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MOS FET MODEL ALGORITHMS FOR POCKET CALCULATOR NONLINEAR CIRCUIT ANALYSIS

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ABSTRACT

Starting with the model of Kohli the present MOSFET models developed by the authors for pocket calculator aided design are summarized and algorithms given for determining the device constants of the main functional relationship. Programs have been developed, for use with the TI-SR-59 calculator, for all expressions allowing complete characterization and determination of the MOSFET models from experimental data, as illustrated by an example.

I. INTRODUCTION

Because of their frequent use in the design of integrated circuits it is important to have models of MOS transistors adequate to fit the needs of the situation. For computer aided design a number of such models exist [1] - [12]. These models are generally extensive which is justifiable for their use on a large scale computer to varify the adequacy of a design. But at the first stages of a design the use of models incorporating high order effects can be confusing and tends to obscure the basic principles of design. Nevertheless it is often just the second order effects which give sought after properties in a design and, thus, models incorporating second order effects are necessary for use in the first instance of design of such circuits. This is the place where pocket calculators can be very effective. For, although for simple designs the designer can carry first order models in his head, the details of second order models are very conveniently carried in small programs suitable for pocket calculators.

Here we summarize the present state of our MOSFET models for pocket calculators, Section II, presenting in Section III a determination of the main device constants giving in Part E complete set of programs suitable for the Texas Instrument SR-59 calculator.

II. MOSFET FUNCTIONAL MODEL FOR POCKET CALCULATORS

Previously Kohli[13 pp. 33-38] [14] has introduced a very simple model for the MOS transistor suitable for pocket calculators. For an n-channel MOS FET in the active region this takes the form

$$i_D = \beta(v_{GS}-V_T)^2 \{1-\exp[-Kv_{DS}/(v_{GS}-V_T)]\} \quad (1)$$

where i_D is the drain current, v_{GS} is the gate-source voltage, V_T is the turn-on (that is, threshold) voltage [a constant for a given device], and β & K are two other constants of a given device. This was arrived at by an ingenious curve-fitting technique of Kohli and, thus, as yet has not been justified in terms of the physics of the device. But the important point to observe is that only simple differences, multiplications, division, and exponentiations are used in which case Kohli's MOSFET law is ideal for pocket calculators.

Although (1) is probably the simplest expression available for overall behavior it should be noted that operation in the square-law region results when v_{DS} is large, in which case the exp[·] term can be dropped. But even in that case other effects can be important for a designer. For example substrate bias effects can control V_T through the substrate-source voltage, v_{SS}. Further high gate-source voltages cause a significant crimping of the square-law behavior due to mobility effects degrading β. Consequently, it is convenient to reexpress (1) in the general form

$$i_D = f(v_{GS}-V_T, v_{DS}) \quad (2a)$$

and add on the relationship

$$V_T = g(v_{SS}) \quad (2b)$$

We comment that if $f(\cdot,\cdot)$ and $g(\cdot)$ are known for a transistor type, say n-channel, then the same $f(\cdot,\cdot)$ and $g(\cdot)$ can be used for the complementary type by replacing all variables by their negatives.

As noted above, in order to be useful for pocket calculator aided design the functions $f(\cdot,\cdot)$ and $g(\cdot)$ should be simply expressible in terms of the normal key stroke functions of standard programmable pocket calculators. Thus, based upon the physics of the devices, the known nature of the v-i curves, and suitability for calculator utilization, we propose in their most elaborate form, generalizing (1),

$$f(x, y) = \beta[1+\alpha x^m]^{-1} x^2 [1-\exp(-Ky/x)] 1(x), \quad (3a)$$

$1(\cdot)$ = unit step

$$g(z) = V_{T_o} + \gamma[(1-\delta z)^{\frac{1}{2}} - 1] \quad (3b)$$

where $\alpha, \beta, \gamma, \delta, K, m, V_{T_o}$, are device constants to be determined for any given transistor. We note that these functions require at most 12 data registers, but that in many cases simpler models suffice which use as little as three data registers (with $\alpha = m = 0$ if there is no deviation from square-law, omission of $\exp(-Ky/x)$ if there is not transitioning through the "resistive" region, and $\gamma = \delta = 0$ if $v_{SS} = 0$; also $1(x) = 1$ can be taken if it is known that $v_{GS} > V_T$ in all calculations).

