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# THE SCATTERING MATRIX OF A SAW SECTION

ABSTRACT- The scattering and transfer scattering matrices are determined as a function of frequency for N equal basic SAW sections. This uses a previously determined equivalent circuit which was analysed on chain and admittance matrix basis. These results allow for a general but quite practical formulation of a theory for SAW circuits including coverage of multiport SAW devices.

## 1. INTRODUCTION

In the last decade considerable work has been done in the surface acoustic wave field, starting from the theory and principles of operation to the design and fabrication techniques. It has been shown that in the design of SAW devices a circuit model representation is its cornerstone. Especially a frequency domain equivalent circuit model [1,2,3] has proven remarkably successful [2], in characterizing the exitation and detection of surface waves. Usually only the transfer characteristic and input admittance at the electric port of the interdigital transducer, as a vital part of every SAW device, are determined - using chain and admittance matrix analysis of the equivalent circuit. However, the reflections at acoustical and, as well, at the electrical port are also of vital importance. There are some papers [3, 4] dealing with reflection coefficients but only at the synchronious frequency. In this paper the scattering and transfer scattering matrices of a SAW device section are determined as a function of frequency. The starting point is the equivalent circuit of a basic one electrode pair section in the most general case of the ratio of the electrode width and spacing. Using the previous chain and admittance matrix results the scattering and transfer scattering matrices of N equal basis sections are determined.

## 2. REVIEW OF EQUIVALENT CIRCUIT SECTION AND SCATTERING MATRICES

The basic structure of a SAW device is one electrode pair section as shown in Fig. 1. Its equivalent model with one electric and two acoustic ports is given in Fig. 2 [1]. The acoustic variables are the velocities and the forces $v_1$ & $v_2$, $F_1$ & $F_2$, and the electric ones are $I_3$ and $V_3$. The relationship between the variables is given by:

(1) $I = [y]\, V$ where:

(2) $I = \begin{bmatrix} v_1 \\ v_2 \\ I_3 \end{bmatrix} \qquad V = \begin{bmatrix} F_1 \\ F_2 \\ V_3 \end{bmatrix}$ and according to [1 ]:

(3) $$[y] = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{11} & y_{13}(-1)^{i-1} \\ y_{13} & y_{13}(-1)^{i-1} & y_{33} \end{bmatrix}$$

(4) $$y_{11} = \frac{1}{jZ_c} \cdot \frac{\cos\theta - \Psi \sin\theta}{\sin\theta + 2\Psi(\cos\theta - 1)}$$

(5) $$y_{12} = \frac{j}{Z_c} \quad \frac{1 - \Psi \sin\theta}{\sin\theta + 2\Psi(\cos\theta - 1)}$$

(6) $$y_{13} = \frac{j\phi}{Z_c} \quad \frac{\cos\theta - 1}{\sin\theta + 2\Psi(\cos\theta - 1)}$$

(7) $$y_{33} = j\omega C_0 \left[ 1 + \frac{2k^2}{\theta} \quad \frac{1 - \cos\theta}{\sin\theta + 2\Psi(\cos\theta - 1)} \right]$$

(8) $$\phi^2 = 2k^2\, C_0\, Z_c v/\gamma\lambda_0$$

(9) $$\theta = \omega\gamma\lambda_0/2v = \gamma\pi\frac{\omega}{\omega_0}$$

(10) $$\Psi = \alpha\frac{k^2}{\theta}$$

(11) $$Z_c = Z_0(1 - \beta k^2) \quad , \quad Z_c = \rho v A$$

(12) $$C_s = \frac{C_0}{\alpha\phi^2}$$

where: $\rho$ = density of the substrate
$A$ = cross section area
$v$ = sound velocity
$K$ = electromechanical coupling constant
$\zeta$ = turns ratio of an acoustic-to-electric circuit transformer
$\alpha$ and $\beta$ are the functions of the ratio $(L_i-G_i)/G_i$,
and i denotes the position of the particular electrode pair.

