PASSIVE CONDITIONS FOR N-PORT

BILINEAR-QUADRATIC SYSTEMS

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Abstract

Necessary and sufficient conditions for passivity of N-port bilinear-quadratic systems are derived. These conditions which are similar to those already obtained for linear N-port systems are of primordial importance in order to synthesize the system. With these conditions, it is now possible to provide an exact synthesis for this type of non-linear N-ports.

1. INTRODUCTION

It is well known that the synthesis of non-linear systems presents a large number of problems whose resolution is not yet known.

The traditional methods used to synthesize non--linear systems usually linearize the input-output map, either by considering a limited band of frequencies, or by taking a piecewise linear approximation of the function to be synthesized.

Our approach leads to an exact synthesis of the system. It follows the line of the synthesis of a more restrict class of systems, these ones represented by

$$\begin{cases} \dot{x} = Fx + Gu + N(x)u \\ y = Hx + Ju + M(x)x \end{cases} \quad (1)$$

which is presented in [1].

However in order to be able to tackle this problem we need, first, to derive conditions for passivity.

In this paper we determine necessary and sufficient conditions for passivity of a certain class of nonlinear systems these ones beeing described by equations of the form

$$\begin{cases} \dot{x} = Fx + Gu + N_1(x)u + N_2(x)x \\ y = Hx + Ju + M_1(x)x + M_2(x)u \end{cases} \quad (2)$$

where $\underline{u}$ and $\underline{y}$ are conjugate n-vectors associated with the ports, F, G, H, J are constant matrices, and N_1, N_2, M_1, M_2 are linear homogeneous matrix functions of the state x with proper dimensions and all quantities assumed real.

For example

$$N_1(x) = [n^1_{ij}(x)] = [\sum_{k=1}^{n} n^1_{ijk} x_k] \quad (3)$$

2. ANALYSIS

Let us consider a system described by a set of state-space equations of the form (2).

We assume that the dimension of the state-space is minimal, i.e., the system is reachable and observable and that $u(t) \in R^m$, $y(t) \in R^m$ and $x(t) \in R^n$.

It has been shown, Moylan [2], that a system described by

$$\begin{cases} \dot{x} = f(x) + G(x)u \\ y = h(x) + J(x)u \end{cases} \quad (4)$$

is passive if and only if there exist real functions $\phi(.)$, $l(.)$, $W(.)$, with $\phi(x)$ continuous and $\phi(x) \geq 0 \; \forall x$, $\phi(0) = 0$ such that

$$\begin{cases} \nabla^T \phi(x)\ f(x) = -1^T(x)\ 1(x) \\ \frac{1}{2}\ G^T(x) \nabla\ \phi(x) = h(x) - W^T(x)\ 1(x) \\ J(x) + J^T(x) = W^T(x)\ W(x) \end{cases} \quad (5)$$

where $\phi(x)$ represents the extractable energy when the system is in state x.

Now, by definition, the stored energy when the system is in state x is

$$\phi_{st} = x^T Kx = \int_{-\infty}^{t} x^T K \dot{x}\, d\xi \quad (6)$$

for some positive definite matrix K, and this energy must be greater or equal than the energy that can be extracted.

So, by the physical meaning of ϕ as an extractable energy

$$\phi_{st} = x^T Kx \geq \phi \quad (7)$$

Therefore, ϕ can contain no higher powers in the components of x than the second and thus must be of the form

$$\phi = x^T Px + P_2 x + P_3 \quad (8)$$

However, if no input is applied the system stays at state x=0 and the available energy is zero, what implies

$$\phi(0) = P_3 = 0 \quad (9)$$

P_2 must also be equal to zero otherwise one could choose an x small enough so the linear term would predominate and have ϕ negative.

Thus

$$\phi(x) = x^T Px \quad (10)$$

3. THEOREMS

Comparing the systems described by equations (4) and (2) we see that

$$\begin{cases} f(x) = Fx + N_2(x).x \\ G(x) = G + N_1(x) \\ h(x) = Hx + M_1(x).x \\ J(x) = J + M_2(x) \end{cases} \quad (11)$$

and from (10) we have

$$\nabla\phi(x) = 2Px \quad (12)$$

Substituting (11) and (12) in (5) we get

$$\begin{cases} 2x^T\ P\ [Fx + N_2(x)x] = -1^T(x)\ 1(x) \\ [G + N_1(x)]^T Px = Hx + M_1(x).x - W^T(x)\ 1(x) \\ [J + M_2(x)] + [J+M_2(x)]^T = W^T(x)\ W(x) \end{cases} \quad (13)$$

From the last equation we have that

$$W(x) = W_o \quad (14)$$

this is, W is a constant matrix, not function of x, otherwise the right-hand side of the third equation (13) would have terms in x^2, while the left-hand side will have terms in x or constants, because $M_2(x)$ is a linear homogeneous matrix in x.

Then from the last equations of (13)

$$J + J^T + M_2(x) + M_2^T(x) = W_o^T\ W_o \quad (15)$$

or

$$J + J^T = W_o^T\ W_o \quad (16)$$

and

$$M_2(x) + M_2^T(x) \equiv 0 \quad (17)$$

From the first two equations of (13)

$$\begin{cases} x^T(PF + F^TP)\ x + x^T(PN_2(x) + N_2^T(x)\ P)\ x = \\ \qquad = -1^T(x)\ 1(x) \\ G^TPx + N_1^T(x)\ Px = Hx + M_1(x)\ x - W_o^T\ 1(x) \end{cases} \quad (18)$$

From the second equation of (18) we see that $1(x)$ is a function of the form

$$1(x) = Lx + x^T L_1\ x \quad (19)$$

because the left-hand side of that equations has only terms in x and x^2.