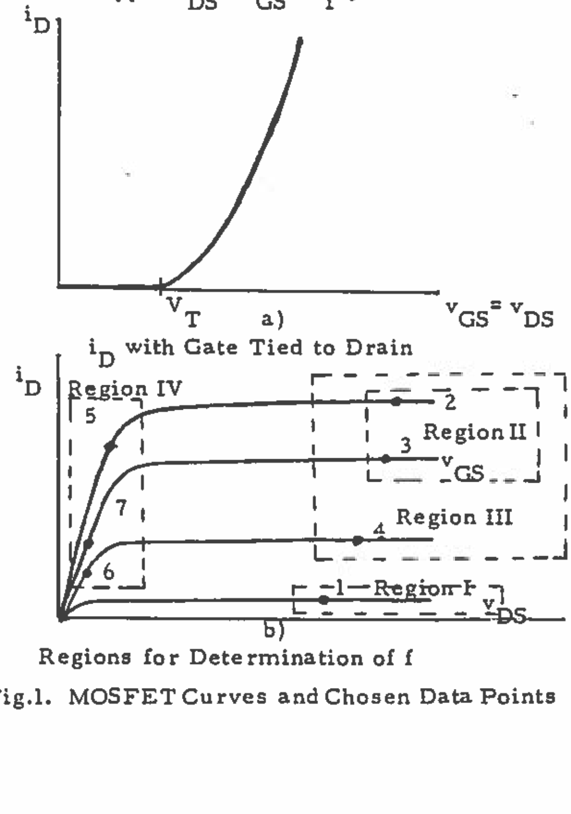
The algorithms for determining the device constants are presented in the next section.

III. ALGORITHMS FOR DETERMINING MODEL CONSTANTS

Before using the f and g of (3) it is necessary to determine the device constants. In [15] we have discussed a method of determining $g(\cdot)$ from experimental data. In this section we do the same for $f(\cdot,\cdot)$.

Figure 1 gives typical curves for n-channel MOSFETS. Using these we will give a technique for determining the device constants by using data at the numbered points. Specifically, we desire to choose α, β, K, m, V_T such that Fig. 1b) is described by

$$i_D = \beta(v_{GS}-V_T)^2[1+\alpha(v_{GS}-V_T)^m]^{-1} \{1-\exp[-Kv_{DS}/(v_{GS}-V_T)]\} \quad (4)$$



Regions for Determination of f

Fig.1. MOSFET Curves and Chosen Data Points

A. Determination of V_T.

V_T is found directly from the curve of Fig.1 a), where $v_{GS} = v_{DS}$, as the turn-on voltage, that is the voltage $v_{GS}(=V_T)$ when $i_D = 0_+$. Because of its effect on other parameters, V_T should be chosen accurately in which case expanded scale measurements around $i_D = 0_+$ are helpful.

B. Determination of β.

If v_{DS} is chosen large and v_{GS} very close to V_T, (4) reduces to $i_D = \beta(v_{GS}-V_T)^2$. Consequently, assuming these conditions for point 1 of Fig. 1b) with

$$v_{GS} = v_1, \; i_D = i_1 \quad \text{at point 1} \quad (5a)$$

we have

$$\beta = i_1[v_1 - V_T]^{-2} \quad (5b)$$

C. Determination of α and m.

Several methods are available for choosing α and m for which it is important to note are "isolated" from K by choosing v_{DS} large. For example the use of points 2 and 3 give two equations in two unknowns which are readily solved. However, it has been found more profitable to use a linear regression, which is preprogrammed within the TI-59. For this we make a change of notation in (4), valid for v_{DS} large,

