Green's function is defined by separate expressions for $z > z'$ and $z < z'$ is irrelevant to the choice of principal volume one is allowed to use in (11) because (11) was derived not from (2) but from (1) where V_J has no principal volume associated with it. If one simply elects to exclude the singularity in (1) by a principal volume, the delta-function term in the Green's function expression (10) will not contribute, yielding bogus results regardless of the shape of the chosen principal volume. On the other hand, if one simply elects to exclude the singularity in the integral in (11) by a principal volume, one does not know (except through the rigorously derived results of the above paper) that a delta function at $\bar{R}' = \bar{R}$ was not overlooked or that the shape of the principal volume must be a pillbox.

Of course, as the above paper proves, $\bar{\bar{S}}^{\pm}$ can be used with an arbitrarily shaped principal volume by merely inserting it under the integral of (2) in conjunction with the source dyadic $\bar{\bar{L}}$ corresponding to the chosen principal volume. To illustrate this point clearly in Section IV-A of the above paper, I purposely chose a somewhat unorthodox "needle-shaped" (thin-cylinder along the z-axis), principal volume, resulting not in the equation (12) of Tai's letter but in the equation

$$\bar{E}(\bar{R}) = i\omega\mu_0 \lim_{\delta \to 0} \int_{V_J - V_{\text{needle}}} \bar{\bar{S}}^{\pm} \cdot \bar{J}\, dv' + \frac{\bar{J}_{\text{transverse}(\text{to}\, z)}}{2i\omega\epsilon_0}.$$

7) Tai takes exception to a statement in footnote 1 of the above paper that there is disagreement in the source region between the results in a *cavity* obtained by Tai and Rozenfeld [17c] of the above paper and that of Rahmat-Samii [19] of the above paper. Let me emphasize that the disagreement occurs only at the source point of the electric dyadic Green's function. Away from the source region there is no question that both expressions should be identical. And indeed, a manipulation of the summations representing the electric dyadic Green's function of Tai and Rozenfeld and that of Rahmat-Samii show that they are identical for $\bar{R}' \neq \bar{R}$. At the source point $\bar{R}' = \bar{R}$, however, a similar manipulation of the summations is not strictly valid because the summations are not only singular but may contain dyadic delta functions at $\bar{R}' = \bar{R}$. Thus the only valid way to compare the two representations at the source point is to substitute them into the (1) for which they were both derived, integrate the explicit dyadic delta function ($-\hat{z}\hat{z}\, \delta(\bar{R}' - \bar{R})/k^2$ in Tai and Rozenfeld's work; $-\bar{\bar{I}}/k^2$ in Rahmat-Samii's work), and evaluate the remaining integral. As discussed in the above paper and comments 3) and 4) above this remaining integral is ill defined unless a principal volume is chosen to exclude the singularity. Moreover, when a specific principal volume is chosen, the resulting electric fields from the representation of Tai and Rozenfeld and from that of Rahmat-Samii are different in the source region. That is, in the form and context which they were originally presented, these two different expressions for the same electric dyadic Green's function in a cavity yield, at best, different results for the electric field in the source region.

Let me conclude this reply to Tai's letter by repeating one of the conclusions in the above paper (see especially Section III-D). The familiar delta function method should not be discarded as a means of finding the electric dyadic Green's function, nor is the $\hat{z}\hat{z}$ source dyadic derived by Tai and others incorrect, provided it is used in the context of (2) with a pillbox (normal to z) principal volume. Nor is any other source dyadic L which can be associated with a specific principal volume incorrect, as long as it is used in conjunction with its associated principal volume; e.g., the $\hat{n}\hat{n}$ source dyadic obtained by Tai [6] gives correct electric fields everywhere for a pillbox principal volume normal to the n direction. However, the delta-function technique as it has been applied in the past to derive (1) does not provide, implicitly or explicitly, this necessary principal volume information either for the $\hat{z}\hat{z}$ source dyadic of a pillbox or for the general $\bar{\bar{L}}$ source dyadic of an arbitrary principal volume.

