MODELING NETWORKS OF MORISHITA NEURONS WITH APPLICATION TO THE CEREBELLUM

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Abstract

A neural network composed of one-dimensional strings of Morishita neurons is presented. Existence and uniqueness of solutions for this network are established, generalizations are given, and a formal description is presented. These techniques are applied to a network topologically similar to the cerebellum for which simulation results are shown.

1. Introduction

In a series of papers [3,7] Morishita, et. al., investigated the behavior of double layer neural networks with mutally inhibiting interconnections. In this paper, we will generalize and apply the above work to neural networks which are topologically similar to the cerebellum.

This work is motivated by the desire to use these results in constructing efficient and versatile computer systems based upon applying engineering principles to the knowledge gained from physiological data of the Central Nervous System. We first begin by defining the Morishita neuron and then present a category of neural networks, those composed of one-dimensional strings of Morishita neurons. Next the existence and uniqueness of solutions for these networks is proven. In developing these neural networks we will state some important connectivity properties extracted from physiological data and will generalize our structure using these connectivity properties.

Finally, we will present simulation results for a network that is topologically similar to the structure of the cerebellum.

2. The Morishita Neuron

Morishita, et. al., [3,7] introduced a continuous variable model of the neuron. We now generalize this to a continuous variable model of the neuron with threshold. This model is given in Fig. 2.1 and consists of an adder that represents the dendrites, a low-pass filter and a nonlinearity that represent the cell body and the axon. The model is described analytically by the following equations

$$\tau \frac{dy(t)}{dt} + y(t) = \sum_{i=1}^{\mu} w_i e_i(t) \qquad (2.1)$$

$$z(t) = [y(t) - \theta] \cdot \phi[y(t) - \theta] \qquad (2.2)$$

where $y(t)$ represents the membrane potential, $e_i(t)$ are the inputs to the neuron (they may come either from other neurons in the network or be external to the network), w_i are the synaptic-weights, $z(t)$ is the spike-frequency on the output axon, θ is the threshold and $\phi[\cdot]$ is the unit step fonction

$$\phi[y] = \begin{cases} 1 & y > 0 \\ 0 & y \le 0 \end{cases} \qquad (2.3)$$

We call any device described by Eqs. (2.1) and (2.2) a "Morishita neuron". Choosing as a new variable

$$f(t) = y(t) - \theta \qquad (2.4)$$

called the activity, we simplify (2.1) and (2.2) to the following form.

$$\tau \dot{f} + f = \sum_{i=1}^{\mu} w_i e_i - \theta \qquad (2.5)$$

$$z(t) = f \cdot \phi[f] \qquad (2.6)$$

As it turns out, in biological systems neurons most often occur in physiologically similar collections. Any such collection will be called a class with the neurons in a given class distinguished by the distribution of nonzero weights w_i and the value of the time constant τ in (2.1). We now proceed to examine networks composed of several neural classes.

3. The Neural Network

Suppose that we have a neural network composed of N classes of one-dimensional strings of Morishita neurons and suppose that each neuron in class π_i can take inputs from neurons distinguished by an index set c_{ji} indexing the $2k_{ji} + 1$ nearest neuron neighbors in class π_j. This set c_{ji} is not empty iff class π_j gives input to class π_i, and then consists of the set of integers in the range $[-k_{ji}, k_{ji}]$. Similarly, for the external inputs we have the set d_{ji} indexing the $2\lambda_{ji} + 1$ nearest input neighbors where d_{ji} is not empty if the class j of the external inputs drives neural class π_i.

For such a network, and using (2.5) and (2.6), we can write the following system of differential equations

$$\tau_x^i \frac{df_x^i(t)}{dt} + f_x^i(t) = \sum_{j=1}^{N} \sum_{y\epsilon x+c_{ji}} w_{yx}^{ji}\, \phi[f_y^j] f_y^j - \theta_x^i + \sum_{j=1}^{M} \sum_{y\epsilon x+d_{ji}} v_{yx}^{ji}\, e_y^j \qquad (3.1)$$

