A MOSFET MODEL WITH SUBSTRATE BIAS FOR THE POCKET CALCULATOR*

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Abstract

A model which takes into account the effects of substrate bias is obtained to describe the static voltage - current characteristics of the MOS transistor operating at various substrate bias potentials. The model uses simple functions which are suitable for evaluation by a pocket calculator. The turn - on voltage model constants are determined by simple experimental measurements. Theoretical results obtained from the model are in close agreement with experimental data.

1. Introduction

Metal Oxide Semiconductor Field Effect Transitors (MOSFETs) form a basis for a large number of circuits including those coming into play for digital and analog filtering [1]. For pocket calculator aided design it is convenient to have simple MOSFET models which are suitable for calculator programming. Such a preliminary model of the MOSFET was recently proposed [2] where the nonlinear DC characteristics of the MOSFET over a useful range of operation were modeled by a single exponential function of two variables, which can be evaluated readily with a pocket calculator. When the transistor is operated with a grounded substrate, the proposed model [2] was found to match the experimental curves reasonably well. However, in circuit design, there is often a need to operate the MOSFET with the source and substrate at different potentials, as for example in CMOS-R filters [1] [3] and modulators [4, p. 59].

In this paper we extend the MOSFET model proposed in [2] so that it describes the voltage - current behaviour of the MOSFET under different substrate bias conditions. Since the effect of the substrate bias is to vary the turn - on voltage of the transistor [5, p. 56], we derive a single analytic function which is incorporated into the model to account for the effect of the substrate bias. Simple measurements to determine the model parameters, which are constants of the describing equations, are outlined. Finally, using a pocket calculator program, the MOSFET turn - on voltages which are determined by the model equations are compared to the actual experimental characteristics, as measured under various substrate and gate bias situations, to show the degree of agreement.

2. Form of the Model

In this section we present simple functions which describe the voltage - current characteristics of the MOSFET operating as a four terminal device with a substrate potential defined independently from the source potential. The presented expressions are for an n - channel MOSFET. The p - channel MOSFET equations are obtained by substituting the negatives of the variable quantities in the equations for the n - channel devices as outlined in section 4.

The symbol for the four - terminal n - channel MOSFET is shown in Fig. 1, where the variables are also defined. The voltage - current relationship for the MOSFET can be characterized by a function f(·,·) of two variables [4, pp. 49-59] such that

$$I_D = f(V_{GS} - V_T, V_{DS}) \qquad (1)$$

where $I_D \geq 0$ is the drain current, V_{GS} (≥ 0 for enhancement devices) is the gate to source voltage, and $V_{DS} \geq 0$ is the drain to source voltage, while V_T is the turn - on, or threshold, voltage ($V_T > 0$ for enhancement devices). Recently [2] the function f was proposed to be of the following form:

$$f(x, y) = \beta x^2[1 - \exp(-Ky/x)]1(x) \qquad (2)$$

where 1(·) is the unit step function and β and K are constants for a given device.

When the substrate is shorted to the source the value of V_T is a determined value V_{To}, typical values of which are listed in data sheets. But when the substrate terminal is not shorted to the source a voltage, V_{BS}, applied on the substrate modulates the turn - on voltage, V_T, so that we can express V_T as a function, g(·), of V_{BS} through

$$V_T - V_{To} = g(V_{BS}) \qquad (3)$$

We obtain the function g for an n - channel MOSFET as follows. From the device physics it is known that the voltage V_T necessary to induce a conducting channel on the surface of a semiconductor is given by an equation of the form [6, p. 323]

$$V_T = V_{FB} + 2|\varphi| + a[2|\varphi| - V_{BS}]^{\frac{1}{2}} \qquad (4)$$

where V_{FB} is the flat-band voltage, φ is a potential of the semiconductor, V_{BS} is the substrate - to -

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source voltage, and a is a constant which depends on the semiconductor material, the device geometry, and doping. For our purposes we express Eq. (4) in the form

$$V_T = V_{To} + \gamma[(1 - \delta V_{BS})^{\frac{1}{2}} - 1] \qquad (5)$$

where we have set

$$V_{To} = V_{FB} + 2|\varphi| + a(2|\varphi|)^{\frac{1}{2}} \qquad (6a)$$

$$\delta = 1/(2|\varphi|) \qquad (6b)$$

$$\gamma = (2|\varphi|)^{\frac{1}{2}}a \qquad (6c)$$

Equation (5) is in a form where the constants V_{To}, γ, and δ can be determined by simple measurements outlined below. Thus Eq. (5) gives the required function $g(V_{BS})$, showing the effect of the substrate bias on the turn - on voltage V_T, and on the MOSFET characteristics when incorporated in Eq. (1).

