SCATTERING MATRIX FOR N-SECTION
CROSSED - FIELD MODEL OF SAW TRANSDUCERS

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Abstract
The scattering matrix of N-equal sections of a crossed - field model for a SAW device is calculated with the aid of chain and admittance matrices.

I. Introduction

In the past decade, due to successful applications to the design of different kinds of filters and delay lines considerable attention has been given to the phenomena of surface acoustic waves (SAW) created on piezoelectric substrates. It has been shown that a circuit model representation can be the key for the development and design of SAW devices. Especially, a frequency domain equivalent circuit model representation[1,2,3] of a transducer has been found to be very successful[2] in characterizing the excitation and detection of surface waves. In these references the transfer characteristic and input admittance at the electric port of the interdigital transducer have been determined with the aid of chain and admittance matrices. Smith, et.al[3], and Gerard[4] have calculated reflection coefficients, but only at the synchronious frequency. Recently, Hribsek and Newcomb[5] have outlined the formalism for obtaining the scattering and transfer scattering matrices of a SAW device section as a function of frequency.

The SAW devices offer special interest to system theorists since they rely upon piezoelectrically generated mechanical waves in realizing practical devices for which the theory is anything but complete. And, of course, critical to the operation are the input and output transducers which can be considered as 3-ports having one electrical port and two mechanical (acoustic) ports (one for forward and one for backward acoustic wave motion). These transducers can be characterized by what have been called "in - line" and "crossed - field" models for the basic sections. This paper is concerned with the scattering matrix for N - sections of a SAW transducer described by the crossed-field model.

The primary objective is to develop a general mathematical technique by using the known admittance matrix of a three-port SAW transducer section. The starting point is the equivalent circuit of a basic one electrode section of the SAW transducer. This is then used to obtain the 3-port admittance matrix of a full (interdigital) transducer by first finding the matrix of one interdigital period and then applying a cascading formalism. The 3-port admittance matrix for an entire N - section transducer is found by connecting the N sections in cascade acoustically and in parallel electrically. Using these chain and admittance matrix results, the scattering matrix of N equal basic sections expressed in terms of the crossed - field model is determined.

II. Admittance Matrix for a Three-port N - Section Network

We consider a three-port section with two identical mechanical ports (of variables $F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$ = force applied, $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ = velocity of material) and one electrical port (of variables V_3 = voltage, I_3 = current applied to the interdigital electrodes), as shown in Fig. 1.

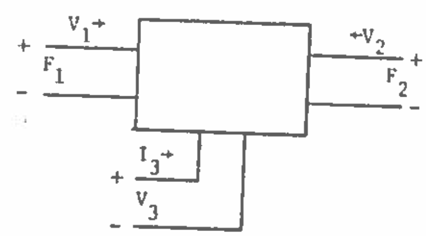

Figure 1. Block-diagram of a three-port Model

Then

$$\begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{11} & y_{13} \\ y_{13} & y_{13} & y_{33} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ V_3 \end{bmatrix} \tag{1}$$

is taken as the admittance matrix y where $y_{22} = y_{11}$ and $y_{23} = y_{13}$ by symmetry. y is symmetric by assumed reciprocity.

We now connect N of these sections in cascade at the mechanical ports and in parallel, with every other one reversed, at the electrical ports as shown in Fig. 2.

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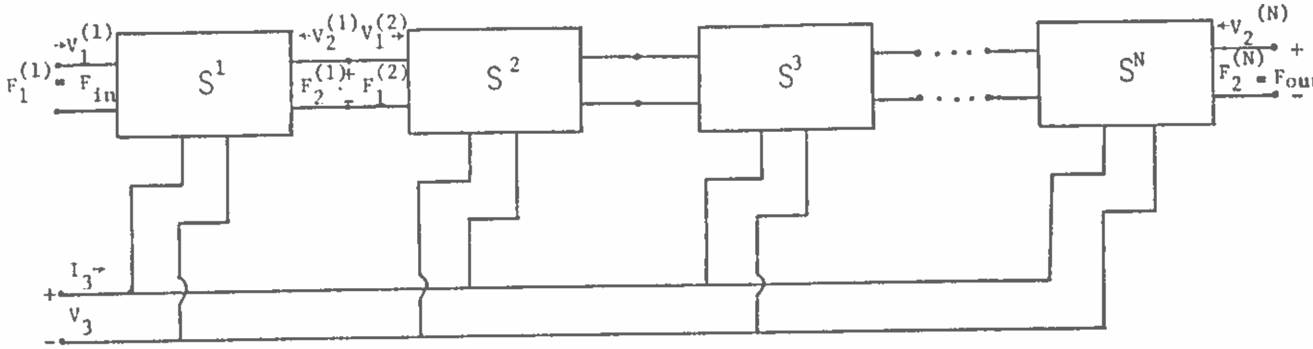

Figure 2. Cascaded one-electrode sections for a SAW transducer.