$i = 1,2, \ldots N$; $x = 1,2, \ldots \nu_i$; where ν_i is the number of neurons in class π_i. The superscripts denote membership in a neural class while the subscripts denote a particular member of a class. e_y^j is the yth external input of the jth input class and v_{yx}^{ji} denotes the weights attached to the input signals. All the other letters have the meaning attached to them in Section 2. Incorporating the two indices into one (i.e., $f_x^i = f_a$ where

$$a = \sum_{\lambda=1}^{i} \nu_\lambda + x;\; w_{yx}^{ji} = w_{ba};\; b = \sum_{\lambda=1}^{j} \nu_\lambda + y), \qquad (3.1a)$$

we can write system (3.1) in the matrix form

$$T \cdot \dot{f} + f = W \cdot \phi[f] \cdot f - \theta + V \cdot e \qquad (3.2)$$

where f is the excitation minus the threshold vector, W is the network connectivity matrix, θ is the threshold vector, V is the external input connectivity matrix, e is the external input vector, $T = \mathrm{diag}[\tau_a; a=1,2, \ldots \sum_{\lambda=1}^{N} \nu_\lambda]$ and

$$\phi[f] = \mathrm{diag}\,[\phi[f_a];\; a=1,2, \ldots \sum_{\lambda=1}^{N} \nu_\lambda]$$

Since there are N classes, we partition the connectivity matrix into N^2 submatrices.

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & \cdot & \cdot & W_{2N} \\ \vdots & & & \\ W_{N1} & \cdot & \cdot & W_{NN} \end{bmatrix}$$

Since the weights are nonzero for $2k_{ji}+1$ neighbors, each of the submatrices w_{ij} has the form

$$w_{ij} = \begin{cases} 0 & \text{if class } \pi_j \text{ does not give input to class } \pi_i \\ {}^{k_{ji}}\!\begin{bmatrix} \ddots & & 0 \\ & \ddots & \\ 0 & & \ddots \end{bmatrix}^{k_{ji}} & \text{if class } \pi_j \text{ gives input to class } \pi_i \end{cases} \qquad (3.3)$$

As an example, for the case of the cerebellum connections are shown in Fig. 3.1 as derived from [2,6]. Hence, the connectivity matrices are

$$W = \begin{bmatrix} 0 & -w_{12} & 0 & w_{14} \\ 0 & 0 & 0 & w_{24} \\ 0 & 0 & 0 & w_{34} \\ 0 & 0 & -w_{43} & 0 \end{bmatrix} \qquad (3.4)$$

$$V = \begin{bmatrix} v_{11} & 0 \\ v_{21} & 0 \\ v_{31} & v_{32} \\ 0 & v_{42} \end{bmatrix} \qquad (3.5)$$

4. Existence and Uniqueness of Solutions

Denote by

$$\omega(f) \triangleq T^{-1} \cdot (-f + W\, \phi[f] \cdot f - \theta + V \cdot e) \qquad (4.1)$$

then (3.2) is written as

$$\dot{f} = \omega(f) \qquad (4.1a)$$

We proceed now to derive a Lipschitz condition for (3.2). Let R^N by an N-dimensional Euclidean space, then $\forall\, (f_1, f_2) \epsilon\, R^N \times R^N$ we have

$$||\omega(f_1) - \omega(f_2)|| = ||T^{-1}(f_2 - f_1) + T^{-1} \cdot W(\phi[f_1] \cdot f_1 - \phi[f_2] \cdot f_2)||$$

$$\leq ||T^{-1}|| \cdot ||f_1 - f_2|| + ||T^{-1} \cdot W|| \cdot ||\phi[f_1] \cdot f_1 - \phi[f_2] \cdot f_2|| \quad (4.2)$$

Observe now that $\phi[f^i] \cdot f^i = \max\{0, f^i\}$ where f^i is the i^{th} component of the vector f. If we chose as a norm $||f|| = \max_i |f^i|$ we have

$$|\phi[f_1^i] \cdot f_1^i - \phi[f_2^i] \cdot f_2^i| \leq |f_1^i - f_2^i| \Rightarrow$$

$$\max_i |\phi[f_1^i] \cdot f_1^i - \phi[f_2^i] \cdot f_2^i| \leq \max_i |f_1^i - f_2^i| \rightarrow \qquad (4.3)$$

$$||\phi[f_1] \cdot f_1 - \phi[f_2] \cdot f_2|| \leq ||f_1 - f_2||$$

combining (4.2) and (4.3) we have

$$||\omega(f_1) - \omega(f_2)|| \leq (||T^{-1}|| + ||T^{-1} \cdot W||)\, ||f_1 - f_2|| \quad (4.4)$$

Thus we have a Lipshitz condition. Considering the topological equivalence of the norms [8, p.58] then certainly (4.4) holds true for every norm.