It should be pointed out that the section and its equivalent model are generally valid for any ratio of electrode width $L_i-G_i$ and electrode spacing $G_i$ [1].

It is well known [5] that the scattering matrix S of a network characterized by its admittance matrix Y can be determied as:

$$S = 1_3 - 2Y_a \tag{13}$$

where $Y_a$ is the admittance matrix of the augmented network which is shown in Fig. 3.

Using $R_1=R_2=R_3=1$, $Y_a$ can be found as:

$$Y_a=Y(1_3+Y)^{-1} \tag{14}$$

Substituting (14) into (13) we get:

$$S = 1_3-2Y(1_3+Y)^{-1} \tag{15}$$

The transfer scattering matrix T, defined as in [5], can be expressed in terms of the $S_{ij}$'s of the S matrix as follows:

$$T = \begin{bmatrix} S_{12} - S_{11}S_{21}^{-1}S_{22} & S_{11}S_{21}^{-1} \\ -S_{21}^{-1}S_{22} & S_{21}^{-1} \end{bmatrix} \tag{16}$$

## 3. THE SCATTERING MATRICES FOR SAW SECTIONS

The goal of this paper is to determine the scattering and transfer scattering matrix of a SAW structure consisting of N equal basic sections connected as shown in Fig.4a. To be able to achieve it, as can be seen from (15) and (16), we can find the admittance of the whole structure. According to Fig.4a and (3), (4), (5) and (6) we find:

$$\begin{bmatrix} F_n \\ v_n \end{bmatrix} = q \begin{bmatrix} F_{n-1} \\ v_{n-1} \end{bmatrix} + \begin{bmatrix} -y_{13}/y_{12} \\ y_{13}y_{22}/y_{12}-y_{13} \end{bmatrix} V_3 \tag{17}$$

where:

$$(18)\quad \text{ч} = \frac{1}{1-\psi\sin\theta}\begin{bmatrix}\cos\theta-\psi\sin\theta & -jZ_c(\sin\theta+2\psi\cos\theta-2\psi)\\ -jZ_c^{-1}\sin\theta & \cos\theta-\psi\sin\theta\end{bmatrix}$$

Taking into account the basic sections in Fig.4a are connected accoustically in cascade and electrically in parallel, and that $y_{31}$ and $y_{32}$ in alternating sections have opposite signs (3), we get:

$$(19)\quad \begin{bmatrix}F_N\\ v_N\end{bmatrix}=\text{ч}^N\begin{bmatrix}F_1\\ v_1\end{bmatrix}+y_{13}\begin{bmatrix}jZ_c\sin N\delta\sin\delta(\sin\theta)^{-1/2}\\ -\cos N\delta(1-\psi\sin\theta)^{-1}-(-1)^{N-1}\end{bmatrix}V_3$$

where $\delta$ is defined by:

$$(20)\quad \sin\delta=(\sin\theta+2\psi\cos\theta-2\psi)^{1/2}(1-\psi\sin\theta)^{-1}(\sin\theta)^{1/2}$$

Diagonalizing ч using a unitary transformation the product is seemed to be:

$$(21)\quad \text{ч}^N=(1-\psi\sin\theta)^{-1}\sin N\delta\begin{bmatrix}\operatorname{cotg}N\delta & -jZ_c(\sin\theta)^{-1/2}\sin\delta(1-\psi\sin\theta)\\ \dfrac{-jZ_c^{-1}(\sin\theta)^{1/2}}{\sin\delta(1-\psi\sin\theta)} & \operatorname{cotg}N\delta\end{bmatrix}$$

