A HYSTERESIS CIRCUIT SEEN THROUGH SEMI-STATE EQUATIONS

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Abstract

Semi-state equations are set up for an hysteresis amplifier showing that multiple valued input-output curves can be described by single-valued semi-state operators. Reduction, experimental verification, and interpretation in terms of bifurcation theory are given.

1. INTRODUCTION

A set of basic equations called semi-state equations (similar to state equations) were recently introduced as an almost universal description of nonlinear circuits [1,2], where the semi-state may be chosen as tree branch voltages and link branch currents in a circuit graph. The semi-state equations have several advantages: They can describe circuits [3, p. 404] for which state equation descriptions do not exist; are obtained directly and simply from the circuit graph; contain state equations as a special case, and allow multi-valued input-output relationships to be represented by single-valued operators.

In this paper we illustrate the use of semi-state equations in handling multi-valued input-output relationships by using semi-state equations to describe through single-valued equations a CMOS circuit which displays hysteresis in its voltage transfer function. The obtained semi-state equations are then reduced by eliminating some of the semi-state variables. The resulting solutions are shown to exhibit hysteresis. These results are then interpreted through bifurcation theory [4].

Hysteresis is of course important in a number of applications, the particular circuit we investigate here finding use as a Schmitt trigger in contact debouncing [5, p. 222]. But details on this circuit do not appear to have been explored in any depth in the literature.

The presentation is organized as follows. In section 2 we give the general form of the semi-state equations for describing general nonlinear circuits and proceed to derive the semi-state equations for describing the CMOS hysteresis circuit. In section 3 the bifurcating solutions of the hysteretic transfer function curve are obtained, and compared with the experimentally measured curve. Finally a discussion of the results follows in section 4.

2. SEMI-STATE EQUATIONS FOR HYSTERESIS CIRCUIT

In [2] it is shown that (almost) every finite circuit can be described by semi-state equations which take the general canonical form

$$A\dot{x}+B(x,t) = Du \tag{1a}$$

$$y = Fx \tag{1b}$$

where u = input vector; y = output vector; A,D,F are linear constant operators, possibly singular; B(·,·) is an operator which is possibly nonlinear; and x = semi-state which can be chosen as the tree branch voltages and link-branch currents. Other choices of the semi-state are possible. Also in some cases, circuit equivalences may have to be used to obtain the equations in the canonical form. Here we derive the semi-state equations for a CMOS hysteresis circuit, selecting the tree branch voltages and the link branch currents as the semi-state variables.

The CMOS hysteresis circuit is formed by adding a feedback resistor and an input resistor to the CMOS non-inverting amplifier circuit of the type used in digital circuits. The CMOS non-inverting amplifier circuit [5, p. 170] is shown in Figure 1(a), and its experimentally obtained nonlinear voltage transfer characteristic is shown in Figure 1(b). When the output of the CMOS noninverting amplifier is resistively coupled to the input, and an input resistor is added, the resulting CMOS circuit shown in Figure 2(a) displays hysteresis [5,p.171] in its voltage transfer function as seen in the experimental curves shown in Figure 2(b).

To derive the semi-state equations for the hysteresis circuit shown in Figure 2(a), we model the CMOS amplifier with a voltage controlled voltage source, $v_2 = f(v_1)$ where $f(\cdot)$ is a nonlinear function representing the amplifier characteristic curve of Figure 1(b). The equivalent circuit of the hysteresis circuit and the corresponding circuit graph are shown in Figure 3, where the input Norton's equivalent current source and the input resistor are treated as one branch, and the tree is formed from branches 1 and 2. We select the tree branch voltage and the link branch currents as the semi-state variables. By using the standard cutset and tie set matrices, or by straight forward analysis of the circuit graph using Kirchhoff's current and voltage laws and the laws of the elements, we write the circuit equations. Then we eliminate all other variables except the semi-state variables from the circuit equations, to obtain the semi-state equations which we express in the matrix form:

$$\begin{bmatrix} C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & R_i & 0 \\ -1 & 1 & 0 & R_f \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -f(v_1) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u \qquad (2a)$$

$$y = [0,1,0,0]x \qquad (2b)$$

where the semi-state is $x = \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{bmatrix}$ and input is $u = [v_i]$; $\dot{x}$ is the time derivative of x. Here, practically C would be a parasitic input capacitance which is included to give dynamic behavior.

Note that all the terms in the above equations are single-valued. These semi-state equations are reduced in the next section. The equilibrium equations derived from the reduced equations are also solved there and shown to exhibit hysteresis.

3. BIFURCATING SOLUTIONS

The semi-state equations (2) derived above, completely describe the circuit shown in Figure 2(a) for given (consistent) initial conditions. Each term in the equations is single-valued. However we show here that with the appropriate choice of parameters, the solutions to the equilibrium equations, obtained by setting to zero the derivative term in equation (2) can be multiple-valued and display hystersis.