Thus for the i^{th} section we have

$$\begin{bmatrix} V_1^{(i)} \\ V_2^{(i)} \\ I_3^{(i)} \end{bmatrix} = \begin{bmatrix} y_{11}^{(i)} & y_{12}^{(i)} & y_{13}^{(i)} \\ y_{12}^{(i)} & y_{11}^{(i)} & y_{13}^{(i)} \\ y_{13}^{(i)} & y_{13}^{(i)} & y_{33}^{(i)} \end{bmatrix} \begin{bmatrix} F_1^{(i)} \\ F_2^{(i)} \\ V_3^{(i)} \end{bmatrix}, \tag{2}$$

while the interconnection laws are (for mechanical ports, and $i > 1$)

$$\left.\begin{aligned} F_1^{(i)} &= F_2^{(i-1)} \\ V_1^{(i)} &= -V_2^{(i-1)} \end{aligned}\right\} \tag{3}$$

where

$$\begin{aligned} F_1^{(1)} &= F_{in} = F_{left\ boundary}, \quad V_1^{(1)} = V_{in} \\ F_2^{(N)} &= F_{out} = F_{right\ boundary}, \quad V_2^{(N)} = V_{out}. \end{aligned} \tag{4}$$

Also for electrical ports with $i \geq 1$, the interconnection laws are

$$\left.\begin{aligned} V_3^{(i)} &= (-1)^{i-1} V_3 \\ I_3 &= \sum_{i=1}^{N} (-1)^{i-1} I_3^{(i)} \end{aligned}\right\} \tag{5}$$

In order to find the interconnected network 3-port description we convert to a reverse - chain matrix type of description. From equation (2) it follows that

$$\begin{bmatrix} F_2^{(i)} \\ -V_2^{(i)} \end{bmatrix} = \psi_R^{(i)} \begin{bmatrix} F_1^{(i)} \\ V_1^{(i)} \end{bmatrix} + A^{(i)} V_3 \tag{6}$$

where

$$\psi_R^{(i)} = -\frac{1}{y_{12}^{(i)}} \begin{bmatrix} y_{11}^{(i)} & -1 \\ (y_{12}^{(i)})^2 - (y_{11}^{(i)})^2 & y_{11}^{(i)} \end{bmatrix}, \tag{7}$$

$$A^{(i)} = \frac{(-1)^i y_{13}^{(i)}}{y_{12}^{(i)}} \begin{bmatrix} 1 \\ y_{12}^{(i)} - y_{11}^{(i)} \end{bmatrix} \tag{8}$$

and

$$I_3^{(i)} = -\frac{y_{13}^{(i)}}{y_{12}^{(i)}} [y_{11}^{(i)} - y_{12}^{(i)},\ 1] \begin{bmatrix} F_1^{(i)} \\ V_1^{(i)} \end{bmatrix} + \frac{(-1)^{i-1}}{y_{12}^{(i)}} [y_{33}^{(i)} y_{12}^{(i)} - (y_{13}^{(i)})^2] V_3, \tag{9}$$

Equation (6) is the key for this analysis. Using the mechanical interconnections, we obtain

$$\begin{bmatrix} F_2^{(1)} \\ -V_2^{(1)} \end{bmatrix} = \psi_R^{(1)} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + A^{(1)} V_3, \tag{10}$$

$$\begin{bmatrix} F_2^{(2)} \\ -V_2^{(2)} \end{bmatrix} = \psi_R^{(2)} \begin{bmatrix} F_1^{(2)} \\ V_1^{(2)} \end{bmatrix} + A^{(2)} V_3 \tag{11}$$