$$v = i_D = \beta u^2[1+\alpha u^m]^{-1}, \quad u = v_{GS} - V_T \quad (6a)$$

or, since $\alpha u^m = (\beta u^2/v) - 1$

$$m \ln u + \ln \alpha = \ln[(\beta u^2/v) - 1] \quad (6b)$$

This expression is linear in m and $\ln \alpha$, of the form $mx + b = y$, in which case linear regression can be applied using the data

$$x_j = \ln[v_{GS_j} - V_T], \quad y_j = \ln\{(\beta[v_{GS_j} - V_T]^2/i_{D_j}) - 1\} \quad (6c)$$

For this as many data points from region III of Fig. 1b) can be used as desired. For example we use points 2, 3, 4 in our calculator program which through linear regression finds an optimal m and $\ln \alpha$.

D. Determination of K.

The purpose of K has been to be able to curve fit in region IV of Fig. 1b). Since K has the most influence for larger v_{GS} for a fixed v_{DS} we choose point 5 for the determination of K which is found by direct solution of (4) as

$$K = -[(v_{GS}-V_T)/v_{DS}] \ln\{1 - i_D[1+\alpha(v_{GS}-V_T)^m][\beta(v_{GS}-V_T)^2]^{-1}\} \quad (7)$$

Using point 5 on the same curve as point 2 yields, from (7) with

$$i_{D_5} = i_5, \; v_{GS_2} = v_2, \; v_{DS_5} = v_5 \quad (8a)$$

$$i_{D_2} \approx i_2 = \beta[v_2 - V_T]^2/[1+\alpha(v_2 - V_T)^m] \qquad (8b)$$

$$K = -[(v_2 - V_T)/v_5]\,\ln[1-(i_5/i_2)] \qquad (8c)$$

which since $i_5 < i_2$ guarantees that K exists with $\ln[1-(i_5/i_2)] < 0$.

E. Programs

The functions $f(\cdot,\cdot)$ and $g(\cdot)$ of (3) which characterize the i-v relationship of the MOSFET, and the expressions which have been given in this section for evaluating the device constants, are all suitable for evaluation by a Programmable pocket calculator because they involve simple additions, subtractions, multiplications, divisions, logarithms and exponentiations only. We programmed these functions and expressions on Texas Instrument's SR-59 pocket calculator, and they used at most 12 registers. These programs are available from the third author on request.