5. Physiological Principles

In deriving the model of our network in Section 4 we implicitely used two principles, namely (i) that there exist classes of neurons and that each neural class may not be connected to all other neural classes, and (ii) that if two neural classes, are connected then this does not mean that every neuron in one of these two classes is connected to every neuron in the other. We can cite numerous examples from physiological data of these two principles [2,6]. However, we will concentrate on the case of the cerebellum where an extensive bibliography exists to substantiate our claims. In the cerebellum [4,2] it is known that there are four different neural classes, namely (1) Purkinje cells, (2) basket cells, (3) Golgi cells and (4) granule cells. The members of each one of these classes are physiologically distinct from those of other classes and the way these classes connect is given in Fig. 3.1. On the other hand, if we take the granule cell-parallel fibre-Purkinjie cell pathway, we have the following for the cerebellum of the cat [4,5].

Granule cells: $2.8 \times 10^6/mm^3$ homogeneously distributed over the $355\ mm^3$ of the granule layer for a total of 2.2×10^9 granule cells. The average thickness of the granule layer is 254.86 μm.

Parallel fibres: Average length of 2 mm synapsing with the dendrites of the Purkinje cells.

Purkinje cell dendrites: Width of 292.5 μm, thickness of 8.9 μm.

Basket cells: Six basket cells per Purkinje cell.

Taking into account the above data and the geometrical arrangement of the cerebellum cells [2,6] we can deduce that each Purkinje cell: (i) is excited (via the parallel fibres) from granule cells included in an orthogonal parallelepiped centered immediately below the Purjinje cell. The dimensions of this parallelepiped are 2mm[length of the parallel fibres]x 0.2925 mm [width of the Purkinje cell dendrites]x 0.25486 mm [thickness of the granule layer]. The volume of such a parallelepiped is $0.15\ mm^3$ or 0.04% of the total granule layer. (ii) is inhibited from basket cells that are included in 2 parallelograms adjacent to the Purkinje cell each one of which has dimensions of 450 μm x 300 μm; these leave a gap of 100 μm between them. The Purkinje cell inhibited by these basket cells is located in the middle of the gap.

Having established the existence of neural classes and the special way these classes connect to form a network we proceed to define in a formal way the neural network.

6. General Neural Network

Let n_i^x stand for the x^{th} neuron in the neural class π_i, which leads to $\pi_i = \{n_i^x \in V \subseteq R^3\}$, where V is a Euclidean vector space that describes the exact arrangement of the neurons n_i^x in a hyperplane of 3-space. With this in hand, given a set of neural classes $\pi = \{\pi_i;\ i=1,2,\ldots N\}$ we give the following definitions.

Definition 1: We call the set $R = \pi \times \pi$ the macroscopic connectivity relation of the set of neural classes π iff $(\pi_i, \pi_j) \in R$ when neurons from the class π_i give input to neurons of the class π_j.

Definition 2: We define the macroscopic connectivity cover $\mathcal{J} = \{G_i \mid G_i \subset \pi \wedge (\pi_j, \pi_i) \in R\ \forall \pi_j \in G_i\}$. For example, from Fig. 3.1 we obtain $G_1 = \{\pi_2, \pi_4\}$.

Turning next to individual neurons we have;

Definition 3: We write $n_x^i \rightarrow n_y^j, n_x^i \in \pi_i, n_y^j \in \pi_j$ to mean that the x^{th} neuron of the neural class π_i gives input to the y^{th} neuron of the neural class π_j.

Definition 4: We say that the neural class π_j induces the microscopic connectivity cover $C_{\pi i}(\pi_j)$ (or C_{ij}) on the class $\pi_i \in G_j$ where

$$C_{\pi i}(\pi_j) = \{N_a^{ij} \subseteq \pi_i \mid \exists \text{ a unique } n_y^j \in \pi_j \wedge n_x^i \rightarrow n_y^j\ \forall n_x^i \in N_a^{ij}\}$$

Definition 5: A member of an induced microscopic connectivity cover is called a block.

Definition 6: We define the distance $d[n_x^i; N_a^{ij}]$ of the neuron $n_x^i \in C_{ij}$ as follows

$$d[n_x^i; N_a^{ij}] = \max_{n_y^i \in N_a^{ij}} \{d[n_x^i; n_y^j]\}$$

where $d[n_x^i; n_y^j]$ is the Euclidean distance between the x^{th} neuron of class i and the y^{th} neuron of class j. We understand now the term "block with the minimal distance" from a given neuron $n_x^i \in \pi_i$ as the block $N_x^{ij} \in C_{ij}$ such that $d[n_x^i; N_x^{ij}] = \min_{N_a^{ij} \in C_{ij}} \{d[n_x^i; N_a^{ij}]\}$.