Or

$$\begin{bmatrix} F_2^{(2)} \\ -V_2^{(2)} \end{bmatrix} = \psi_R^{(2)} \psi_R^{(1)} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + \{\psi_R^{(2)} A^{(1)} + A^{(2)}\} V_3 \qquad (12)$$

In general for N Sections

$$\begin{bmatrix} F_{out} \\ -V_{out} \end{bmatrix} = \begin{bmatrix} F_2^{(N)} \\ -V_2^{(N)} \end{bmatrix} = \psi_R^{(N)} \begin{bmatrix} F_1^{(N)} \\ V_1^{(N)} \end{bmatrix} + A^{(N)} V_3$$

$$= \psi_R^{(N)} \psi_R^{(N-1)} \cdots\cdots \psi_R^{(2)} \psi_R^{(1)} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix}$$

$$+ \{A^{(N)} + \psi_R^{(N)} A^{(N-1)} + \cdots \psi_R^{(N)} \psi_R^{(N-1)} \cdots$$

$$\cdots \psi_R^{(3)} A^{(2)} + \psi_R^{(N)} \psi_R^{(N-1)} \cdots$$

$$\cdots \psi_R^{(3)} \psi_R^{(2)} A^{(1)}\} \qquad (13)$$

Thus for the entire cascaded system

$$\begin{bmatrix} F_{out} \\ -V_{out} \end{bmatrix} = \Psi_R \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + A V_3 \qquad (14)$$

where

$$\Psi_R = \prod_{i=1}^{N}{}_R \; \psi_R^{(i)} \qquad (15)$$

$$A = \sum_{i=1}^{N} \left(\prod_{k=i}^{N-1}{}_R \; \psi_R^{(k+1)} \right) A^{(i)} \qquad (16)$$

The notation $\prod_R$ means multiply lower indexed terms on the right and $\prod_{k=N}^{N-1} \equiv$ identity. If all sections are identical then it follows from (7), (15) and (16) that

$$\Psi_R \equiv (\psi_R^{(1)})^N = \frac{(-1)^N}{y_{12}^N} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix}^N \qquad (17)$$

and

$$A = \sum_{i=1}^{N} (\psi_R^{(1)})^{N-i} (-1)^{i-1} A^{(1)}$$

$$= \sum_{k=0}^{N-1} (\psi_R^{(1)})^{k} (-1)^{N-k} \frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} \qquad (18)$$

where $N-i \equiv k$.

Continuing with the identical section case we have for each section

$$\psi_R = -\frac{1}{y_{12}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix},$$

$$A = -\frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} \qquad (19)$$

so that

$$\psi_R^2 = \frac{1}{y_{12}^2} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix} \qquad (20)$$

Or

$$\psi_R^2 = \frac{1}{y_{12}^2} \begin{bmatrix} 2y_{11}^2 - y_{12}^2 & -2y_{11} \\ 2y_{11}(y_{12}^2 - y_{11}^2) & 2y_{11}^2 - y_{12}^2 \end{bmatrix} \qquad (21)$$

Thus it leads to the recursion relation

$$\psi_R^2 = -\psi_R^0 - \frac{2y_{11}}{y_{12}} \psi_R^1, \qquad (22)$$

where

$$\psi_R^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (23)$$

Relation (22) can be used to reduce the higher powers of ψ_R to the first.

We next proceed to find the admittance matrix of the overall 3-port network. Explicit expansion of (14) yields

$$\begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} -\Psi_{R_{12}}^{-1} \Psi_{R_{11}} & \Psi_{R_{12}}^{-1} & -\Psi_{R_{12}}^{-1} A_{11} \\ -\Psi_{R_{12}} + \Psi_{R_{22}} \Psi_{R_{12}}^{-1} \Psi_{R_{11}} & -\Psi_{R_{22}} \Psi_{R_{12}}^{-1} & \Psi_{R_{22}} \Psi_{R_{12}}^{-1} A_{11} - A_{21} \end{bmatrix} \times \begin{bmatrix} F_{in} \\ F_{out} \\ V_3 \end{bmatrix} \qquad (24)$$

where $\Psi_{R_{11}}$, $\Psi_{R_{12}}$, $\Psi_{R_{22}}$ etc. represent the corresponding elements of the final matrix Ψ_R for the entire cascaded network system.

