TIME-INVARIANT ADJOINT NETWORKS: GENERALIZED DESCRIPTION FORMULATION*

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ABSTRACT

The adjoint network is developed through the allowed pair formulation of networks and shown to yield a general description representation in the time-invariant case.

I. INTRODUCTION

Since Bordewijk [1] introduced the concept of interreciprocity as applied to electrical networks, the adjoint network has played an important role in Computer Aided Design [2] and network sensitivity [3]. Its description has recently been developed in terms of general descriptions on graphs by Desoer [4].

In this paper a formulation of the adjoint network N^a is outlined in terms of the definition [5] of an n-port network N through its allowed pairs of voltage and current, $[v, i] \in N$. This is developed for any linear time-invariant network having the frequency domain general description $AV = BI$, amplifying the idea of Desoer. Moreover, the general description for the adjoint network N^a is found.

II. FORMLATION OF THE ADJOINT NETWORK

Given an n-port network defined through its set of allowed time-domain voltage - current pairs $[v, i] \in N$, [5, p.7], with v and i the n-vectors of port voltages and currents, we define the adjoint n-port network N^a by $[v^a, i^a] \in N^a$ if and only if for all $[v, i] \in N$ and all t and τ

$$[\tilde{v}(t), \tilde{i}(t)]\begin{bmatrix} i^a(\tau) \\ -v^a(\tau) \end{bmatrix} = 0 \qquad (1)$$

where $\tilde{}$ denotes the transpose. Let $t = x - \tau$ and and integrate over τ to get convolution in (1). On taking Laplace transforms this gives in the frequency domain

$$[\tilde{V}(p), \tilde{I}(p)]\begin{bmatrix} I^a(p) \\ -V^a(p) \end{bmatrix} = 0 \qquad (2)$$

If N is linear and characterized in the frequency domain by the general description

$$AV = BI \qquad (3)$$

then we can similarly characterize N^a as follows: Equation (3) implies

$$[A, -B]\begin{bmatrix} V \\ I \end{bmatrix} = 0_{n\times 1}$$

or equivalently on transposing

$$[\tilde{V}, \tilde{I}]\begin{bmatrix} \tilde{A} \\ -\tilde{B} \end{bmatrix} = 0_{1\times n} \qquad (4)$$

Multiplying by an arbitrary n-vector X, Equation (4) gives

$$[\tilde{V}, \tilde{I}]\begin{bmatrix} \tilde{A} \\ -\tilde{B} \end{bmatrix} X = 0 \qquad (5)$$

On identifying with Equation (2)

$$I^a = \tilde{A}X \qquad (6a)$$

$$V^a = \tilde{B}X \qquad (6b)$$

III. GENERAL DESCRIPTION OF THE ADJOINT NETWORK

By finding a common left multiple M of $\tilde{A}$ and $\tilde{B}$ for Equations (6), following [6, p.35,36] (as illustrated by the example below), we obtain

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$$[B^a, A^a]\begin{bmatrix}\tilde{A}\\ -\tilde{B}\end{bmatrix} = 0_{nxn} \quad (7a)$$

or

$$M = B^a\tilde{A} = A^a\tilde{B} \quad (7b)$$

where $[B^a, A^a]$ has rank n, the maximum possible rank. Thus, for these matrices B^a and A^a in p, Equations (6) give

$$B^aI^a = B^a\tilde{A}X = MX = A^a\tilde{B}X = A^aV^a \quad (8)$$

The general description of the adjoint network N^a is thus established as

$$B^aI^a = A^aV^a \quad (9)$$

which completely describes the adjoint network in view of the rank n condition satisfied by $[B^a, A^a]$ as mentioned above.

__Example__

A current controlled current source has

$$AV = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix} = \begin{bmatrix}0 & 0\\ \beta & -1\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} = Bi$$

We form, with the W matrix nonsingular, using elementary row and column operations

$$\begin{bmatrix}W_{11} & W_{12}\\ W_{21} & W_{22}\end{bmatrix}\begin{bmatrix}\tilde{A} & 0\\ -\tilde{B} & 0\end{bmatrix} = \begin{bmatrix}D & 0_n\\ 0_n & 0_n\end{bmatrix}$$

from which

$$W_{21}\tilde{A} = W_{22}\tilde{B}$$

In the case of our example

$$\left[\begin{array}{cc|cc}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ \hline 0 & 0 & -1 & -\beta\\ 0 & 1 & 0 & 0\end{array}\right]\left[\begin{array}{cc|cc}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \hline 0 & -\beta & 0 & 0\\ 0 & 1 & 0 & 0\end{array}\right] = \left[\begin{array}{cc|cc}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ \hline 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$$

or, extracting the 2x2 (2,1) submatrix,

$$\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}-1 & -\beta\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & \beta\\ 0 & -1\end{bmatrix}$$

Multiplying by X on the right gives, from (6)

$$\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}i_1^a\\ i_2^a\end{bmatrix} = \begin{bmatrix}-1 & -\beta\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_1^a\\ v_2^a\end{bmatrix}$$

which is a (reversed) voltage controlled voltage source. The circuits are as shown in Figure 1.

IV. CONCLUSIONS

We have extended in a straightforward manner the approach of Desoer for formulating the adjoint network. This establishes the constraint $B^aI^a = A^aV^a$ for N^a directly from the port variable definition of the original network N in the frequency domain without reference to the network graph. The time-variable case will be developed elsewhere because it is too lengthy to present here. The formulation here, though, when applied to the special cases of Z, Y, and H descriptions, gives the classical results, $Z^a = \tilde{Z}$, $Y^a = \tilde{Y}$, and $H^a = \tilde{H}$, respectively.

V. REFERENCES

[1] J. L. Bordewijk, "Interreciprocity Applied to Electrical Network," Applied Science Research, V. B6, 1957, pp. 1 - 74.

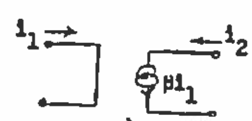
[2] S. W. Director and R. A. Rohrer, "On the Design of Resistance n-Port Networks by Digital Computer," IEEE Transactions on Circuit Theory, Vol. CT-16, No. 3, August 1969, pp. 337 - 346.

[3] S. W. Director and R. A. Rohrer, "The Generalized Adjoint Network and Network Sensitivity," IEEE Transactions on Circuit Theory, Vol. CT-16, No. 3, August 1969, pp. 318 - 323.

[4] C. A. Desoer, "On the Description of Adjoint Networks," IEEE Transactions on Circuits and Systems, Vol. CAS-22, No. 7, July 1975, pp. 585 - 587.

[5] R. W. Newcomb, "Linear Multiport Synthesis," McGraw-Hill Book Co., New York, 1966.

[6] C. C. McDuffee, "The Theory of Matrices," Chelsea Publishing Co., New York, 1956.



a)
Original Network

b)
Adjoint Network

Figure 1
Example Circuits

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