After this series of definitions we see that the topology of a neural network is defined through the connectivity relation R. And with the understanding that each neuron gets inputs only from blocks with minimal distances from it, we are in position now to formally define a neural network.

Definition 8: We define as a neural network the quadruple $\langle \pi, \mathcal{J}, C, O \rangle$

where
- π is the set of all the neural classes
- $\mathcal{J}$ is the macroscopic connectivity cover
- C is the set of all the induced connectivity covers
- O is the set of all the operators describing the operation of the neural elements.

Then for such a network we can write the following set of equations

$$f_x^i(t) = O^i\{\mathcal{J}^{ix}(t); \mathcal{E}^{ix}(t)\} \qquad (6.1)$$

where $f_x^i(t)$ is the activity of neuron $n_x^i \in \mathcal{N}_i$ at time t, $\mathcal{O}^i$ is the operator describing the action of the neurons in class i, $\mathcal{J}^{ix}(t)$ is the set of activities of all the neurons giving input to neuron $n_x^i \in \mathcal{N}_i$, that is

$$\mathcal{J}^{ix}(t) = \{f_j^y(t) \mid \mathcal{N}_j \in G_i \wedge n_j^y \in N_x^{ij}\} \qquad (6.2)$$

and, finally, $\mathcal{E}^{ix}(t)$ is the set of all the external inputs to the neuron n_x^i at time t.

7. Simulation Results

A general program was written to solve Eq. (3.2). This program uses a Runge-kutta iterative method and can handle systems with up to 100 differential equations and up to 100 external inputs.

The program generates the connectivity matrices W and V through the set of parameters: (i) κ_{ij}, λ_{ij} (both having the meaning presented in Section 3); (ii) ω_{ij}, ν_{ij} (weights by which the submatrices W_{ij} and V_{ij} are multiplied); (iii) z_{ij} (which are the number of zero elements across the diagonals of the submatrices W_{ij}) (see the case of Basket-Purkinje cell synapse in Section 4).

This program was used to simulate the behavior of neural networks that are topologically similar to the cerebellum. The parameters for the cases studied are given in Table I while the results are presented in Figs. 7.1-7.3. Three important results appear through the cases studied. (1) The output is a short pulse appearing on the onset of the input (some kind of a leading edge detector), (2) The output is spatially confined to an area specified by the input stimulus, (3) The network is stable (Fig. 7.2) a result analytical proven elsewhere [1].

8. Conclusions

In this paper we presented a mathematical description for a generalized neural network. A model was derived for one-dimensional strings of neurons and the existence and uniqueness of solutions were proven for this model.

Through simulation of networks that are topologically similar to the cerebellum, we found that these networks behave like leading edge detectors, that the output is confined to a region defined by the input, and that these networks are stable. Research investigating the behavior of different network topologies with various sets of operators, $\mathcal{O}$, is continuing.

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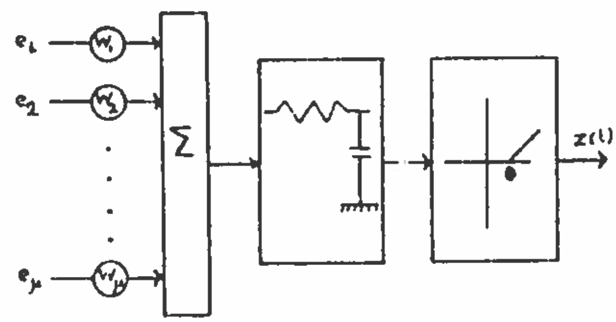


fig. 2.1

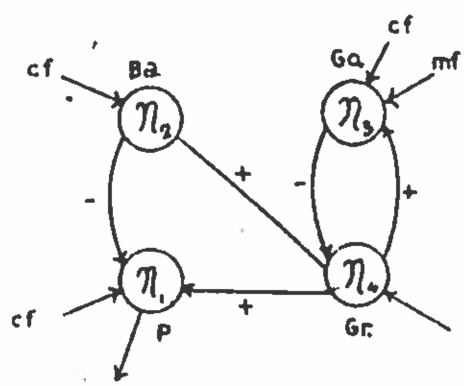


Fig. 3.1. P, Purkinje cells; Ba, Basket cells; Go, Golgi cells; Gr, Granule cells; Mf, Mossy Fibres; cf, Climbing Fibres.