3. Determination of Model Constants

Equation (5) shows the effect of a substrate bias, $V_{BS} < 0$, on the turn - on voltage, $V_T > 0$ of an n - channel MOSFET. This is reflected in the experimental curves of Fig. 2, these being curves of I_D versus V_{GS} for various V_{BS} when $V_{DS} = V_{GS}$ is set, where we note that V_T is the intersection on the horizontal axis, and that this intersection increases as V_{BS} is made more negative. The three constants V_{To}, γ, and δ in Eq. (5) are to be obtained from this type of curve in a manner outlined in this section.

Observe from Eq. (5) that V_{To} is the turn - on voltage when the substrate bias is zero, as seen by setting $V_{BS} = 0$ in the equation. Thus, to determine V_{To} we short the substrate terminal to the source, short the drain terminal to the gate, and then display the curve of the drain current, I_D, versus the gate - to - source voltage, V_{GS}. V_{To} is the value of V_{GS} at the point where the curve intersects the V_{GS} axis, which is read off the curve.

To obtain the constants γ and δ we disconnect the substrate terminal from the source in the above experiment, apply a negative voltage, V_{BS1}, on the substrate terminal while leaving the drain terminal shorted to the gate, display the I_D versus V_{GS} curve again, and read off a new turn - on voltage value, V_{T1}, on the V_{GS} axis intercept. We repeat the experiment with another substrate bias voltage, $V_{BS2} < V_{BS1}$, and obtain a corresponding turn - on voltage V_{T2}. From these measurements the constants γ and δ are determined as follows. By transposing terms and squaring, Eq. (5) can be rewritten in the form

$$(V_T - V_{To})^2 + 2\gamma(V_T - V_{To}) + \gamma^2\delta V_{BS} = 0 \qquad (7)$$

Substituting the two pairs of measured values of V_T and V_{BS} in Eq. (7) we get two equations in the two unknowns γ and δ as follows.

$$(V_{T1} - V_{To})^2 + 2\gamma(V_{T1} - V_{To}) + \gamma^2\delta V_{BS1} = 0 \qquad (8a)$$

$$(V_{T2} - V_{To})^2 + 2\gamma(V_{T2} - V_{To}) + \gamma^2\delta V_{BS2} = 0 \qquad (8b)$$

Equations (8) are linear in δ. We eliminate δ from the equations to get a single equation linear in 2γ which we solve to get γ; then by back substitution in one of the equations in (8) we obtain δ. The solutions obtained are:

$$2\gamma = \frac{(V_{T1} - V_{To})^2 V_{BS2} - (V_{T2} - V_{To})^2 V_{BS1}}{(V_{T2} - V_{To})^2 V_{BS1} - (V_{T1} - V_{To})^2 V_{BS2}} \qquad (9a)$$

$$\delta = \frac{-[(V_{T2} - V_{To})^2 + 2\gamma(V_{T2} - V_{To})]}{\gamma^2 V_{BS2}} \qquad (9b)$$

Example

As an example for determining the V_T model constants we took measurements for the RCA CA3600E n - channel enhancement transistor. Figure 2 shows the curves of the drain current I_D versus the gate to source voltage, V_{GS}, taken at various substrate bias voltages, $V_{BS} < 0$, with the drain shorted to the gate. From these curves we read the data shown in Table 1. At $V_{BS} = 0$ we obtain $V_{To} = 1.75$ volts. Using the values of V_T at $V_{BS1} = -2.5$, and $V_{BS2} = -3$ respectively in Eqs. (9) we obtain $\gamma = 1.625$ and $\delta = 1.325$ for the "best case" column. Thus, from Eq. (1), (2), and (5) the characteristics of the n - channel CA3600E MOSFET are given by the equations

$$I_D = \beta(V_{GS} - V_T)^2[1 - \exp(-KV_{DS}/(V_{GS}-V_T)]1(V_{GS}-V_T) \qquad (10a)$$

$$V_T = 1.75 + 1.625[(1 - 1.325V_{BS})^{\frac{1}{2}} - 1] \qquad (10b)$$

Using Eq. (10b) the remaining values of V_T for the "best case" column of Table 1 were calculated in which it can be seen that a maximum deviation from measured data at other points is about 2.1%. Also in Table 1 we give several other results for a perfect match of experimental and calculated values at other values of V_{BS1} and V_{BS2} from which it is seen that the overall matches are not as good as with the "best case."

