#209

MOSFET STATIC MODEL FROM SIMPLIFIED MEASUREMENTS

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Abstract

It is shown that MOS diode characteristics can be used to determine the device constants for the corresponding MOS triode or tetrode. Using the model of Kohli a least p^{th} fit is developed and it is shown that close fits to experimental data can result if the proper local minima of the optimization is used.

1. INTRODUCTION

The MOS (metal oxide silicon) transistor has turned out to be a very versatile device which is now used in many situations. Thus, one can buy separate transistors, groups of integrated pairs, VLSI chips, or if the equipment is on hand one can design the transistors for a specific need. This being the case it is not surprising that models for the MOS transistor have become important and that a number have appeared. The majority of these are based upon the device physics [1]-[9] and consequently are of most use in the design of the transistors themselves; these are especially important for integrated circuit layouts. Unfortunately many of these are mathematically quite cumbersome and their accuracy is seldom to within 10% of actual characteristics. Consequently, recently more analytic models have appeared which use curve fitting techniques and hence yield a higher accuracy when applied to a specific transistor [10]-[13].

The techniques used to this point in the curve fitting rely upon a full set of "3-dimensional" curves of drain current i_d versus gate-source voltage v_{gs} and drain-source voltage v_{ds}. These measurements are made upon the 3-terminal device which in general requires considerable sophistication of measuring techniques when compared to 2-terminal device measurements. Here we show that the 2-terminal device characteristics of the MOS transistor connected as a diode (gate tied to drain) completely characterize the 3-terminal behavior. Mathematically, this amounts to "lifting" a function of one variable up to a function of two variables raising some interesting practical and theoretical questions.

Already this idea has occurred in the literature. Thus, Meyer has introduced an Ebers-Moll "like" model containing field-effect diodes. For this model he comments that only one nonlinear diode characteristic is needed to specify the full v - i characteristics of the transistor [7, p. 47]. As this too has been our view we here present some results on this determination. To do this we proceed in Section 2 through a functional model, that of Kohli, to show that the same device constants determine the diode and triode characteristics. Using a least p^{th} fit to experimental data we show the results in Section 3 of least square fits. The results are discussed in the final section.

2. THE METHOD

In this section we discuss the technique which extracts a characterization of the 3-terminal MOS transistor from its characterization when connected as a diode.

Consider the typical MOS curves of Fig. 1a), b) where the diode curves result from the triode curves by setting $v_{gs} = v_{ds}$ according to the connection tying the drain to the source. Consequently the triode curve of Fig. 1a) contains all the information concerning the diode curve of Fig. 1b); to obtain Fig. 1b) from Fig. 1a) one chooses v_{ds}, goes to the $v_{gs} = v_{ds}$ curve and reads i_d for the chosen v_{ds} repeating for each v_{ds}. However, to go from the diode curve, Fig. 1b), to the triode curve, Fig. 1a), is not so straightforward nor even obvious that it can be done. Our method, given as follows, rests upon a functional characterization of i_d versus v_{gs}, v_{ds}.

Although several characterizations are available [1] [6] [9] we choose the one of Kohli [11] since mathematically it is the most tractible. In the form giving the best fits to experimental data this takes the form, for $v_{ds} \geq 0$,

$$i_d = K_0 \frac{(v_{gs} - V_T)^2}{1 + \theta (v_{gs} - V_T)^m} \times$$

$$\{1 - \exp[-K \frac{v_{ds}}{v_{gs} - V_T}]\} 1(v_{gs} - V_T) \quad (2.1)$$

where $1(\cdot)$ is the unit step function. The constants V_T, K_0, θ, K, and m are constants which characterize a given transistor and are to be determined from curves like those of Fig. 1. In order to better conceptualize this functional description we set

$$v_{ds} = x, \quad v_{gs} - V_T = \bar{x}, \quad i_d = y \quad (2.2)$$

Thus

$$y = f(x, \bar{x}) =$$

$$K_0 \bar{x}^2 (1 + \theta \bar{x}^m)^{-1} \{1 - \exp[-Kx/\bar{x}]\} 1(\bar{x}) \quad (2.3)$$