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Further Comments on "Integrable Insensitive Subaudio-Sine-Wave Generator Using DVCVS/DVCCS"

ROBERT A. PEASE

I now see that in the above letter[1] Nandi has a circuit for a sine-wave oscillator which can be tuned from 60 Hz all the way down to 10 Hz, although I would hesitate to claim that as "subaudio." But Nandi has not chosen to tell us what is the material or the lossiness of the capacitor he has chosen to operate at 10 Hz. He has not indicated that he considers this capacitor "integrable." He has not shown how or whether the circuit which gave the waveform in Fig. 1. does start oscillating by itself, nor has he indicated what he would do if the oscillator's amplitude failed to start promptly. Again, it is not what he says, but what he does not say, that causes concern.

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Hysteretic System for Neural-Type Circuits

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***Abstract*—A first-order system is introduced which through the incorporation of hysteresis yields results of the type recently found in neural-type circuits. Analog computer experiments verify the properties of the system.**

I. Introduction

Neural-type systems are systems that possess selected properties of neural physiological systems. Previously an MOS neural circuit [1] was shown [2] to possess an important property of nerve axons in that repetitive action-potential like pulses result for inputs held constant above a threshold. Experiments have verified that this repetitive response is a consequence of a hysteresis observed in the CMOS transistor pair feedback used [3, p. 53]. Here we abstract this concept by presenting a very simple first-order system with binary hysteresis feedback which exhibits similar properties.

II. The System

We will introduce a state-variable description of the system, equation (1) below. For this we note that since hysteresis incorporates a

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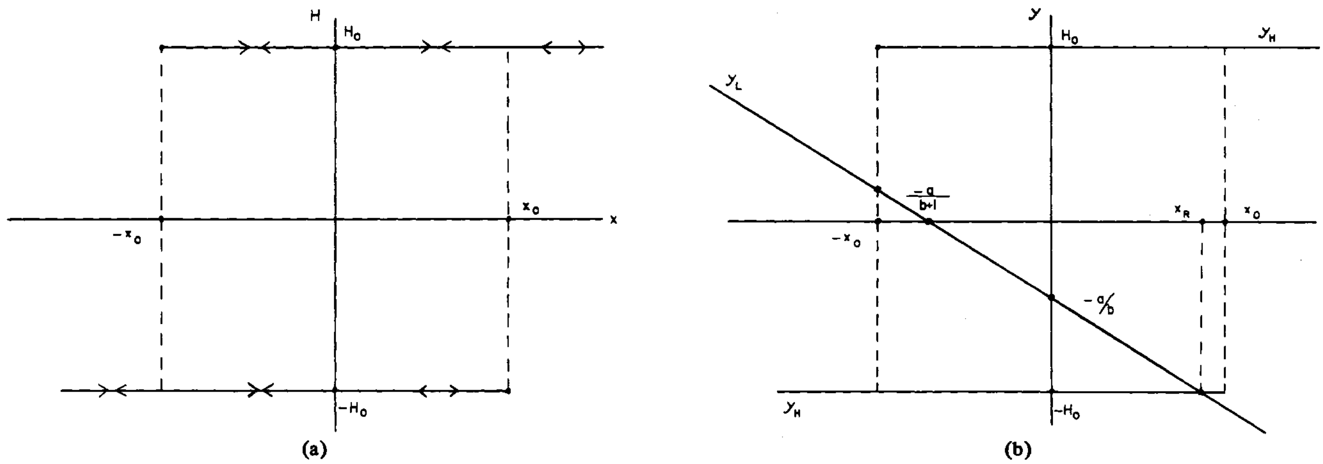

Fig. 1. Hysteresis with load. (a) Binary hysteresis. (b) Basic load line on hysteresis showing resting state x_R.