However the left-hand side of the first equation of (18) has only terms in x^2 and x^3.

Thus we must have

$$1(x) = Lx \quad (20)$$

and substituting in (18) we get

$$x^T(PF + F^TP)\ x + x^T\ [PN_2(x) + N_2^T(x)\ P]\ x = -x^T\ L^T\ L\ x \quad (21)$$

$$G^T P x + N_1^T(x)\, P x = H x + M_1(x)\, x - W_o^T L x \tag{22}$$

or, because x is an arbitrary state,

$$\begin{cases} PF + F^T P = - L^T L \\ PN_2(x) + N_2^T(x)\, P \equiv 0 \\ G^T P = H - W_o^T L \\ N_1^T(x)\, P \equiv M_1(x) \end{cases} \tag{23}$$

Thus, combining (16), (17) and (23) we have:

THEOREM 1:

A N- Port Bilinear-Quadratic System described by a set of state-space equations of the form

$$\begin{cases} \dot{x} = Fx + Gu + N_1(x)\, u + N_2(x)\, x \\ y = Hx + Ju + M_1(x)\, x + M_2(x)\, u \end{cases} \tag{24}$$

where $u \in R^m$, $y \in R^m$, and $x \in R^n$ (minimal is passive if and only if there exist three real matrices P, L, W_o and P positive, definite (symmetric) such that

$$\begin{cases} PF + F^T P = - L^T L \\ G^T P = H - W_o^T L \\ J + J^T = W_o^T W_o \end{cases} \tag{25}$$

$$\begin{cases} N_1^T(x)\, P \equiv M_1(x) \\ PN_2(x) + N_2^T(x)\, P \equiv 0 \\ M_2(x) + M_2^T(x) \equiv 0 \end{cases} \tag{26}$$

Comments: Equations (25) are the conditions for passivity in the linear case while equations (26) are the conditions imposed by the nonlinear terms.

In order to synthesize the system we need to have these conditions under a new form. For this we need the following theorem:

THEOREM 2:

With T any matrix such that $T^T\, T = P$ the system (24) can be represented by

$$\begin{cases} \dot{z} = \bar{F}\, z + \bar{G}\, u + \bar{N}_1(z)\, u + \bar{N}_2(z)\, z \\ y = \bar{H}\, z + J\, u + \bar{M}_1(z)\, z + \bar{M}_2(z)\, u \end{cases} \tag{27}$$

with

$$\begin{cases} \bar{F} = T\, F\, T^{-1} & \bar{G} = TG \\ \overline{N_1}(z) = T\, N_1(T^{-1}z) & \overline{N_2}(z) = TN_2(T^{-1}z)\, T^{-1} \\ \bar{H} = H\, T^{-1} & \overline{M_1}(z) = M_1(T^{-1}z)\, T^{-1} \\ \overline{M_2}(z) = M_2(T^{-1}z) & \end{cases} \tag{28}$$

and satisfying, in the passive case,

$$\begin{cases} \bar{F} + \bar{F}^T = - L_1^T L_1 \\ \bar{G}^T = \bar{H} - W_o^T L_1 \\ J + J^T = W_o^T W_o \end{cases} \tag{29}$$

$$\begin{cases} \overline{N_1}(z)^T \equiv \overline{M_1}(z) \\ \overline{N_2}(z) + \overline{N_2}(z)^T \equiv 0 \\ \overline{M_2}(z) + \overline{M_2}(z)^T \equiv 0 \end{cases} \tag{30}$$

with

$$L_1 = L\, T^{-1} \tag{31}$$

Proof:

With $z = Tx$ and T such that $T^T\, T = P$ from (24) we have

$$\begin{cases} \dot{z} = T\, \dot{x} = TFT^{-1}z + TGu + TN_1(T^{-1}z)u + \\ \qquad + TN_2(T^{-1}z)\, T^{-1}z \\ y = HT^{-1}z + Ju + M_1(T^{-1}z)\, T^{-1}z + M_2(T^{-1}z)u \end{cases} \tag{32}$$

So,

$$\begin{cases} \bar{F} = TFT^{-1} & \bar{G} = TG \\ \overline{N_1}(z) = TN_1(T^{-1}z) & \overline{N_2}(z) = TN_2(T^{-1}z)\, T^{-1} \\ \bar{H} = HT^{-1} & \overline{M_1}(z) = M_1(T^{-1}z)\, T^{-1} \\ \overline{M_2}(z) = M_2(T^{-1}z) & \end{cases} \tag{33}$$

For the passive case, substitution of (33) in (25) and (26) leads to (29) and (30) with $L_1 = LT^{-1}$.

4. CONCLUSIONS

Starting with a N-Port bilinear-quadratic system we found necessary and sufficient conditions for passivity. These conditions were not, however, in proper form to proceed with the synthesis so we presented equivalent conditions. These latter ones are necessary to synthesize by an N-Port any system of this type whose variables are current (or voltage) inputs and respectively voltage (or current) outputs.

5. REFERENCES

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6. BIOGRAPHIES

Carlos Mendonça-e-Moura was born in Porto, Portugal in 1949. He received the Electrical Engineering Diploma from the Universidade do Porto, Porto, Portugal in 1971 and the M.S. and Ph.D. degrees from the University of Maryland, College Park, Md. U.S.A., in 1974 and 1978, respectively.

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