Equation (2.3) characterizes Fig. 1a) and when subjected to the constraint $x = \bar{x} + V_T$, Fig. 1b). Thus Fig. 1b) is described by

$$y = h(\bar{x}) = f(\bar{x} + V_T, \bar{x}), \quad x = \bar{x} + V_T \quad (2.4)$$

$$= K_0 \bar{x}^2 (1 + \theta \bar{x}^m)^{-1} \{1 - \exp[-K(\bar{x} + V_T)/\bar{x}]\} 1(\bar{x}) \quad (2.5)$$

Comparing $h(\bar{x})$ with $f(x, \bar{x})$ it is clear that a knowledge of the constants V_T, K_0, θ, m, and K from $h(\bar{x})$ completely determines $f(x, \bar{x})$. In other words, determination of the MOS device constants from the MOS diode curve completely characterizes the MOS transistor as a triode.

Our method then is to use eq. (2.5) to curve fit the diode curve through choice of the device parameters and thus determine $f(\cdot, \cdot)$ of eq. (2.3) which gives, as eq. (2.1), the 3-terminal MOS transistor characterization $i_d = f(v_{ds}, v_{gs} - V_T)$.

To determine the device constants we do a least p^{th} approximation. For this let i_{d-M_i} denote the current measured at the i^{th} data point, $P_i = (v_{gs_i}, i_{d-M_i})$, $i = 1, \ldots, n$, and E the error function using the calculated $i_{d-C_i} = h(\bar{x}_i)$.

Thus

$$E = \sum_{i=1}^{n} (i_{d-C_i} - i_{d-M_i})^p, \quad p \text{ even},$$

$$i_{d-C_i} = h(v_{gs_i} - V_T) \quad (2.6)$$

If we set (knowing V_T from Fig. 1b) directly)

$$a_1 = K_0, \; a_2 = \theta, \; a_3 = K, \; a_4 = m \quad (2.7)$$

then our problem is to find the set of a_i such that E is minimized over the set of all possible a_i. This is the standard minimization problem [14] and hence standard techniques can be applied. For example the gradient method is convenient since the partial derivatives $\partial h/\partial a_i$ are readily calculated and evaluated.

Of course the real question is "Does a determination of constants a_i over the diode curves determine the constants a_i over the triode curves sufficiently accurately?" The results of the next section indicate yes if properly restricted.

3. Results

Because the function $f(\cdot,\cdot)$ of eq. (2.3) involves only operations which can simply take place in a programmable pocket calculator the method of the last section is amenable to use of the pocket calculator. Consequently, a Texas Instrument 59 has been programmed according to the flow-chart of Fig. 2. Table 3.1 shows the results.

For Table 3.1 v_{gs} and v_{ds} were specified and i_d calculated, called i_{d-C}, and compared with i_d measured (given in the middle column as i_{d-M}, in two cases for an n-channel transistor (type MC14007CP)). The two cases presented are for a choice of device parameters which give in Case 1 a local minima and in Case 2 a global minima, using the gradiant method, to the error E of eq. (2.6). It is seen quite clearly that even though the error function is much smaller on the diode curve for the global minimum, being $E = 0.00669$ versus $E = 0.80513$ for the local minima parameters, the error off of the diode curves is much more (being -34.1% in i_d at $v_{gs} = 4$, $v_{ds} =$ [illegible] versus -4.92%). However, the choices of parameters for the global minimum are not very reasonable; for example one expects $K \approx 0.$[illegible], [illegible] ≈ 0.1, $K \sim 4$, $m = 1$ according to the physical meaning of the parameters.