Now, using (21) the admittance matrix Y of the whole structure shown in Fig. 4a and defined according to Fig. 4b can be calculated as follows:

$$(22)\quad Y=\begin{bmatrix}\dfrac{-j\cot N\delta}{Z_c(1-2\psi\operatorname{tg}\frac{\theta}{2})^{1/2}} & \dfrac{j(1-\psi\sin\theta)}{Z_c\sin N\delta(1-2\psi\operatorname{tg}\frac{\theta}{2})^{1/2}} & y_{13}\\ \dfrac{j(1-\psi\sin\theta)}{Z_c(1-2\psi\operatorname{tg}\frac{\theta}{2})^{1/2}\sin N\delta} & \dfrac{-j\operatorname{cotg}N\delta}{Z_c(1-2\psi\operatorname{tg}\frac{\theta}{2})^{1/2}} & (-1)^{N-1}y_{13}\\ y_{13} & (-1)^{N-1}y_{13} & Ny_{33}\end{bmatrix}$$

where $y_{13}$ and $y_{33}$ are given by (6) and (7).

Substituting (22) into (15) we can calculate $S_{ij}$'s and, therefore, T at any frequency. The calculation is straightforward but the use of a computer is helpful since the expressions are rather bulky.

## 4. DISCUSSION

In the above we have determined the scattering and transfer scattering matrices of SAW sections using the equivalent circuit available in the literature. It can be seen that the cases discussed by Smith et al. in [3, p. 860] are only the special cases of $S_{11}$ and $S_{12}$ when $R_2=R_1=R_0$, $f=f_0$, $R_3 = jY_L$ and $\alpha = 0$ ("cross-field" model or $\alpha = 1$ ("in line" model).

Of particular importance are the 2-port purely mechanical transfer scattering matrices since these simply multiply when SAW sections are cascaded.

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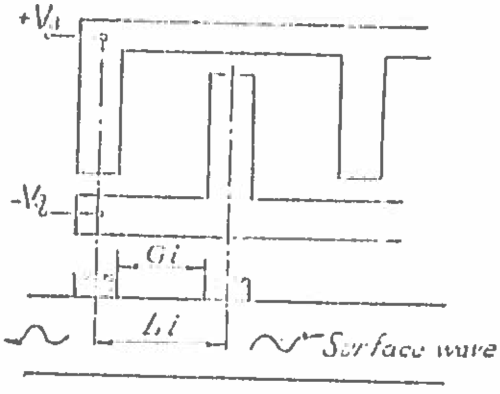

Fig. 1

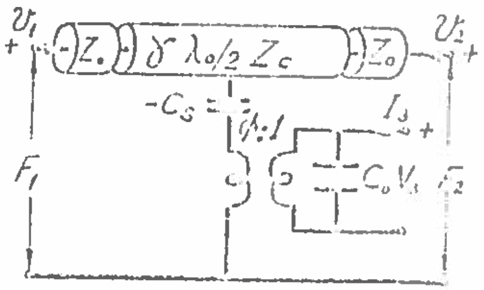

Fig. 2

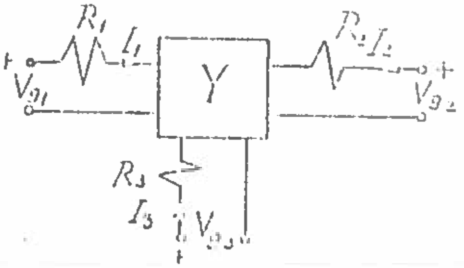

Fig. 3

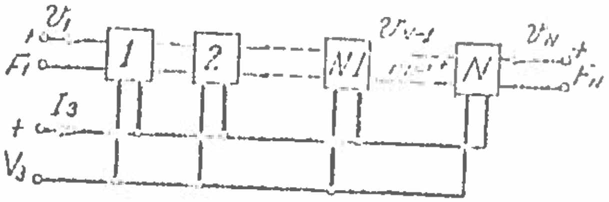

Fig. 4.a

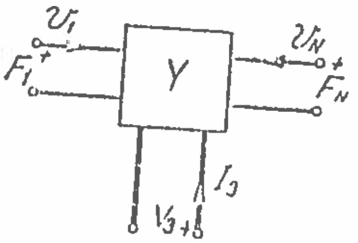

Fig. 4.b

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