type of memory its value at a given instant depends upon all past values. Thus, if we let $x = x(\cdot)$ denote the state function, we have need for its value at a given instant t, $x(t)$, and its values up until t, denoted as the function $x_t = x_t(\cdot)$. With this notation in hand we take as our system that described by the first order nonlinear state-variable equations

$$\dot{x} = -(b+1)x - a - bH(x_t) + bu \tag{1a}$$

$$y = x \tag{1b}$$

where a and $b > 0$ are constants, u is the input, y is the output, and H is the binary hysteresis shown in Fig. 1(a) and described by

$$H(x_t) = \begin{cases} H_0, & \text{if } x(t) \geqslant x_0 \\ -H_0, & \text{if } x(t) \leqslant -x_0 \\ \begin{Bmatrix} H_0 \text{ if } H(x_{t\_}) = H_0 \\ -H_0 \text{ if } H(x_{t\_}) = -H_0 \end{Bmatrix}, & \text{if } -x_0 < x(t) < x_0 \end{cases} \tag{2}$$

where H_0 and x_0 are positive constants (and $t\_$ is t approached from the left [= instant before t]).

Before discussing responses of the system, we establish a convenient resting state. This is found, as the zero input equilibrium "state," by setting $u = \dot{x} = 0$ in (1a), in which case

$$-\left(1 + \frac{1}{b}\right)x - \frac{a}{b} = H(x_t). \tag{3}$$

Thus, the equilibrium "state" is found as the intersection of the straight line $y_L = -(1 + 1/b)x - (a/b)$ and the hysteresis curve $y_H = H$. As seen from Fig. 1(b), where a single valued intersection is shown, this intersection can be double valued or "valueless." For a single valued resting state x_R of the type shown in Fig. 1(b), geometry requires $y_L(x_0) < -H_0 < y_L(-x_0) < H_0$ or, $-(b+1)x_0 - a < -bH_0 < (b+1)x_0 - a < bH_0$; from the left and the right inequalities $-a < (b+1)x_0 - bH_0 \triangleq a_1$ and $-a < -(b+1)x_0 + bH_0 = -a_1$. For design purposes we will choose b such that $a_1 < 0$ in which case

$$b > \frac{x_0}{H_0 - x_0}, \quad b > 0 \text{ and } a > bH_0 - (b+1)x_0 > 0 \tag{4a}$$

are required. The resting "state" x_R resulting is

$$x_R = \frac{1}{b+1}[bH_0 - a]. \tag{4b}$$

III. Principle of Operation

Operation of the system can be seen by referring to (1) and Fig. 1(b). In the absence of an input the system rests in the constant state x_R, as found above from the intersection of the load line y_L with the hysteresis curve H. If then a positive input u is applied such as to force $x > x_0$, then a jump from the bottom hysteresis line ($-H_0$) to the top ($+H_0$) will immediately occur; the needed value of u is the threshold of excitation, u_{th}, and found from (1a) as $\dot{x} = 0 = -(b+1)x_0 - a - bH(x_R) + bu_{th}$ or

$$u_{th} = \frac{(b+1)}{b}[x_0 - x_R]. \tag{5}$$

The rise of x above x_0 causes an immediate jump in H from $-H_0$ to $+H_0$ forcing $\dot{x}$ quite negative, by (1a), in turn lowering x along the H_0 portion of the hysteresis. Under control of $\dot{x}$ the hysteresis curve again participates in a jump from $+H_0$ to $-H_0$ as x continues negative; $\dot{x}$ consequently becomes quite positive and the hysteresis curve is traced on its $-H_0$ portion as x continues positively beyond x_R. If in the meantime the input is turned off the system returns to its resting state while if the input remains present various types of repetition occur depending upon the size and nature of the input. Further if a positive input pulse occurs during the middle of the cycle just described there are small effects, generally speeding up the cycle, such that a refractory period is noted until the derivative of x returns close enough to zero as determined by the initial input.