Following the use of induction, an explicit evaluation of the matrix elements in (24) gives the following results for all N = 1, 2, 3, ...

$$Y_{13} = y_{13} \qquad (25)$$

$$Y_{23} = (-1)^{N-1} y_{13} \qquad (26)$$

Y_{ij} are the matrix elements of the final admittance matrix Y for the N cascaded sections, and y_{ij} are the matrix elements for the single electrode section. It remains to calculate I_3 to complete the admittance matrix. We have from (5)

$$I_3 = \sum_{i=1}^{N} (-1)^{i-1} I_3^{(i)}$$

$$= \sum_{i=1}^{N} (-1)^{i-1} y_{13}^{(i)} [F_1^{(i)} + F_2^{(i)}] + \sum_{i=1}^{N} y_{33}^{(i)} V_3 \qquad (27)$$

where

$$F_1^{(i)} = F_2^{(i-1)}, \qquad (28)$$

$$\begin{bmatrix} F_2^{(i)} \\ -V_2^{(i)} \end{bmatrix} = \prod_{j=1}^{i} \psi_R^{(j)} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + \sum_{j=1}^{i} \left(\prod_{k=j}^{i-1} \psi_R^{(k+1)} \right) A^{(j)} V_3, \qquad (29)$$

and

$$\begin{bmatrix} F_1^{(i)} \\ -V_2^{(i-1)} \end{bmatrix} = \prod_{j=1}^{i-1} \psi_R^{(j)} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} + \sum_{j=1}^{i-1} \left(\prod_{k=j}^{i-2} \psi_R^{(k+1)} \right) A^{(j)} V_3 \qquad (30)$$

for which the (1,1) entries are to be summed and the expression for V_{in} substituted.

For identical sections, it turns out that

$$\begin{bmatrix} F_2^{(i)} \\ -V_2^{(i)} \end{bmatrix} = \frac{(-1)^{i-1}}{y_{12}^{i-1}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix}^{i-1} \cdot \frac{(-1)}{y_{12}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix}$$

$$+ \left\{ \sum_{k=0}^{i-2} (\psi_R^{(1)})^k (-1)^{i-k} + (\psi_R^{(1)})^{i-1} (-1)^{i-(i-1)} \right\} \cdot \frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} V_3, \qquad (31)$$

$$\begin{bmatrix} F_1^{(i)} \\ -V_2^{(i-1)} \end{bmatrix} = \frac{(-1)^{i-1}}{y_{12}^{i-1}} \begin{bmatrix} y_{11} & -1 \\ y_{12}^2 - y_{11}^2 & y_{11} \end{bmatrix}^{i-1} \begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix}$$

$$+ \sum_{k=0}^{i-2} (\psi_R^{(1)})^k (-1)^{i-1-k} \frac{y_{13}}{y_{12}} \begin{bmatrix} 1 \\ y_{12} - y_{11} \end{bmatrix} V_3 \qquad (32)$$

After some algebric manipulation of (31) and (32), we find an expression for $[F_1^{(i)} + F_2^{(i)}]$. On substitution of this expression for $[F_1^{(i)} + F_2^{(i)}]$ into (27) we get

$$I_3 = \sum_{i=1}^{N} y_{33} V_3$$

$$+ \sum_{i=1}^{N} (-1)^{i-1} \frac{y_{13}}{y_{12}} [\{(y_{12} - y_{11} - \Psi_{R_{12}}^{-1} \Psi_{R_{11}})(\psi_R^{(1)^{i-1}})_{11}$$

$$-(y_{12} - y_{11})[y_{12} + y_{11} - (-\Psi_{R_{12}}^{-1} \Psi_{R_{11}})](\psi_R^{(1)^{i-1}})_{12}\} F_{in}$$

$$+\{(\psi_R^{(1)^{i-1}})_{11} + (y_{12} - y_{11})(\psi_R^{(1)^{i-1}})_{12}\} \Psi_{R_{12}}^{-1} F_{out}] \qquad (33)$$

From this result for N = 1, 2, 3 the expected reciprocity results, $Y_{31} = Y_{13}$ and $Y_{32} = Y_{23}$, can be easily varified. By induction it follows that the reciprocity relations are valid for an arbitrary N.