TABLE 1

IN ALL CASES THE NUMBER OF CLASSES IS 4 WITH 25 NEURONS IN EACH CLASS		
CASE #1	CASE #2	CASE #3
$\kappa_{12} = 5$ $\omega_{12} = -1$ $z_{12} = 3$	$\kappa_{12} = 5$ $\omega_{12} = -1$ $z_{12} = 3$	$\kappa_{12} = 5$ $\omega_{12} = -1$ $z_{12} = 3$
$\kappa_{14} = 5$ $\omega_{14} = 5$ $z_{14} = 0$	$\kappa_{14} = 5$ $\omega_{14} = 5$ $z_{14} = 0$	$\kappa_{14} = 5$ $\omega_{14} = 5$ $z_{14} = 0$
$\kappa_{24} = 5$ $\omega_{24} = 3$ $z_{24} = 0$	$\kappa_{24} = 5$ $\omega_{24} = 3$ $z_{24} = 0$	$\kappa_{24} = 5$ $\omega_{24} = 3$ $z_{24} = 0$
$\kappa_{34} = 5$ $\omega_{34} = 2$ $z_{34} = 0$	$\kappa_{34} = 5$ $\omega_{34} = 2$ $z_{34} = 0$	$\kappa_{34} = 5$ $\omega_{34} = 2$ $z_{34} = 0$
$\kappa_{43} = 5$ $\omega_{43} = 2$ $z_{43} = 0$	$\kappa_{43} = 5$ $\omega_{43} = 2$ $z_{43} = 0$	$\kappa_{43} = 5$ $\omega_{43} = 2$ $z_{43} = 0$
$\lambda_{11} = \lambda_{21} = \lambda_{31} = 1$, $\nu_{11} = \nu_{21} = \nu_{31} = 4$ } climbing fibres	$\lambda_{32} = \lambda_{42} = 4$, $\nu_{32} = \nu_{42} = 2$ } mossy fibres	$\lambda_{32} = \lambda_{42} = 4$, $\nu_{32} = \nu_{42} = 2$ } mossy fibres
$\lambda_{32} = \lambda_{42} = 4$, $\nu_{32} = \nu_{42} = 2$ } mossy fibres		

