method. The value of $K_s$ was obtained to be 11.5 and the value of $\sigma_0$ agreed to within 10 per cent of the dc value.

It may be noted that the proposed method requires comparatively simple experimental arrangement, namely a SWR detector and a variable-frequency signal source only. The temperature control may also be conveniently introduced by enclosing the sample holder inside a bath. A detailed description of the method for measuring the conductivity and dielectric constant at different temperatures and the experimental results will be published later.

The authors wish to express their appreciation of the constant encouragement received from Prof. J. N. Bhar and their thanks to other colleagues of the laboratory for their kind cooperation.

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## Reciprocal Ferrite Phase-Shifter Measurements*

The results of high-power (spin-wave) experiments on microwave ferrites are generally published in the form of $y=f(h)$, where $y$ is the loss component of permeability or susceptibility and $h$ is the microwave magnetic field. Similar results for the real component of the complex permeability can also be plotted but are rarely published. These graphs represent important intrinsic characteristics of the ferrites but unfortunately mean little to the general microwave practitioner who prefers to see graphs of phase shift and attenuation in a given geometry. It must be realized, however, that phase shift cannot be entirely ascribed to the real, and loss to the imaginary component of the complex permeability.

In the course of some experiments on reciprocal phase shifters it was found convenient to plot their behavior in the form of Smith charts, in which the two components mentioned above can easily be recognized. The experiments were performed at 9 Gc in rectangular large $X$-band waveguide. The test circuits for low- and high-power measurements are shown in Figs. 1 and 2, respectively. In the latter case the incident power was determined by replacing the ferrite phase shifter with a solid short.

The difference in behavior between the MgMn ferrite "Ferramic R1"[1] and the Ni ferrite "Airtron C3P50"[2] is striking. The former exhibits an obvious increase of its loss component in higher microwave fields, while the reactive component remains almost constant, as shown in Fig. 3. The magnitude of the RF field was calculated at about 30 oersteds for the maximum power input of 100 kw. In the case of the Ni ferrite, on the other hand, the two components of the complex permeability seem to have switched roles. Fig. 4 shows that at 1200 oersteds the loss component experiences little change with increasing power, while the phase-shifting ability is noticeably reduced. At 700 oersteds neither component exhibits a significant change. Because of this unexpected behavior the phase shifter was cold-tested again after the high-power experiments; no significant deviation from the original results were observed.

Although the graphs of Figs. 3 and 4 do not represent intrinsic material characteristics it is felt that such a presentation conveys more information to the microwave engineer than the usual susceptibility plots. If some standard configuration could be agreed upon, the Smith-chart presentation should be quite sufficient for most power applications of ferrites.

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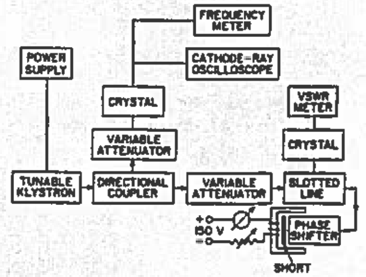


Fig. 1—Cold-test circuit.

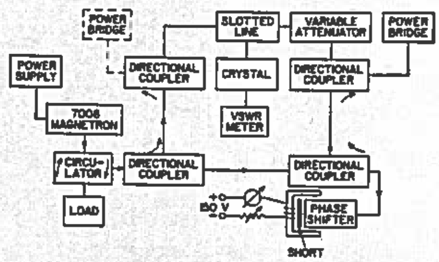


Fig. 2—High-power test setup.

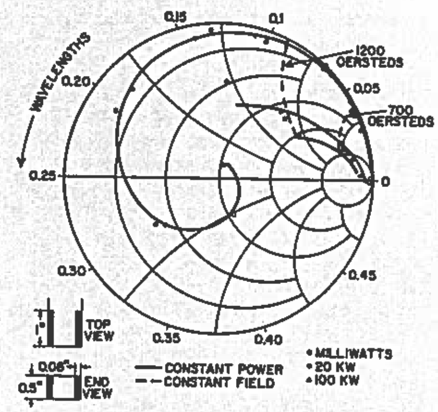


Fig. 3—Phase-shifter characteristics with Ferramic R1.

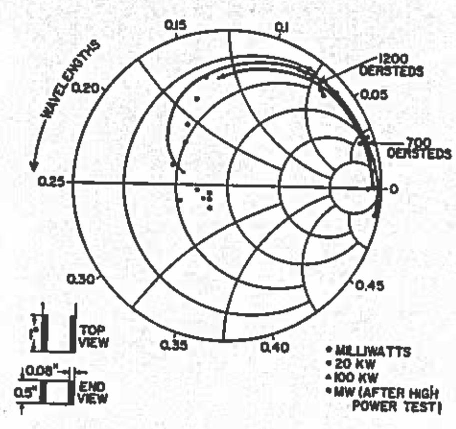


Fig. 4—Phase-shifter characteristics with Airtron C3P50.