Therefore, the final simplified form of the admittance matrix Y for N equal section case is given by

$$Y = \begin{bmatrix} -\Psi_{R_{12}}^{-1} \Psi_{R_{11}} & \Psi_{R_{12}}^{-1} & y_{13} \\ \Psi_{R_{12}}^{-1} & -\Psi_{R_{12}}^{-1} \Psi_{R_{11}} & (-1)^{N-1} y_{13} \\ y_{13} & (-1)^{N-1} y_{13} & N y_{33} \end{bmatrix} \qquad (34)$$

This is the most general form of the admittance matrix for the "N cascaded" network shown in Fig. 2. The combined network structure consequently may be defined by a single section as shown in Fig. 3.

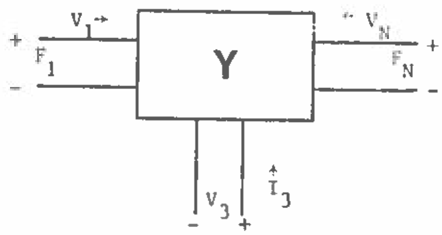

Figure 3. Combined Network Structure

III. Equivalent Circuit of the Crossed-field Model and the Admittance Matrix

The development of the interdigital transducer transfer function begins with the one dimensional crossed-field model for a single electrode section[1, p. 74]; this is shown for the sinusoidal steady state in Fig. 4. The model of a single

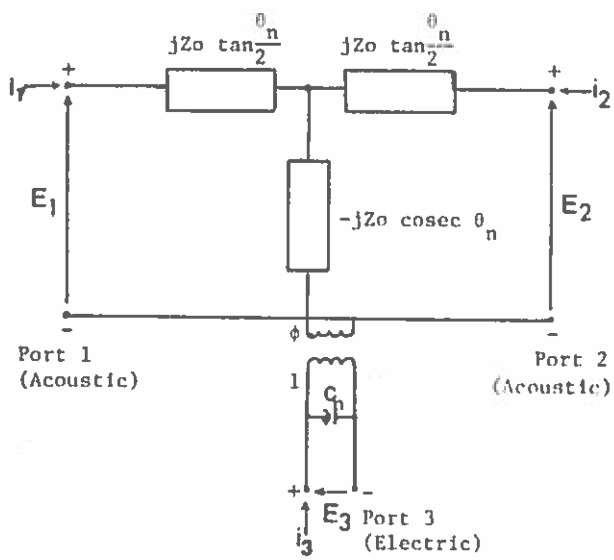

Figure 4. Equivalent Circuit of the Crossed-field Model for one-electrode Section.

electrode section has three ports, corresponding to the two symmetric acoustic ports and the electric port. The stresses and particle velocities at the acoustic ports are represented by equivalent voltages and currents, respectively, to which they are numerically equal. A convenient method for describing the relationship between these currents and voltages, $i_j = y_{jk} E_k$, is in terms of the admittance matrix. The y_{jk}'s for the n^{th} section are the matrix elements in the equation

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_n = \begin{bmatrix} -\frac{j}{Zo}\,\mathrm{Cot}\ \theta_n & \frac{j}{Zo}\,\mathrm{Cosec}\ \theta_n & -\frac{j}{Zo}\,\phi\tan\frac{\theta_n}{2} \\ \frac{j}{Zo}\,\mathrm{Cosec}\ \theta_n & -\frac{j}{Zo}\,\mathrm{Cot}\ \theta_n & -\frac{j}{Zo}\,\phi\tan\frac{\theta_n}{2} \\ -\frac{j}{Zo}\,\phi\tan\frac{\theta_n}{2} & -\frac{j}{Zo}\,\phi\tan\frac{\theta_n}{2} & j\left(\omega C_n + \frac{2\phi^2}{Zo}\tan\frac{\theta_n}{2}\right) \end{bmatrix} \times \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}_n \qquad (35)$$

where

$\theta_n = \frac{l_n}{v} \cdot 2\pi f$, is the acoustic transit angle of the n^{th} section; f = frequency

$\phi = \left(\frac{k^2 C_n\ Zo\ V}{l_n}\right)^{1/2}$, is the transformer turns ratio

Zo = acoustic characteristic impedance

k^2 = piezo-electric coupling coefficient

C_n = capacitance of the nth section

V = velocity of acoustic propagation and

l_n = length of nth section.