IV. Experimental Results

Fig. 2 shows the analog computer simulation used to experimentally study the behavior of the system. An EAI581 Analog Computer was programmed as shown with x scaled to a possible peak value of 5 and $\dot{x}$ scaled to a peak of 10. The values of x_0, H_0, a, and b are set by potentiometers with the input u controlled by potentiometer 20 when a step value of u is desired. In the case of short duration inputs, a pulse generator (HP8002A) is directly connected to the amplifiers (8 and 19) fed by u. Rather than switch between $\pm H_0$, it was more convenient to switch between f_1 and f_2 where

$$f_1 = x - H_0 - u, \; f_2 = x + H_0 - u \tag{6}$$

this switching being accomplished through the function relay which is controlled by comparators 00 and 04 [4].

Fig. 3 shows results as displayed on an x-y recorder for the values $a = 2.1, b = 1.5, x_0 = 1, H_0 = 3$. The arrows in Figs. 3(a), 3(b), and 3(c) indicate the start of the external input u. Fig. 3(a) shows typical subthreshold and above threshold responses to a square input pulse of width 1.6 s at the point K in Fig. 2 ($u_{th} = 0.067$ V from (5)). Fig. 3(b) shows the x output. Note that all perturbations do come to rest at the resting potential ($x_R = 0.96$ V from 4(b)). Fig. 3(c) shows the refractory nature of the system since repeated application of the input pulse does not trigger the system until it has sufficiently recovered. Finally, in Fig. 3(d) trains of output pulses are seen to be generated in response to a constant input of sufficient magnitude.

V. Conclusion

The proposed system has the basic properties of interest in neural-type electronic circuits: a threshold of input u_{th}, for output triggering, an action-potential type of response, a refractory period, and repetitive outputs for constant inputs. The nature of the hysteresis lends itself to CMOS realization for integrable structures [3] while the simple first-order form of the describing equations lends itself advantageously to the study of large interconnected systems [5].

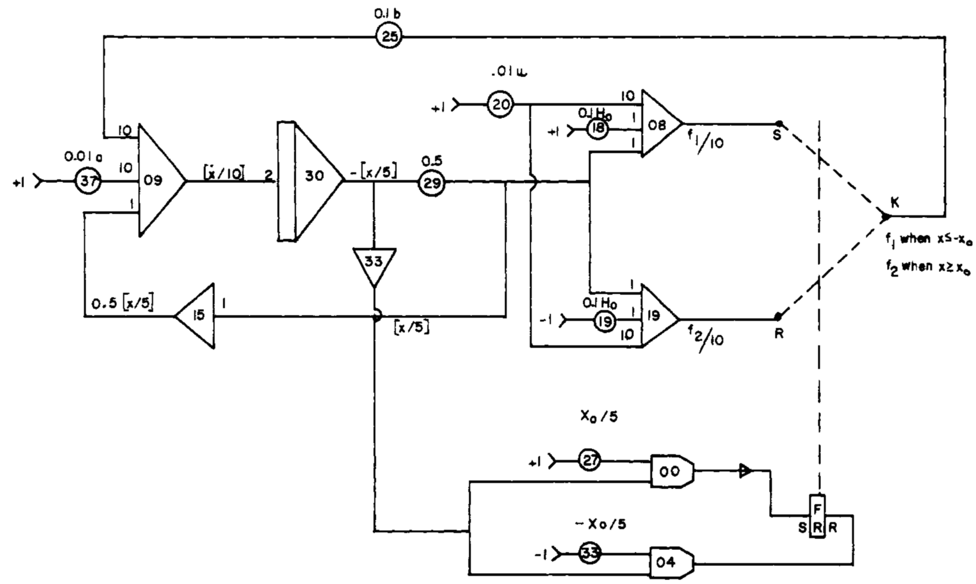

Fig. 2. Analog computer setup for (1).