4. Discussion

It has been shown that the MOS transistor can be characterized by a determination of the device constants solely from the curves measured with the transistor connected as a diode. These latter curves are very easily determined on an oscilloscope, without the need for elaborate curve tracing equipment. The resulting data obtained can then be put into a programmable calculator to yield the device constants. The accuracy obtained is variable and depends upon which of several local minima of the error function are used. If the global minima is used in matching to the diode curve probably no degrees of freedom are available for matching the triode curves. Consequently, as shown in Section 3, it is best to choose parameters through the use of a local minima where device constants are close to physically meaningful values. When physically meaningful device constants result Table 3.1 shows that the results match experimental data as closely as anything so far reported in the literature.

Of course the technique described does not depend upon the particular function $f(\cdot,\cdot)$ used. We chose the function of Kohli because of its analytic nature but we have obtained similar results with other functional models. Too, we have concentrated upon i_d for fixed V_T. As is known, V_T varies with substrate to source voltage v_{ss},

$$V_T = g(v_{ss}) \qquad (4.1)$$

and this dependence again shows up very characteristically on the diode curves, V_T being the breakpoint shown in Fig. 1b). Consequently $g(\cdot)$ can be similarly determined from a set of diode curves parametrized by v_{ss} [13].

It should be noted that the technique used for determining the device constants from the diode curves can also be used on the triode curves. Thus, the flow-chart of Fig. 2 still applies except that $x = v_{ds} / v_{gs}$ and $\overline{x} = v_{gs} - V_T$ are independent variables.

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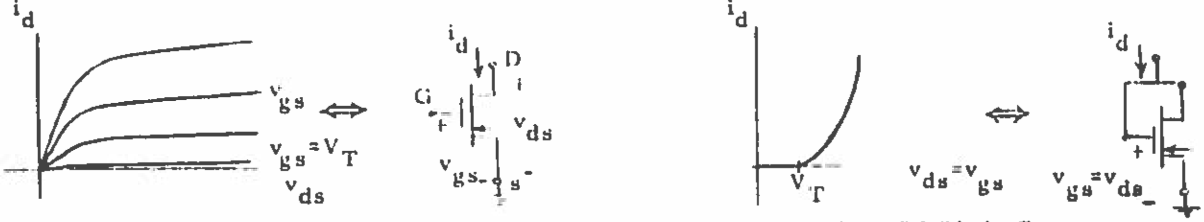
Table 3.1

Calculated vs. Measured Results

MC14007 CP n-channel transistor

		Choice 1 - Local Minima V_T = 2 (v) K_0 = 0.772 (ma) θ = 0.199 K = 3.700 m = 1.001				Choice 2 - Global Minima V_T = 2 (v) K_0 = 1.586 (ma) θ = 1.558 K = 2.21 m = 0.3	
v_{gs}	v_{ds}		i_{d-C}	% error	i_{d-M} ma	% error	i_{d-C}
2.5	2.5		0.175	- 2.55	0.18	- 2.79	0.175
3	3		0.643	7.21	0.6	3.17	0.619
3.5	3.5		1.336	2.76	1.3	- 1.13	1.285
4	4		2.204	4.97	2.1	2.25	2.147
4.5	4.5		3.213	0.40	3.2	- 0.40	3.187
5	5		4.33	- 1.51	4.4	- 0.14	4.394
5.5	5.5		5.55	- 4.37	5.8	- 0.73	5.758
6	6		6.84	- 6.36	7.3	- 0.37	7.273
6.5	6.5		8.19	- 8.00	8.9	0.38	8.93
6	1		4.14	- 2.56	4.25	-24.62	3.204
5	3		4.24	0.84	4.2	- 4.46	4.013
5	1		3.08	- 3.22	3.18	-26.12	2.349
4	1		1.86	0.48	1.85	-21.43	1.454
4	0.5		1.33	-4.92	1.4	-34.10	0.923
5.5	8		5.56	-1.56	5.65	4.49	5.904