4. Discussion

We tested our model on the RCA 3600E n - channel MOSFET whose experimental curves are shown in Fig. 2, these being obtained as the substrate bias voltage is varied. The experimental values of the turn on voltage as read at the intersection of the curves of Fig. 2 with the horizontal axis, and the theoretical values of the turn - on voltage as calculated by a pocket calculator from our model (Eq. (10b)), are tabulated in Table 1. The theoretical results in the "best case" are in quite close agreement with the experimental data. The maximum deviation in fact of the "best case" theoretical value from the experimental value was 2.1% for this transistor.

We found Eqs. (9) and (10) easy to evaluate by a programmable pocket calculator [we used the TI-59 calculator]. In determining the model constants γ and δ of Eq. (5) using Eqs. (8) the best results were obtained by choosing V_{T1} and V_{T2} as far away as possible from V_{To}. One might think that a further improvement could be made by evaluating several pairs of γ and δ using several combinations of the curves and taking the averages. However, just the opposite was true. γ and δ have a form of an inverse relationship, that is, when γ is large δ is small and vice versa.

γ and δ should be positive for an n - channel enhancement mode device. From Eq. (9b) δ will be positive if γ is positive, since $V_{T2} > V_{To}$ and $V_{BS2} < 0$. However, deviations in data may result in a negative value of γ in Eq. (9a), as for example with the data of Table 1 using $V_{BS1} = -0.5$ and $V_{BS2} = -1.5$. In such a case different combinations of the curves should be chosen to give a positive γ, as we have done in Table 1, while a check on the accuracy of the data should be also made. A simple check of "realizability" to yield positive γ can be set up in terms of ratios of terms in Eq. (9a) so that questionable pairs of data can be readily identified.

The MOSFET model of Eqs. (1) and (2) is most accurate at low drain currents [2]. For large drain currents a less accurate match is obtained [2] because the model does not account for the decrease in mobility at large drain currents [6, p. 376]. The decreasein mobility affects the β in the model. This effect is handled in a separate paper [7] where β is obtained as a function of the gate - source voltage, V_{GS}, to extend the range of validity of the model, and where an algorithm is given for determining the constants in the model.

The functions given in this paper are for the n - channel MOSFET. The expressions for a p - channel device are obtained by substituting the negatives of the variables in the n - channel device expressions. Thus, for the p - channel device Eqs. (1) and (5) become

$$-I_D = f(-[V_{GS} - V_T], -V_{DS}) \qquad (11a)$$

$$V_T - V_{To} = -g(-V_{BS}) \qquad (11b)$$

where V_T, V_{To} and $-V_{BS}$ are negative quantities for p - channel (enhancement) devices.

5. References

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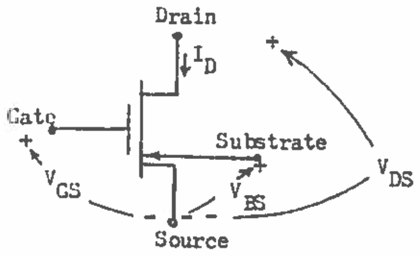

Figure 1
n - Channel MOSFET Symbol and Variables

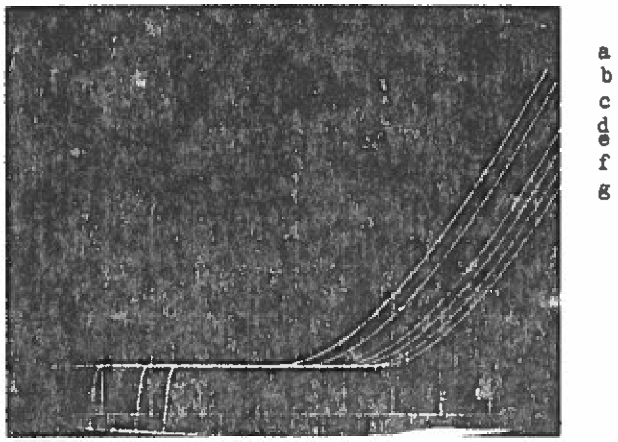

Figure 2
I_D versus V_{GS} when $V_{GS} = V_{DS}$ for an n - channel MOSFET Type CA3600E Showing the Effects of a Bias, V_{BS}, between the Source and the Substrate.

Horiz. = 1 v/div (zero at 3 divs. from left)
Vert. = 1 ma/div (zero at 1 div. from bottom)

Curve a for $V_{BS} = 0$; Curves b, c, d, e, f, g in -0.5 v steps up through -3.0 v.