CASE #1

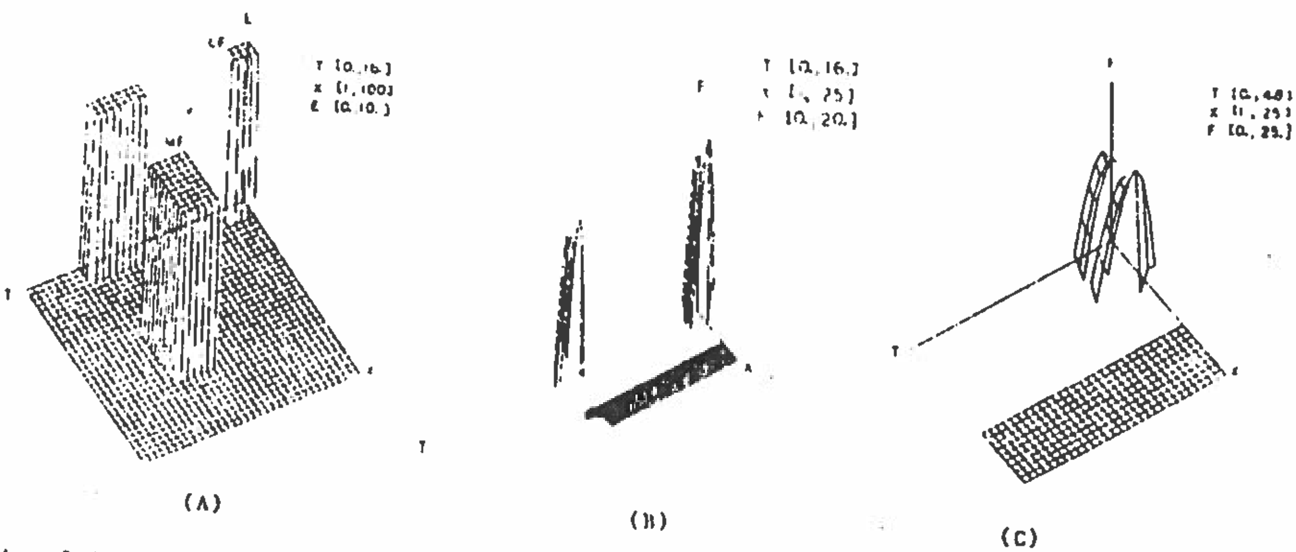
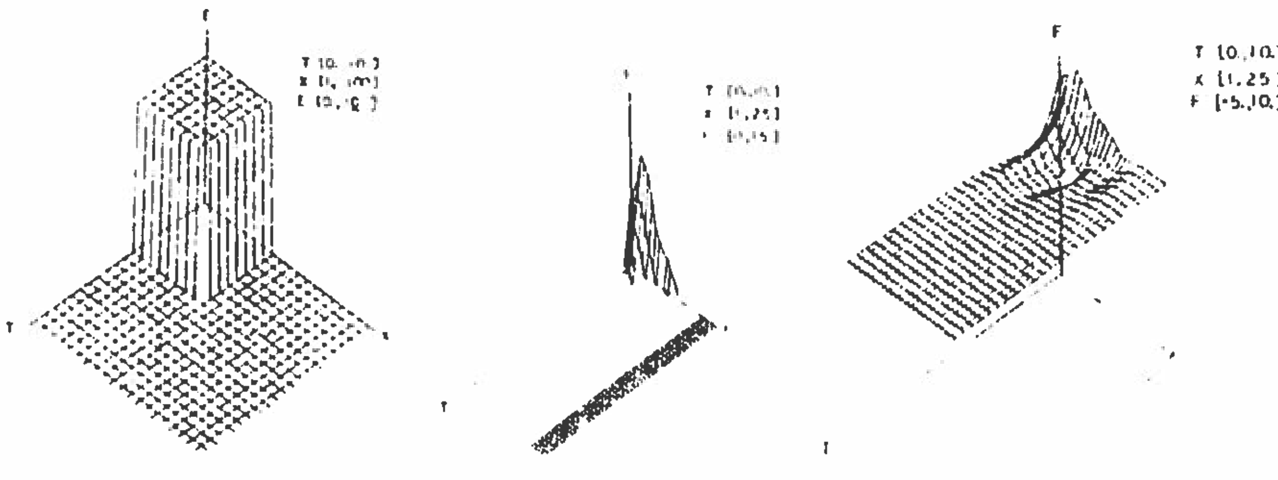


Fig. 7.1. A. Input (MF = Mossy Fibres, CF = Climbing Fibres); B. Output from the Purkinje cells; C. Output from the Purkinje cells (detail).

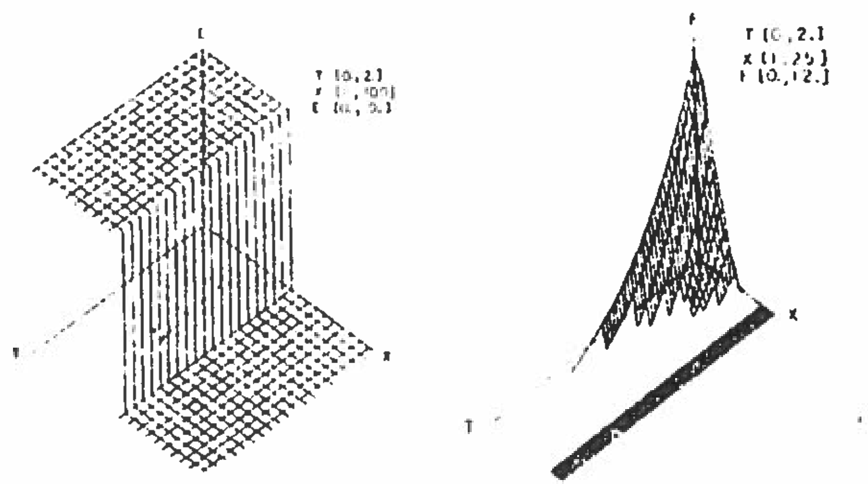
CASE #2



(A) (B) (C)

Fig. 7.2 A. Input (Mossy Fibres); B. Output from the Purkinje cells; C. Output from the Purkinje cells origin shifted to demonstrate stability.

CASE #3



(A) (B)

Fig. 7.3. A. Input (Mossy Fibres); B. Output from the Purkinje cells.

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