a) MOS Triode Curves

b) MOS Diode Curves

Figure 1

MOS n-Channel Transistor Curves

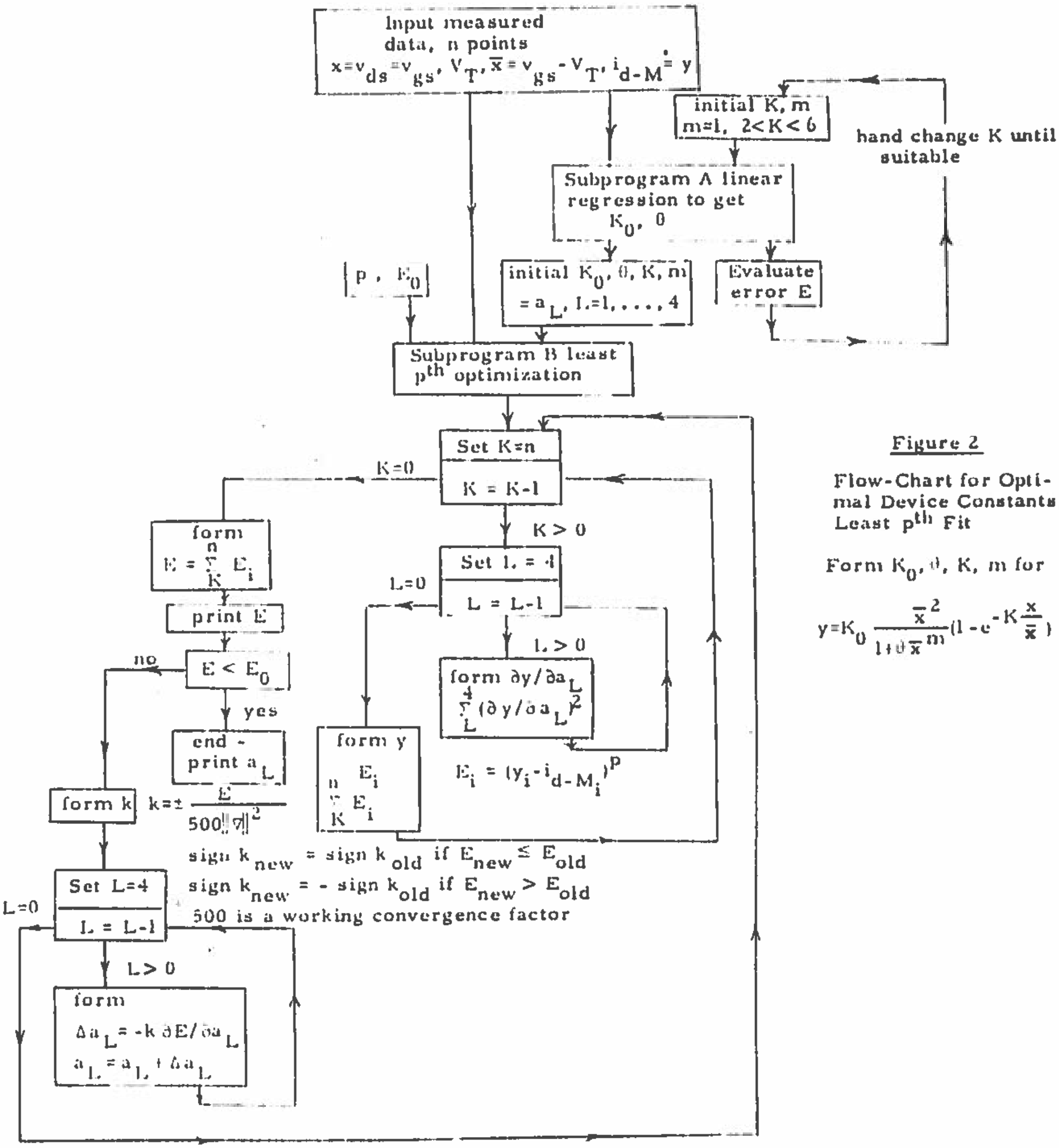


Figure 2

Flow-Chart for Optimal Device Constants, Least p^{th} Fit

Form K_0, θ, K, m for

$$y = K_0 \frac{\overline{x}^2}{1 + \theta \overline{x}^m}\left(1 - e^{-K\frac{x}{\overline{x}}}\right)$$

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