Table 1
Calculated and Measured V_T Values for the Transistor of Figure 2

V_{BS}	V_T Experimental	V_T Theoretical		
		Best Case	Case Two	Case Three
0	1.75	1.75	1.75	1.75
-0.5	2.20	2.220	2.218	2.199
-1.0	2.60	2.603	V_{T1} = 2.60	V_{T1} = 2.60
-1.5	2.95	2.934	2.931	2.964
-2.0	3.30	3.230	3.228	V_{T2} = 3.30
-2.5	3.50	V_{T1} = 3.50	3.499	3.6138
-3.0	3.75	V_{T2} = 3.75	V_{T2} = 3.75	3.909
	γ =	1.625	1.666	3.192
	δ =	1.325	1.218	0.604

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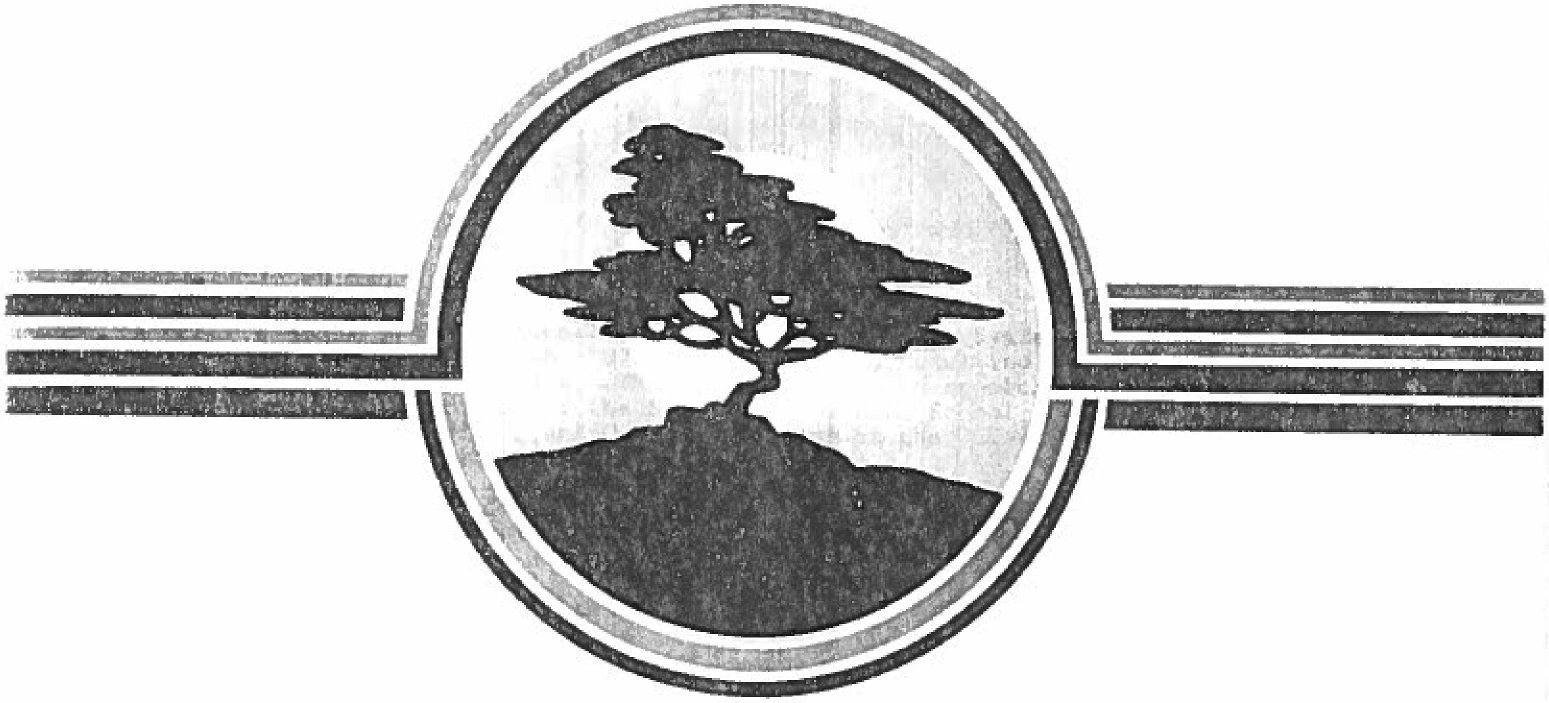

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