IV. Scattering Matrix for N - Section Crossed - Field Model

It is well known [6] that the scattering matrix of a network characterized by its admittance matrix Y is

$$\underset{\sim}{S} = \underset{\sim}{1}_3 - 2\underset{\sim}{Y}_a\ , \qquad (36)$$

where $\underset{\sim}{Y}_a$ is the admittance matrix of the augmented network (shown in Fig. 5).

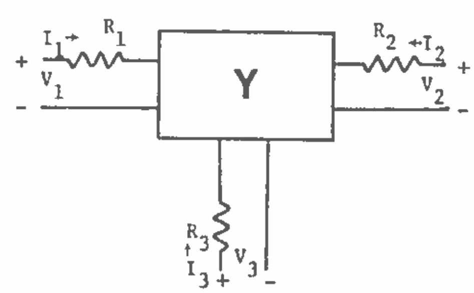

Figure 5. Augmented three-port Network

Using $R_1 = R_2 = R_3 = 1$, Y_a can be found in the form

$$\underset{\sim}{Y}_a = \underset{\sim}{Y}(\underset{\sim}{\mathbb{1}}_3 + \underset{\sim}{Y})^{-1} \quad (37)$$

Results (36) and (37) lead to the expression for the scattering matrix

$$\underset{\sim}{S} = \underset{\sim}{\mathbb{1}}_3 - 2\underset{\sim}{Y}(\underset{\sim}{\mathbb{1}}_3 + \underset{\sim}{Y})^{-1} \quad (38)$$

where $\underset{\sim}{\mathbb{1}}_3$ is the unit matrix. This equation (38) will be used to find the scattering matrix for the N - Section crossed-field model.

Application of a unitary transformation for diagonalization of $\psi_R^{(i)}$ given by (7) gives the product matrix Ψ_R for the N - section crossed-field model as

$$\Psi_R \equiv (\psi_R^{(1)})^N = \text{Sin } N\theta \begin{bmatrix} \text{Cot } N\theta & -jZo \\ -\frac{j}{Zo} & \text{Cot } N\theta \end{bmatrix} \quad (39)$$

Using (34) and (39) the admittance matrix Y of the whole structure shown in Fig. 2 is obtained in the form

$$Y = \begin{bmatrix} -\frac{j}{Zo}\text{ Cot } N\theta & \frac{j}{Zo}\text{ Cosec } N\theta & y_{13} \\ \frac{j}{Zo}\text{ Cosec } N\theta & -\frac{j}{Zo}\text{ Cot } N\theta & (-1)^{N-1}y_{13} \\ y_{13} & (-1)^{N-1}\, y_{13} & Ny_{33} \end{bmatrix} \quad (40)$$

Substituting the values of y_{13} and y_{33} from (35) into (40), the final admittance matrix for the N - section crossed-field model can be rewritten as

$$Y = \begin{bmatrix} -\frac{j}{Zo}\text{ Cot } N\theta & \frac{j}{Zo}\text{ Cosec } N\theta & -\frac{j}{Zo}\,\phi\tan\frac{\theta}{2} \\ \frac{j}{Zo}\text{ Cosec } N\theta & -\frac{j}{Zo}\text{ Cot } N\theta & (-1)^N\frac{j}{Zo}\,\phi\tan\frac{\theta}{2} \\ -\frac{j}{Zo}\,\phi\tan\frac{\theta}{2} & (-1)^N\frac{j}{Zo}\,\phi\tan\frac{\theta}{2} & j(\frac{2N}{Zo}\,\phi^2\tan\frac{\theta}{2} + \omega C_T) \end{bmatrix}, \quad (41)$$

where $C_T = NC_n$.