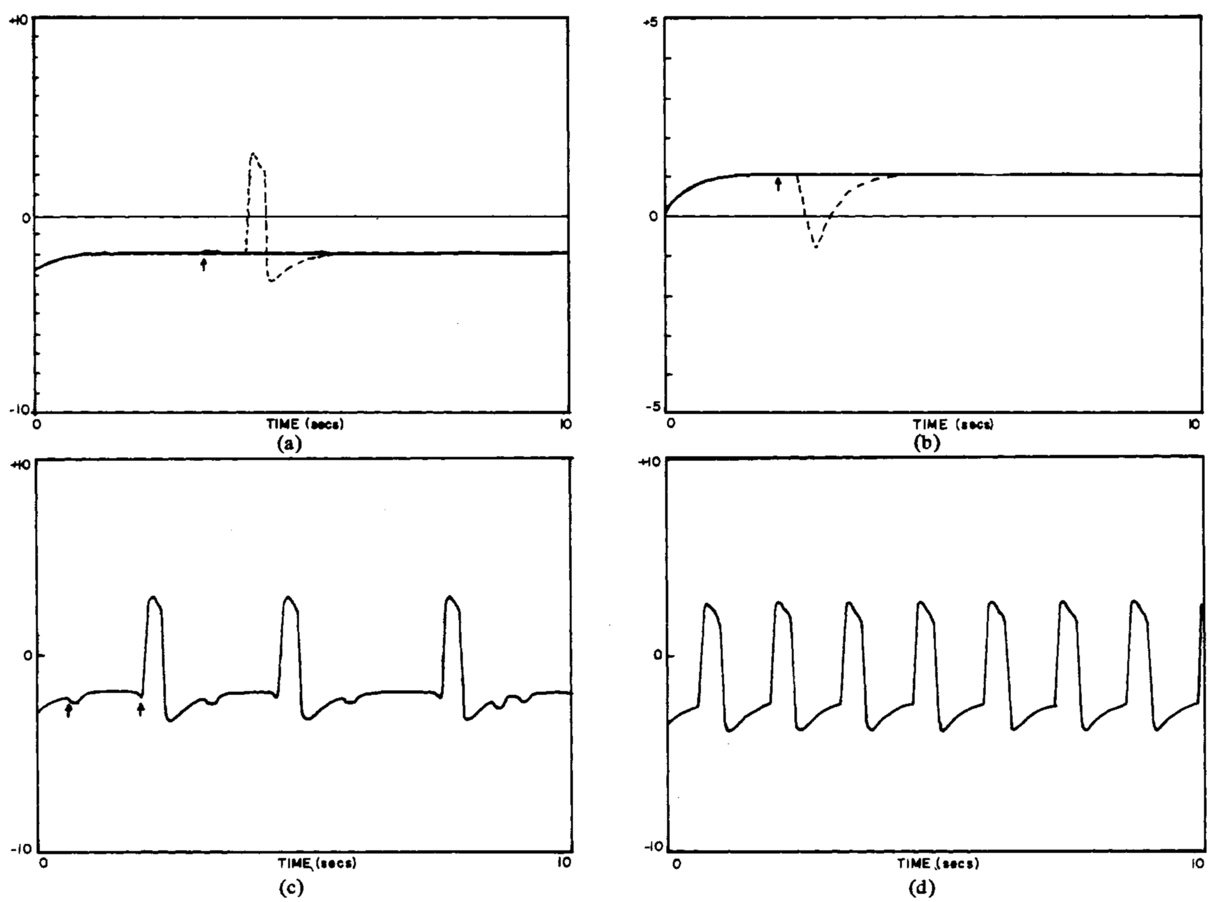

Fig. 3. Typical responses. (a) Subthreshold (solid) and above threshold (dashed) response to a square pulse input u of height 0.060 V and 0.067 V, respectively (pulse width 1.6 s), output of point K. (b) Subthreshold (solid) and above threshold (dashed) response to a square pulse input u of height 0.060 V and 0.067 V, respectively (pulse width 1.6 s), output of x. (c) Refractory response to a pulse input of height 0.428 V and pulse width 0.165 s. (d) A train of output pulses for a step input u of height 0.5 V.

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