* Received May 16, 1962.
[1] Indiana General Co., Keasby, N. J.
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## Hilbert Transforms and Positive-Real Functions*

In a recent communication[1] Papoulis has given a proof of the angle constraint for positive-real functions along the lines of that which Bayard attributes to Leroy.[2] The method is based upon the Hilbert transform and consequently bypasses the lengthy arguments based upon Schwarz's lemma.

Since the proof based upon the Hilbert transform is less general than that using Schwarz's lemma, it is of interest to know the limitations of the first method. For instance, the Hilbert transform method cannot be directly applied to the positive-real functions

$$F(p) = p,$$

$$F(p) = +\sqrt{p},$$

$$F(p) = \sum_{m=1}^{\infty} [p/(m!)]/[p^2 + (1/m)^2].$$

In contrast, the Schwarz's lemma proof holds for all positive-real functions.

To see the limitations, we realize that the Hilbert transform proof rests upon equations (5) and (6) of Papoulis

$$R(\omega) = \int_{-\infty}^{\infty} R(y)\delta(\omega - y)dy \qquad (5)$$

$$F(p) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{R(y)}{p - jy}\, dy. \qquad (6)$$

* Received June 11, 1962; revised manuscript received, June 25, 1962.
[1] A. Papoulis, "Hilbert transforms and positive-real functions," Proc. IRE (*Correspondence*), vol. 50, p. 470; April, 1962.
[2] M. Bayard, "Théorie des réseaux de Kirchhoff," Éditions de la Revue d'Optique," Paris, France, p. 259; 1954.

For (5) to be valid we require $R(y)$ to be a distribution, and for (6) to be valid we require that $R(y)$ and $X(y)$ be distributions which can be convoluted with $1/\omega$ and

$$R(\omega) = \lim_{\substack{\sigma \to 0 \\ \sigma > 0}} \operatorname{Re} F(p), \qquad X(\omega) = \lim_{\substack{\sigma \to 0 \\ \sigma > 0}} \operatorname{Im} F(p)$$

where the limit is taken in the distributional sense.[3] For this we assume, along with Papoulis, that Re $p>0$ and that the imaginary constant at infinity, which generally occurs in (6), is zero, since we are interested in positive-real $F(p)$.

The limitations placed upon the Hilbert transform proof are then

1) $R(\omega)$ and $X(\omega)$ be distributions defined by the above limit, and
2) $R(\omega)$ and $X(\omega)$ be convolutable with $1/\omega$.

If these restrictions are satisfied, the proof given by Papoulis is valid. ($F$ need not be rational.) Note that the three functions given above don't satisfy these restrictions. The beauty of the proof of Papoulis lies in the interpretation of (6) as a convolution, to which engineering concepts such as the impulse response can be applied.

It should be emphasized that the real part of $F(p)=K/(p-j\omega_0)$ is undefined. If in this latter expression one approaches $j\omega_0$ by letting $0<\sigma\to 0$, then one gets $R(\omega) = K\pi\delta(\omega-\omega_0)$ for the "real part." However, if one approaches $j\omega_0$ by letting $0>\sigma\to 0$, then one gets $-K\pi\delta(\omega-\omega_0)$ as the "real part." If one lets $\sigma\to 0$ through both positive and negative $\sigma$, then an indeterminate expression results.[3]

By using Schwarz's lemma, one easily shows that, $\measuredangle$ = angle of,

$$|\measuredangle F(p)| \leq |\measuredangle p| \quad \text{in Re } p > 0$$

if $F(p)$ is any positive-real function; conversely if $F(p)$ is analytic in Re $p>0$ and satisfies this constraint, it is positive-real.[4] We wish to point out that if $F(p)$ is a symmetric, positive-real, $n\times n$ matrix,[5] then $\tilde{x}F(p)x$ is a positive-real scalar for every real $n$ vector $x$; tilde = transpose. Consequently an $n\times n$, symmetric matrix, $F(p)$, is positive-real if and only if

1) $F(p)$ is analytic in Re $p>0$
2) $|\measuredangle \tilde{x}F(p)x| \leq |\measuredangle p|$ in Re $p>0$ for every real $n$-vector $x$.

We also wish to point out that, as Prof. Kuh has suggested, the angle constraint for nonreciprocal networks is still unknown. In this latter connection, Belevitch has some interesting results which are soon to appear.[6]

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[3] R. W. Newcomb, "Hilbert Transforms—Distributional Theory," Stanford Electronics Lab., Stanford University, Calif., Tech. Rept. No. 2250-1, p. 2; February, 1962.

[4] D. F. Tuttle, Jr., "Network Synthesis," John Wiley and Sons, New York, N. Y., vol. I, pp. 115–121; 1958.

[5] D. C. Youla, L. J. Castriota, and H. J. Carlin, "Bounded real scattering matrices and the foundations of linear passive network theory," IRE Trans. on Circuit Theory, vol. CT-6, pp. 102–124; March, 1959. (See p. 122, def. 21; note that $F(p)$ need not be rational.)

