theory for magnetic field SAW's.

#### 5. Applications

Because of their economics the future of the SAW field may well depend upon finding applications which can be filled uniquely, one such may be as a microprobe for physiological measurements, such as for ultrasonic mapping in real-time of single cells.

#### V. Summary

The SAW field is a rapidly developing one which practically, and even moreso theoretically, is extremely fascinating and challenging. Here in order to understand some of the more intriguing open problems the basics of the ideas behind SAW device operation and characterization have been presented. For sure the literature does go well beyond these ideas, and any inference that it does not would be mistaken; indeed the two books [21][22] and the three good survey articles [23] [24][25[ develop other ideas and give further referencing. Still once the developments of this paper are understood the whole fascinating field should be open, and hopefully attractive, to mathematically oriented systems theorists as well as to circuit designers.

Epilogue: "And of the tears that over Poland flood
On their cheeks the tears shall leave no stain
Where happiness may be for every Pole
As holy as a first love and as pure."

[0, pp. 284-255]

# References

- [0] A. Mickiewicz, "Pan Tadeusz", Everyman's Library, Dent, London, 1966.
- [1] J. Tsantes, "Surface-Acoustic-Wave Devices Replace Conventional Filter Components", EDN, November 20, 1979, pp. 69-78.
- [2] J. Sapriel, "Acousto-Optics", John Wiley & Sons, Chichester, New York, 1972.
- [3] B. A. Auld, "Acoustic Fields and Waves in Solids", Volume I, John Wiley & Sons, New York, 1973.
- [4] W. P. Mason, "Electromechanical Transducers and Wave Filters," Second Edition, D. VanNostrand Co., Princeton (New Jersey), 1948.
- [5] R. F. Milsom and M. Redwood, "Interdigital Piezoelectric Rayleigh-Wave Transducer: An Improved Equivalent Circuit", Electronics Letters, Vol. 7, No. 9, May 6, 1971, pp. 217-218.
- [6] L. A. Coldren and R. L. Rosenberg, "Scattering Matrix Approach to SAW Resonators", 1976 Ultrasonics Symposium Proceedings, IEEE Cat. #76- CH1120-5SU, 1976, pp. 266-271.
- [7] M. Hribsek and R. Newcomb, "The Scattering Matrix of a SAW Section" Proceedings of the Fourth International Symposium on Network Theory, Ljubljana, September 1979, pp. 293-298.
- [8] N. C. Debnath, R. C. Ajmera, and R. W. Newcomb, "Scattering Matrix for N-Section Crossed-Field Model of SAW Transducers", Proceedings of the 12th annual Southeastern Symposium on System Theory, Virginia Beach, May 1980, pp. 80-86.
- [9] R. W. Newcomb, "Linear Multiport Synthesis", McGraw-Hill Book Co., New Yor, 1966.
- [10] M. S. Livśic, "Operators, Oscillations, Waves (open systems)", Translations of Mathematical Monographs, Vol. 34, American Mathematical Society, Providence (Rhode Island), 1973.
- [11] N. K. Bose, Editor, "Multidimensional Systems: Theory and Applications", TEEE Press, New York, 1979.
- [12] F. D. Murnaghan, "Finite Deformation of an Elastic Solid", Dover Publications, New York, 1967.
- [13] C. S. Hartmann, D-L. T. Bell, Jr., and R. C. Rosenfeld, "Impulse Model Design of Acoustic Surface-Wave Filters", IEEE Transactions on Microwave Theory and Techniques, April 1973, pp. 162-175.

- [14] R. F. S. Hearman, "Third-Order Elastic Coefficients", Acta Crystallographics, Vol. 6, part 4, April 10, 1953, pp. 331-340.
- [15] R. W. Newcomb, "Nonlinear Differential Systems: A Canonic, Multivariable, Theory", Proceedings of the IEEE, Vol. 65, No. 6, June 1977, pp. 930-935.
- [16] E. G. H. Lean, C. G. Powell and L. Kuhn, "Acoustic Surface Wave Mixing on α-Quartz", <u>Applied Physics Letters</u>, Vol. 15, No. 1, July 1, 1969.
- [17] P. H. Carr, "Second-Harmonic Generation of Microwave Phonons in Quartz and Sapphire", Physical Review, Vol. 169, No. 3, May 15, 1968, pp. 718-729.
- [18] D. H. McMahon, "Acoustic Second-Harmonic Generation in Piezo-electric Crystals", The Journal of the Acoustical Society of America, Vol. 44, No. 4, October 1968, pp. 1007-1013.
- [19] P. O. Løpen, "Second-Harmonic Generation in an Elastic Surface Wave in  $\alpha$ -Quartz", Journal of Applied Physics, Vol. 39, No. 12, November 1968, pp. 5400-5404.
- [20] S. Bhagavantam, "Crystal Symmetry and Physical Properties", Academic Press, London, 1966.
- [21] H. Matthews, Editor, "Surface Wave Filters; Design, Construction, Use", John Wiley & Sons, New York, 1977.
- [22] A. A. Oliner, Editor, "Acoustic Surface Waves", Springer-Verlag, Berlin, 1978.
- [23] K. Dransfeld and E. Salzman, "Excitation, Detection, and Attenuation of High-Frequency Elastic Surface Waves," in Physical Acoustics, Edited by W. P. Mason, Academic Press, New York, 1970, pp. 219-272.
- [24] H. Sabine and P. H. Cole, "Acoustic Surface Wave Devices: A Survey", Proceedings of the Institution of Radio and Electronics Engineers, Australia, Vol. 32, No. 12, December 1971, pp. 445-458.
- [25] A. J. Slobodnik, Jr. T. L. Szabo, and K. R. Laker, "Miniature Surface-Acoustic-Wave Filters", <u>Proceedings of the IEEE</u>, Vol. 67, No. 1, January 1979, pp. 129-146.

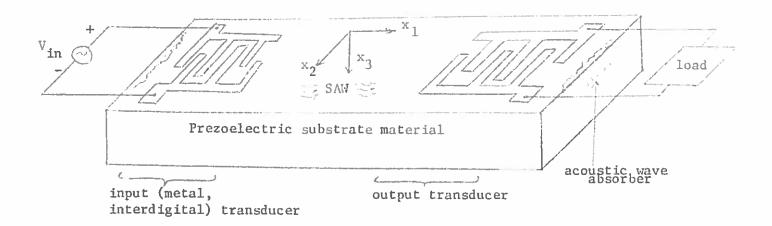


Figure 1
The Basic SAW Device

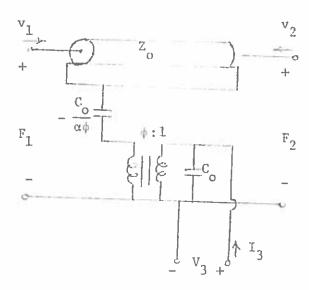


Figure 2
SAW Section Equivalent Circuit

# ECCTD'80

# 1980 EUROPEAN CONFERENCE ON CIRCUIT THEORY AND DESIGN

**Proceedings** 

Vol.

Warsaw, Poland. September 2-5, 1980