Finally, direct substitution of Y from (41) into (38) yields the explicit form of the scattering matrix for the N - section crossed-field model. A straightforward calculation gives the scattering matrix elements as

$$S_{11} = S_{22} = \frac{1}{|M|}\left[(1 + Y_{33})(1 - Y_{11}^2 + Y_{12}^2) + 2Y_{13}^2\,\{Y_{11} + (-1)^N\,Y_{12}\}\right] \quad (42)$$

$$S_{12} = S_{21} = -\frac{2}{|M|}\left[Y_{12}\,(1+Y_{33}) + (-1)^N\,Y_{13}^2\right] \quad (43)$$

$$S_{13} = S_{31} = -\frac{2}{|M|}\left[Y_{13}\,\{1+Y_{11} + (-1)^N\,Y_{12}\}\right] \quad (44)$$

$$S_{23} = S_{32} = \frac{2}{|M|}\left[Y_{13}\,\{(1+Y_{11})(-1)^N + Y_{12}\}\right] = (-1)^{N-1}\,S_{13} \quad (45)$$

$$S_{33} = \frac{1}{|M|}\left[(1-Y_{33})\,\{(1+Y_{11})^2 - Y_{12}^2\} + 2Y_{13}^2\,\{1+Y_{11} + (-1)^N\,Y_{12}\}\right] \quad (46)$$

where

$$|M| = \det(\mathbb{1}_3 + Y) = (1+Y_{33})\left[(1+Y_{11})^2 - Y_{12}^2\right] - 2Y_{13}^2\left[1 + Y_{11} + (-1)^N\,Y_{12}\right] \quad (47)$$

With the aid of (41), the explicit and final form of the elements of the scattering matrix in terms of known parameter is obtained as

$$S_{11} = S_{22} = \frac{1}{|M|}\left[(1-\frac{1}{Zo^2}) + j\left\{(1-\frac{1}{Zo^2})(\omega C_T + \frac{2N}{Zo}\,\phi^2\tan\frac{\theta}{2}) + \frac{2\phi^2}{Zo^3}\tan^2\frac{\theta}{2}\,(\cot N\theta + (-1)^{N+1}\text{cosec } N\theta)\right\}\right] \quad (48)$$

$$S_{12} = S_{21} = \frac{2}{|M|}\left[\left(\frac{\phi^2}{Zo^2}\tan\frac{\theta}{2}\,\{(-1)^N\tan\frac{\theta}{2} + 2N\text{ cosec } N\theta\} + \frac{\omega C_T}{Zo}\text{ cosec } N\theta\right) - \frac{j}{Zo}\text{ cosec } N\theta\right] \quad (49)$$

$$S_{13} = S_{31} = \frac{2}{|M|}\left[\frac{\phi}{Zo^2}\tan\frac{\theta}{2}\,\{\cot N\theta + (-1)^{N+1}\text{cosec } N\theta\} + \frac{j}{Zo}\,\phi\,\tan\frac{\theta}{2}\right] \quad (50)$$

and

$$S_{33} = \frac{1}{|M|}\left[\left\{(1+\frac{1}{Zo^2})-\frac{2\phi^2}{Zo^2}\tan^2\frac{\theta}{2} - \frac{2}{Zo}\cot N\theta \; (\omega C_T + \frac{2N}{Zo}\phi^2 \tan\frac{\theta}{2})\right\} + j\left\{\frac{2\phi^2}{Zo^3}\tan^2\frac{\theta}{2}(\cot N\theta + (-1)^{N+1}\operatorname{cosec} N\theta) - (1+\frac{1}{Zo^2})(\omega C_T + \frac{2N}{Zo}\phi^2\tan\frac{\theta}{2}) - \frac{2}{Zo}\cot N\theta\right\}\right] \tag{51}$$

where

$$|M| = \left[\left\{(1+\frac{1}{Zo^2}) + \frac{2}{Zo}\cot N\theta \; (\omega C_T + \frac{2N}{Zo}\phi^2 \tan\frac{\theta}{2}) + \frac{2\phi^2}{Zo^2}\tan^2\frac{\theta}{2}\right\} + j\left\{(1+\frac{1}{Zo^2})(\omega C_T + \frac{2N}{Zo}\phi^2\tan\frac{\theta}{2}) - \frac{2}{Zo}\cot N\theta - \frac{2\phi^2}{Zo^3}\tan^2\frac{\theta}{2}(\cot N\theta + (-1)^{N+1}\operatorname{cosec} N\theta)\right\}\right] \tag{52}$$

Therefore, the scattering matrix for the N-section crossed-field model takes a symmetric form

$$\underline{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & (-1)^{N-1}S_{13} \\ S_{13} & (-1)^{N-1}S_{13} & S_{33} \end{bmatrix} \tag{53}$$

where the elements S_{ij} are given by the expressions (48) - (51). Although these expressions are rather bulky, they should be useful for computer aided analysis.

