LUMPED MODEL FOR SINGLE SILICON BEAM FILTERS*

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Abstract

A linearized lumped model is presented for resonant beams etched from Silicon. From this, the deflection, for small motions, versus input voltage is determined. Discussion is given on the use for filter transfer function calculation and extensions to multibeam structures.

1. Introduction

In the last two decades considerable effort has been made to solve the problem of high Q selective circuits for low frequencies in integrated circuit technology. However, none of the proposed solutions [1] [2] is completely good. Some of the proposed circuits can have high Q but their sensitivity to the circuit elements is too high making them impractical. Another group of the proposed solutions, where this is not the case, has another drawback: the ratio of the maximal to the minimal values of the elements is proportional to the Q factor [2]. This has suggested the use of mechanical resonators [3]. Some of these electromechanical filters are bulky and nonintegrable [4], but one of them the Silicon Beam Filter [5] is compatible with integrated circuit technology. The resonant element of the Silicon Beam Filter is a cantilevered silicon beam etched from the silicon substrate [5]. In order to find the transfer function of the filter it is necessary to find the deflection of the beam as a function of input voltage. Since the beam is a distributed mass system its motion is represented by a partial differential equation of the fourth order, which is not very easy to solve. However, if the deflection is small, a linearized lumped equivalent model for the system, as presented here, will give sufficiently good results for the deflection with less complicated calculations. From this the use of the model in the transfer function is discussed.

2. The Model

A lossless single silicon beam clamped at the left is shown in Fig. 1 for which the lumped equivalent model is presented in Fig. 2 where k is the equivalent spring constant and m the equivalent mass. The spring constant is almost equal to the static spring constant and is given by [6, p.92]:

$$k = 1.03\,\frac{ET^3W}{4L^3} \qquad (1)$$

where E is Young's modulus in dynes/cm^2
T is thickness of the beam in cm
L is length of the beam in cm
W is width of the beam in cm.

The equivalent mass m of the vibrating beam is one fourth of the total mass and is given by [6, p.92]:

$$m = 0.25\rho LWT \qquad (2)$$

where ρ is the density in gm/cm^3.

By setting up the corresponding differential equation for the system shown in Fig. 2, the resonant frequency is calculated giving:

$$f_1 = \frac{1}{2\pi}\left(\frac{k}{m}\right)^{\frac{1}{2}} \qquad (3)$$

Substituting (1) and (2) into (3) gives

$$f_1 = 0.1615\; TL^{-2}\left(\frac{E}{\rho}\right)^{\frac{1}{2}} \qquad (4)$$

Equation (4) is identical to the expression for the fundamental natural frequency obtained from the partial differential equation of the vibrating beam shown in Fig. 1 [5].

The deflection of the free end of the beam, denoted as y(L,t) can also be calculated using the lumped equivalent model. If the force applied at the free end of the beam has the form $F\exp(st)$, $s=j\omega$, then the differential equation of motion for the system shown in Fig. 2 is

$$m\ddot{y}(L,t) + ky(L,t) = Fe^{st} \qquad (5)$$

The solution of (5) is found to be [5]:

$$y(L,t) = \frac{3.884\;FL^3}{EWT^3}\,\frac{1}{\left(1+\frac{s^2}{\omega_1^2}\right)}\,e^{st} \qquad (6)$$

where $\omega_1 = 2\pi f_1$.

It can be shown [5, p.23] that (6) is almost identical to y(L,t) calculated using the partial differential equations. The lumped model gives only the deflection of the free end and not the deflection as a function of the distance from the clamped end. However, if the deflection is small, the ratio of the deflection y(x,t) at the distance x from the clamped end to y(L,t) can be approximated through the following ratio of statical deflections [7] [8, p.80]

$$\frac{y(x,t)}{y(L,t)} = \frac{x}{3L} + \frac{x^2}{L^2} - \frac{x^3}{3L^3} \qquad (7)$$

3. The Transfer Characteristic

The results of the previous section can be used for the calculation of the transfer characteristic $T'(s) = y(x_B)/V_i$, where x_B denotes the position of the output transducer, see Fig. 1, and V_i is the amplitude of the input voltage. Since T'(s) can be presented in the form

$$T'(s) = \frac{y(x_B)}{V_i} = \frac{y(x_B)}{F}\cdot\frac{F}{V_i} \qquad (8)$$

and $y(x_B)/F$ is known from (6) and (7) we need only find

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F/V_i. The term F/V_i represents the transfer characteristic of the input transducer. If it is of electrostatic type, as in Fig. 1, then the force f produced by the voltage v_i applied between the two metal plates separated by air is given by [9, pp.63-66]:

$$f = v_i^2 \frac{A\epsilon_o}{2a^2} \qquad (9)$$

where A is the area of the plates
a is the distance between the plates
ϵ_o is the permitivity of air.