First we reduce equations (2a) by eliminating i_3 and i_4 to get the two (still semi-state) equations:

$$c\dot{v}_1 + \left(\frac{1}{R_f} + \frac{1}{R_i}\right) v_1 - \frac{v_i}{R_i} - \frac{v_2}{R_f} = 0 \qquad (3a)$$

$$v_2 - f(v_1) = 0 \qquad (3b)$$

By setting C=0, for the nondynamical case, or $\dot{v}_1=0$ for (constant, steady-state) equilibrium solutions we rewrite equations (3) in the form

$$v_2 = (1+\frac{R_f}{R_i})v_1 - \frac{R_f}{R_i} v_i \quad (4a)$$

$$v_2 = f(v_1) \quad (4b)$$

Equations (4) can be solved graphically by drawing the linear loadline of eq. (4a) (with v_i as a parameter) on the nonlinear curve shown in Figure 1(b), and represented by the functional equation 4(b). Here we use the loadline graphical method [6] to illustrate the hysteretic qualitative behavior of the solution and then solve the equations analytically for the quantitative solution.

Figure 4(a) shows the loadline drawn on the non-inverting amplifier curve with the input voltage v_i as the parameter. As v_i is varied from a sufficiently negative to positive value the loadline shifts horizontally, for example, from loadline a to loadline e on the figure. The loadline will intersect the curve at one, two, or three points depending on the value of v_i and the slope of the loadline. We observe that if the slope of the loadline is either negative or greater than the maximum slope of the curve (which occurs at the point of inflection of the amplifier curve), the loadline intersects the amplifier curve in one point only for all values of the input voltage v_i, and there would result a unique (non-hysteretic) solution to equations (4). If the slope of the loadline is positive but less than the maximum slope of the amplifier curve there is a range of input values v_i which give multiple solutions, which we observe from the diagram are the values of v_i which give loadlines between the two bifurcation loadlines b and d. The bifurcation loadlines are the loadlines which are tangential to the curve at one point and intersect the curve at one or more other points. At the bifurcation loadline such as b or d the unique solutions such as 1 or 9 bifurcate into two solutions such as 2 with 3 or 7 with 8 and then into three solutions such as 4,5,6. By plotting the solution values of v_2 versus v_i we obtain the hysteresis curve shown in Figure 4(b) for the two stable solutions. The solutions follow the curve as shown by arrows in Figure 4(b) in accordance with models in catastrophe theory [7,p. 70]. We now quantitatively determine the conditions for hysteresis to occur, and obtain the bifurcation loadline as follows: By curve fitting techniques we can approximate the voltage transfer function of the type shown in Figure 1(b) very closely with the hyperbolic tangent function:

$$v_2=f(v_1)=\frac{V_{DD}+V_{SS}}{2}\tanh[K(v_1-V_x)]+V_y \quad (5)$$

where (V_x,V_y) are the coordinates of the point of inflection of the curve, and K is an empirical constant chosen for a close fit. Using eq. (5) we can now solve equations (4) analytically. The maximum slope of the amplifier curve occurs at the point of inflection (V_x,V_y) to the curve. Differentiating equation (5) we get

$$\frac{\partial f}{\partial v_1} = \frac{\frac{1}{2}K(V_{DD}+V_{SS})}{[\cosh[K(v_1-v_x)]]^2} \quad (6)$$

Evaluating this at the point $v_1=V_x, v_2=V_y$, we obtain the maximum slope m of the amplifier curve as:

$$m = \frac{\partial f}{\partial v_1}\Big|_{(V_x,V_y)} = \frac{1}{2}K(V_{DD}+V_{SS}) \quad (7)$$

A necessary condition for hysteresis to occur is therefore:

$$(1+\frac{R_f}{R_i}) < m = \frac{1}{2}K(V_{DD}+V_{SS}) \quad (8)$$

We obtain the bifurcation points by equating the gradient of the loadline equation (4a) to the gradient of the amplifier curve, equation (6), and solving the resulting equation, which gives two values for v_1.

$$v_{1b\pm} = V_x \pm \frac{1}{2K} \ln(h-1+\sqrt{h^2-2h});\; h = \left(\frac{KR_i}{R_i+R_f}\right)(V_{DD}+V_{SS}) \quad (9a)$$

In the limit of a step-function transfer characteristic, $K\to\infty$, the bifurcation points are thus seen to be

$$(V_{1b\pm}, V_{2b\pm}) = (V_x, V_y \pm \tfrac{1}{2}[V_{DD}+V_{SS}]) \quad (9b)$$

with equation (4) giving the bifurcating input values

$$V_{ib\pm} = (1+\frac{R_i}{R_f})V_x - \frac{R_i}{R_f}(V_y \pm \tfrac{1}{2}[V_{DD}+V_{SS}]) \quad (9c)$$

For reasonably large K, the width of the hysteresis curve is then [5,p. 222]

$$V_H = V_{ib_-} - V_{ib_+} \frac{R_i}{R_f} [V_{DD}+V_{SS}] \quad (10)$$

Experimentally the results were checked using Motorola's CMOS package MC14007UB with V_{DD}=3v, V_{SS}=3v, for which K=20 gave a 3% maximum error in fitting equation (5) to the measured amplifier gain curve (with V_y=0, V_x=0.55) of Figure 1(b). For R_i= 100KΩ and R_f=100KΩ, equation (10) gives V_H=6v, while its measured value is V_{Hmeas}= 6.4, as seen from Figure 2(b).