[6] V. Belevitch, private correspondence; May 2, 1962.

## Selectivity and Sensitivity in Functional Blocks*

The problem of obtaining a narrow bandpass (*i.e.*, selective) frequency response without inductance has long been of interest. The main area of application of such selective networks has been at low frequencies where the inductors that would be required are undesirably large. Presently a selective network is being sought that can be built into a solid functional block where appreciable inductance is not obtainable. The purpose of this communication is to point out the seriousness of the sensitivity to component variations which is inherent and unavoidable in most of the solutions that are being proposed. Although this sensitivity has been known for years,[1–4] it is often either ignored or else treated as something which can be overcome by some slight refinement.

This discussion pertains to all linear networks which achieve selectivity by using an active element to counterbalance losses in the remainder of a feedback loop. Included are all amplifiers using an RC null circuit or phase shifter in the feedback path and all of the various forms of simulated inductance in negative resistance devices where feedback is inherent in the bilateral negative resistance.[5–9] The resonant response of such networks is dominated by a conjugate pair of simple poles of the transfer function lying very close to the $j\omega$ axis. If a $Q$ of 10 is arbitrarily accepted as the lower boundary for "selective" response, the $Q$ resulting from a pair of poles at $s_p=\sigma_p\pm j\omega_p$ is

$$Q = \frac{\omega_p}{-2\sigma_p}. \tag{1}$$

The selective region of the $s$ plane for which $Q\geq 10$ then extends only $\tan^{-1} -\sigma_p/\omega_p \approx 3^0$ to the left of the $j\omega$ axis. Any error in a component which causes the poles to move to the left of their desired location causes loss of selectivity, while a small error to the right causes loss of bandwidth. Of course the network becomes self-oscillatory if the poles touch the $j\omega$ axis.

It is not difficult to show quantitatively how sensitive $Q$ is. The sensitivity of the dominant pole is given by

$$S_x^{s_p} = \frac{ds_p}{s_p}\cdot\frac{x}{dx} = \frac{(d\sigma_p + jd\omega_p)}{(\sigma_p + j\omega_p)}\cdot\frac{x}{dx} = \frac{(\sigma_p d\sigma_p + \omega_p d\omega_p) + j(\sigma_p d\omega_p - \omega_p d\sigma_p)}{\sigma_p^2 + \omega_p^2}\cdot\frac{x}{dx}. \tag{2}$$

This sensitivity may be evaluated from the characteristic equation of the network and is equal to the percentage variation in the pole caused by a 1 per cent error in some element, $x$. The $Q$ sensitivity is then

$$\begin{aligned} S_x^Q &= \frac{dQ}{Q}\cdot\frac{x}{dx} = \frac{\omega_p d\sigma_p - \sigma_p d\omega_p}{-\sigma_p\omega_p}\cdot\frac{x}{dx} \\ &= 2\,\frac{\omega_p}{-2\sigma_p}\cdot\frac{\omega_p d\sigma_p - \sigma_p d\omega_p}{\omega_p^2}\cdot\frac{x}{dx} \\ &\approx -2Q \operatorname{Im} S_x^{s_p} \quad \text{for } \omega_p \gg -\sigma_p. \end{aligned} \tag{3}$$

The real part of the pole sensitivity results in a radial motion of the pole which changes the resonant frequency but does not affect $Q$. On the other hand, the $Q$ sensitivity is the imaginary part of the dominant pole sensitivity magnified by a factor of $2Q$.

If (2) and (3) are evaluated for any passive selective network (which must be absolutely stable) such as an $LC$ tank circuit, the imaginary part of the pole sensitivity is found to be inversely proportional to $Q$. Therefore, the $Q$ sensitivity is independent of $Q$ and is not excessive.

However, it can be shown that the imaginary part of the pole sensitivity of an active feedback network which achieves selectivity by approaching the verge of oscillation does not change appreciably across the narrow selective region. Therefore, the higher the $Q$ that is achieved, the more sensitive $Q$ is to element variations. As an example, if a standard parallel-$T$ null circuit is used in the feedback path of a high-gain amplifier to obtain selectivity, the imaginary parts of the pole sensitivities for the various elements range from $\frac{1}{8}$ to $\frac{1}{4}$. To attain a $Q$ of 50 within $\pm 10$ per cent would then require that the components in the null circuit maintain tolerances of about $\pm 0.1$ per cent over the entire range of operating conditions. Similar conclusions apply to all other ways of achieving selectivity by using either an active feedback loop or negative resistance.

Although these concepts are not new to circuit theorists, they do not seem to have been fully appreciated by those who are seeking solutions to the solid-state tuning problem. A realistic approach to selectivity requires an understanding of the related problems of sensitivity. The desired selectivity must not only be *attained*, but also *maintained* within reasonable limits. On this basis a proposed solution is worthy of continued investigation only if the associated tolerances are likely to be achievable.

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* Received May 14, 1962.

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