If $v_i = E_o + V_i \cos\omega t$ and $V_i \ll E_o$ so that the system can be linearized, the signal component of the force is

$$F\cos\omega t = \frac{A\epsilon_o E_o}{a^2} V_i \cos\omega t$$

or

$$\frac{F}{V_i} = A\epsilon_o E_o a^{-2} \qquad (10)$$

Finally, using (6), (7), (8), and (10) we find:

$$\frac{y(x)}{V_i} = \frac{3.884\ L^3}{EWT^3a^2} \frac{AN\epsilon_o E_o}{(1 + \frac{s^2}{\omega_1^2})} \left(\frac{x}{3L} + \frac{x^2}{L^2} - \frac{x^3}{3L^3} \right) \qquad (11)$$

where N is a fractional coefficient which takes care of the fact that the force F does not act at the very end of the beam, but on the length $L - L_1$ as shown in Fig. 1. The value of N is calculated as [10, p.100case 1&4]

$$N = \frac{3}{8}\left(\frac{L_1^2}{L^2} + \frac{L_1}{L} + 1 - \frac{L_1^3}{3L^3} \right) \qquad (12)$$

4. Discussion

It has been shown that the lumped equivalent model of the single silicon beam gives accurate results for frequency, deflection, and transfer characteristics with very simple calculations. The mathematics involved is far less complicated than the mathematics utilized in the partial differential equation theory of vibrating bars. The transfer characteristic found here can be used for the calculation of the transfer function defined as V_o/V_i where V_o is the output voltage and V_i is the input voltage of the device. Since the transfer characteristic is given by $y(x_B)/V_i$ it should be multiplied by $V_o/y(x_B)$ to give the transfer function. The term $V_o/y(x_B)$ represents the transfer characteristic of the output transducer.

The lumped equivalent model is most convenient for multibeam filter calculations [5], which would be otherwise impossible. Namely, if the lumped equivalent model is not used, the closed form solution even for the natural frequencies of the multibeam structure cannot as yet be found. It is also shown [5] that the model allows us to find the transfer function of any n-beam structure.

5. References

[1] W. E. Newell, "Tuned Integrated Circuits - A State of the Art Survey," Proc. IEEE, Vol. 52, Dec. 1964, pp. 1603 - 1608.

[2] M. Hribšek, "Contribution to Second Order Filter Synthesis Using Operational Amplifiers," MS Thesis, Belgrade 1974 (in Serbo-Croatian).

[3] W. E. Newwll, "Ultrasonics in Integrated Electronics," Proc. IEEE, Oct. 1965, Vol. 53, pp. 1305 - 1309.

[4] J. J. O'Connor, "A 400-cps Tuning Fork Filter," Proc. IRE, Vol. 49, Nov. 1960, pp. 1857 - 1865.

[5] M. Hribšek, "High Q Selective Filters Using Mechanical Resonance of Silicon Beams," Ph.D. Thesis, University of Maryland, 1976.

[6] W. P. Mason, "Electromechanical Transducers and Wave Filters," D. Van Nostrand, New York, 1948.

[7] C. M. Harris, C. E. Crede, "Shock and Vibration Handbook," Vol. 1, McGraw Hill, New York, 1961.

[8] Civil Engineering Handbook, Part I (in Serbo-Croatian), Gradevinska Knjiga, Belgrade, 1954.

[9] G. M. Carter, "The Electromagnetic Field in its Engineering Aspects," American Elsevian Publishing Co., New York, 1967.

[10] R. Y. Roark, "Formulas for Stress and Strain," McGraw Hill, New York, 1954.

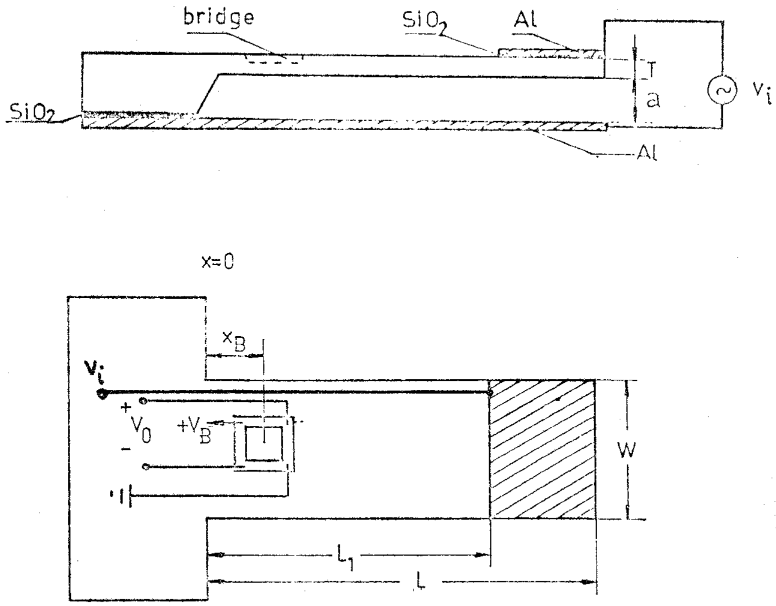

Fig. 1

Single silicon beam filter

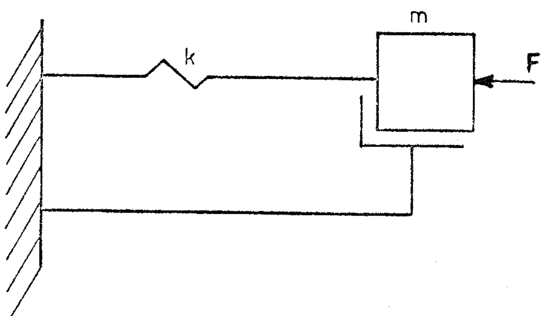

Fig. 2

Lumped equivalent model for a lossless cantilevered silicon beam