4. DISCUSSION

In this paper we have shown that the semi-state equations can be used to describe a hysteresis circuit in such a way that the multi-valued input-output relationship of the hysteresis circuit is represented by single-valued operators in the semi-state equations. The semi-state equations were solved graphically for the qualitative behavior. An analytical solution of the equations was also obtained by fitting the nonlinear transfer curve (Figure 1(b)) of the amplifier by the hyperbolic tangent function. The hyperbolic tangent function provides a very accurate description of the curve when the value of the empirical constant K is well chosen large in equation (5).

In the above we have treated only the cases of nondynamics, C=0, or equilibrium solutions, $\dot{v}_1$=0 (at equation (3)). Should C be nonzero, as we believe is always the case with C representing parasitic capacitance, this means that the treatment is valid only for slowly varying signals. Since the hysteresis curve of Figure 2(b) was recorded for sinusoidal inputs, the highest frequency of validity is important, this being determined to be about 400Hz in our experiments. If the resistors R_i=R_f are lowered from 100KΩ to 20KΩ the upper frequency for equilibrium solutions moves up (as the time constants lower) to about 2.5KHz while lowering further to 500Ω moves the corresponding frequency to about 10KHz. The value of R_i=R_f=100KΩ was chosen to give a sharp (we may call "binary") hysteresis curve. Since the responses are actually single-valued for a given input, the importance of dynamics should be noted. Following the technique used at equation (9) more detailed calculations for V_H can be given; these and other aspects we hope to treat further.

5. REFERENCES

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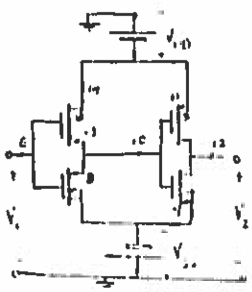

Fig. 1a) CMOS Noninverting Amplifier (with MC14007 package pin numbers)

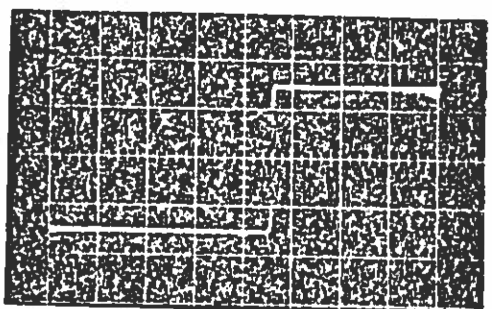

Fig. 1b) Typical Measured Noninverting Amplifier Characteristic
Horizontal = v_1, 1 volt/div.
Vertical = v_2, 2 volt/div.
(0,0) at center graticule point.

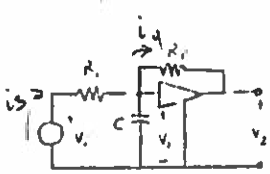

Fig. 2a) CMOS Hysteresis Circuit

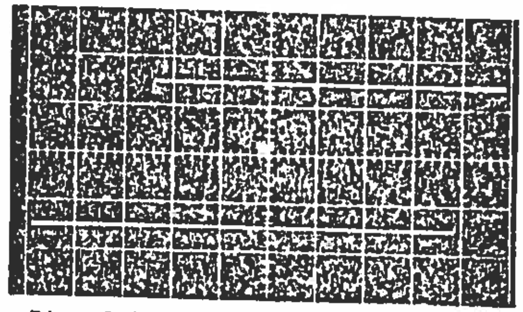

Fig. 2b) Equilibrium Voltage Transfer Characteristic
Horizontal = v_1, 1 volt/ div.
Vertical = v_2, 2 volt/ div.
(0,0) at center graticule point
$R_i = R_f = 100K\Omega$.

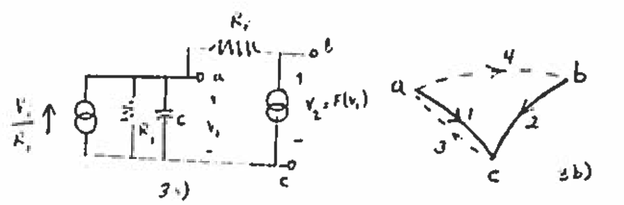

Fig. 3a) Equivalent (nonlinear) circuit of the CMOS Amplifier

Fig. 3b) Circuit Graph (tree = branches 1 & 2)

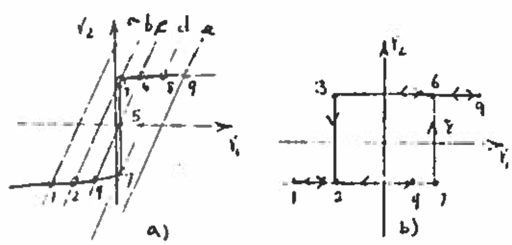

Fig. 4 Tracing Equilibrium Solutions (showing bifurcation points 3 & 7) of Equation (2) with v_1 as Parameters.

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