V. Conclusion

Here we have given, for sinusoidal steady state behavior, explicit formulas for the scattering matrix entries of a SAW interdigital transducer based upon the crossed-field model of a one-electrode section, these formulas appearing in equations (48) - (52). In the form presented they are suitable for computer aided design of SAW transducers while their existence saves the way for more extensive design formalisms.

References

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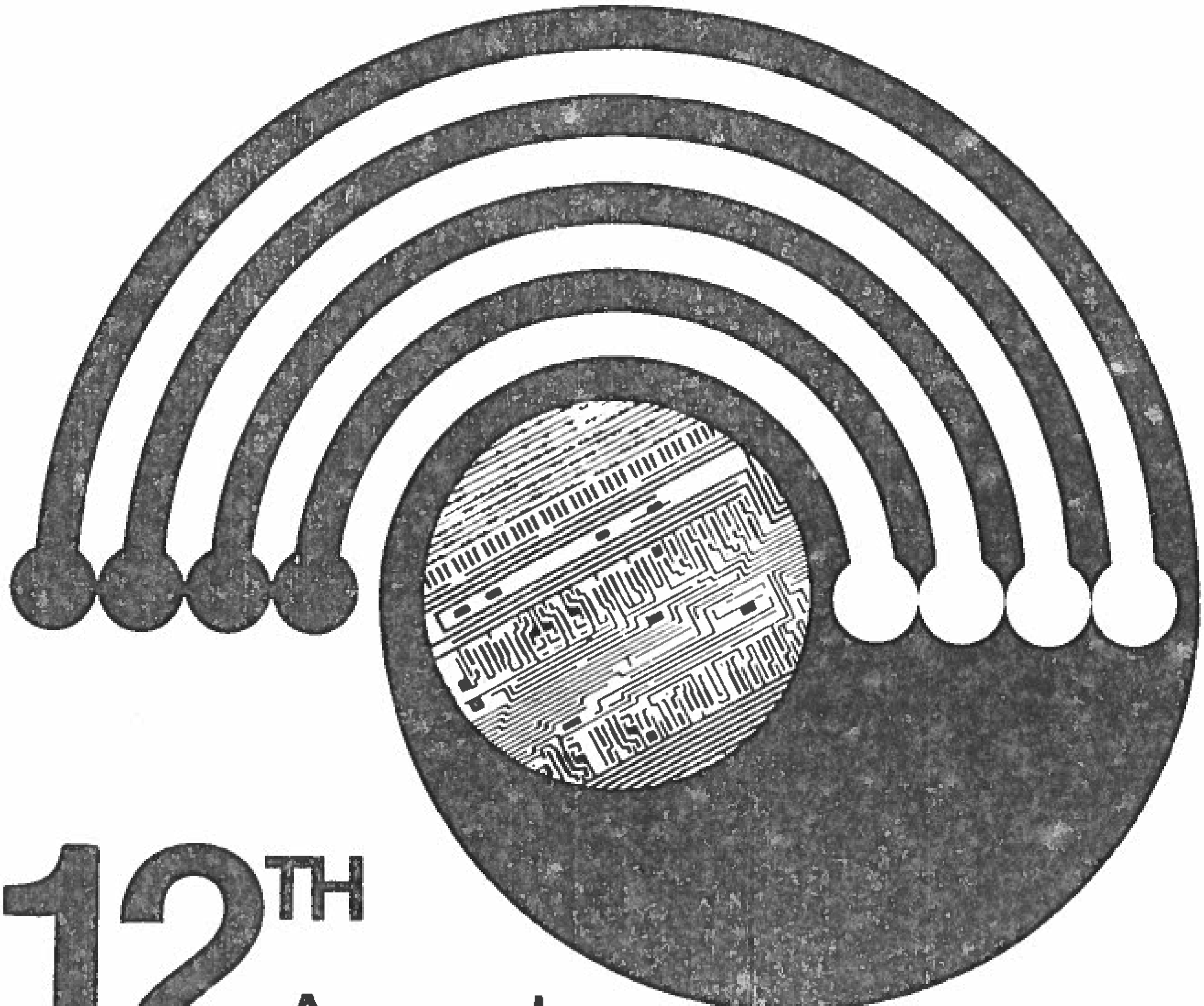

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