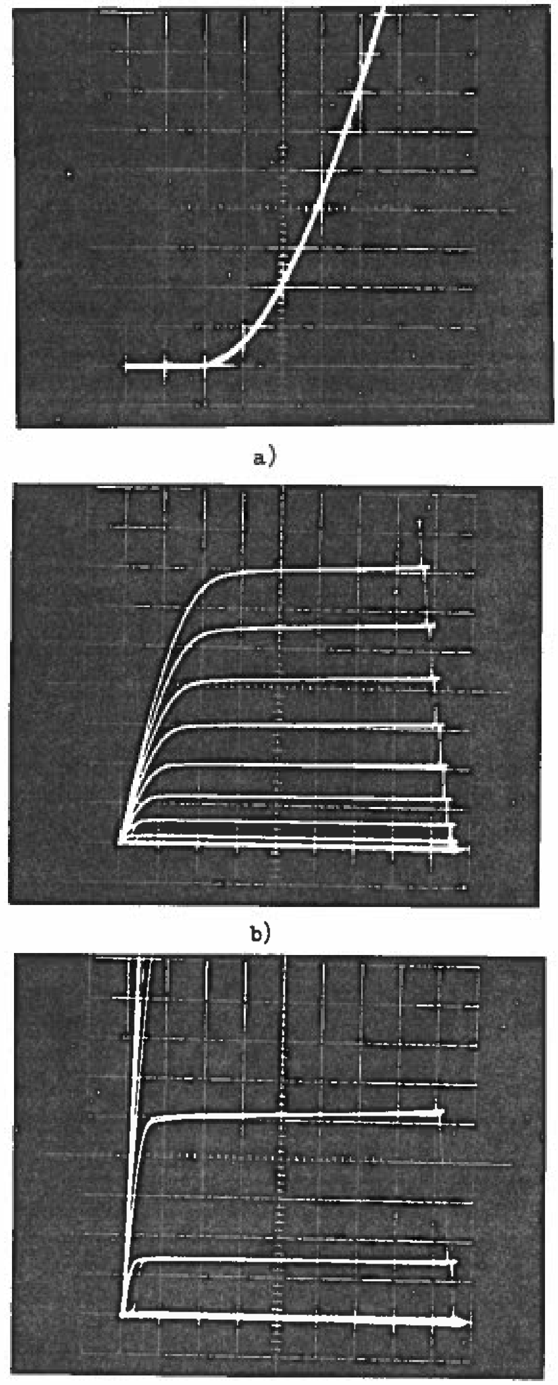
F. Example

Figure 2 (see next column) gives photographs of curves taken on the Tektronix 575 for the Motorola MC 14007CP n-channel transistor. Table 1 (see end of paper) summarizes the data used from these curves while giving the results of the programs to find the device constants for f and for evaluating i_D.

IV. DISCUSSION

Pocket calculators are now beginning to be realized as an excellent tool for the design of electronic circuits. Since among these MOS circuits are of considerable importance it is particularly vital to have models of them suitable for pocket calculators. Here we have presented such a model for the resistive behavior of MOS devices which can account for second order nonlinear effects and which by choosing various device constants zero (or in the case of K, infinity) can reduce to very simple first order models.

In using the model it is necessary to know the device constants for any particular transistor being used. Section III has presented a method for determination of these device constants for f of (4) given experimental data of the type represented by Fig. 1. The n-channel example of Sect. III shows how effective this model determination can be. However, the importance of having point 1 of Fig. 1b) lie on a horizontal curve needs to be emphasized. If no such flat curve near $i_D = 0_+$ can be obtained, as we found to be the case with a number of p-channel transistors, the method of Section III is ineffective. For this reason other techniques are under investigation [16]. The model of (4) though still appears to be a proper and effective one necessitating only an alternate means of device constant determination.



a)

b)

c)

Figure 2
MC14007 CP n-channel MOSFET Measured Curves
Horizontal, all curves = 1v/div. Vertical, a) and b) = 1 ma/div. c) = 0.1ma/div.
a) $v_{GS} = v_{DS}$ curve, b) $v_{GS} = 0.5$v/step; $v_{GSmax} = 6$ v, c) Expanded i_D Scale for curve b) v_{GS}=0.5v/step; v_{GS}=3 at i_D=0.5ma.

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Table 1

Data and Results for MOSFET for Figure 2

Measured Data

Point	i_D, ma	v_{GS}	v_{DS}
1	0.145	2.5	9
2	7.15	6	8
3	5.75	5.5	8
4	1.25	3.5	8
5	4.25	6	1

Resulting Device Constants: Calculator Readout

α = 0.0433234661
β = 0.6
K = 3.707760361
m = 1.53707124
V_T = 2

Calculated Response

v_{GS}	v_{DS}	Calculator Readout $i_D = i_{DC}$ ma	Measured $i_D = i_{DM}$ ma	% Error = $\frac{i_{DC} - i_{DM}}{i_{DM}} \times 100$
2.5	9	0.1477936499	0.15	-1.5
3	3	0.5750768201	0.58	-0.8
3.5	9	1.249079781	1.25	-0.1
4	0.5	1.288207036	1.4	-8
4	1	1.798031535	1.85	-2.8
4	4	2.130672966	2	+6.5
5	1	3.103297646	3.18	-2.4
5	3	4.267023798	4.2	+1.6
5.5	9	5.665802305	5.75	-1.5
6	1	4.25	4.25	0

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