High-Q Selective Filters Using Mechanical Resonance of Silicon Beams

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***Abstract*—The theory of integrated high-Q selective filters based on the mechanical resonance of single silicon beams is presented. The method for transfer function calculation is developed, and it is shown that the silicon beam filters have high-Q selective characteristics. A construction procedure based on anisotropic etching of silicon and classical integrated technology is proposed.**

I. Introduction

IN THE last two decades considerable effort has been made to solve the problem of high-Q selective circuits. A wide variety of different approaches to the solution of this problem exists. They can be divided into two main categories, those for low frequencies (<10 kHz) and those for high frequencies (>10 kHz). For high frequencies up to 10 MHz and high-Q factors the problem is solved by using quartz and ceramic piezoelectric resonators [1]. Quartz filters are very well known as precision tuning elements in the frequency range from 10^4 to 10^7 Hz with Q factors between 50 and 1000 [2]. The main disadvantage of these filters is their high cost and relative fragility. Ceramic filters, which have been developed in recent years, can have central frequencies up to 7 MHz and Q factors up to 300 [3]. However, for low frequencies, where crystal filters are impractical, the problems still exist. There are many proposed solutions with active integrated circuits (IC) [1], [4], but there is no complete solution. Some of the proposed circuits can have high Q but their sensitivity to the circuit elements is too high making them impractical. As a matter of fact in most proposed configurations the sensitivity of the Q factor is proportional to the Q factor itself. Where this is not the case, some of the proposed circuits have another drawback, which makes them inconvenient for integration when high Q is desired: the ratio of the maximum to the minimum value of many elements is proportional to the Q factor [4]. Therefore, it can be concluded that it is hard to obtain entirely integrable filters with a Q factor greater than 50 for low frequencies. This has suggested the use of mechanical resonators [3]. The resonant frequency of these resonators is determined by their dimensions and physical properties. Some of these electromechanical filters are bulky and nonintegrable [5], but one of them, the resonant gate transistor [6], [7] is compatible with integrated circuit technology. The resonant element in the resonant gate transistor is a gold beam positioned over an MOS transistor. In this paper a new integrable electromechanical filter is developed in which the resonator is a cantilevered silicon beam etched from the silicon "substrate" [8]. The idea is illustrated in Fig. 1. The electrostatic force produced by an input signal v_i will cause vibrations of the silicon beam. The vibrations are sensed by the piezoresistive bridge placed at the foot of the beam. The output signal of the bridge is appreciable only at the resonant frequency of the beam, as is shown in Section II. The derivation of the transfer function and discussion on the expected values of Q are also presented.

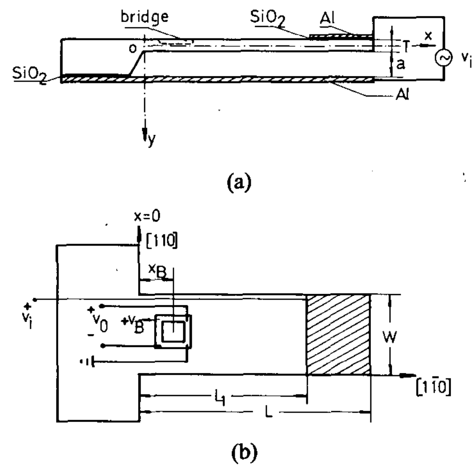

Fig. 1. Single silicon beam device.

In Section III a possible construction procedure of the silicon beam filters is discussed.

II. Transfer Function of a Single Silicon Beam Filter

The basic structure of silicon beam devices has been shown in Fig. 1 It consists of a thin silicon beam ($T=$ 10–15 μm) cantilevered over a metal plate. On the top side of the beam at its free end an aluminium film is deposited while at its fixed end four equal resistors are formed into a bridge. The resistors can be placed either completely or partially on the beam as shown in Fig. 2(a) and (b). Two of the resistors are in the direction parallel to the length of

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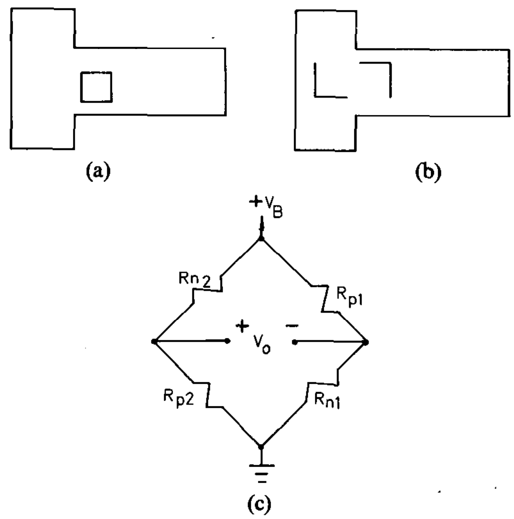

Fig. 2. Piezoresistive bridge positioning and connection.

the beam and the other two are perpendicular to it as suggested by Professor V. Vaganov [8]. The resistors are connected to form a simple piezoresistive bridge, shown in Fig. 2(c). The bridge, which represents the output transducer of the device, is in balance when the beam does not vibrate. If the beam vibrates, the resistances, due to the piezoresistive properties of silicon, will change differently, and an output voltage will occur. The vibrations of the beam can be caused by mechanical disturbance (e.g. sound) or an electrostatic force produced by a variable voltage applied between the two metal plates. In both cases the deflection of the beam, and consequently the output voltage of the bridge, will be maximal when the frequency of the signal is equal to the mechanical resonant frequency of the beam. At all other frequencies the output voltage will be much smaller. To be able to prove this, we will find the transfer function of the device. Considering only the case where the excitation is an input voltage, as shown in Fig. 1., the transfer function is defined as

$$T(s)=\frac{V_o}{V_i} \tag{1}$$

where V_o is the output voltage of the bridge, and V_i is the applied input voltage. Since it is difficult to find the transfer function directly we will express it as a product of three terms, each term corresponding to one of the three parts of the device. The first part of the device is the input transducer which converts the input voltage into mechanical force. The second part is the vibrating beam, and the third part is the bridge which converts the mechanical vibration into the output voltage. Since the bridge is a stress sensing device we will express $T(s)$ in the following form:

$$T(s)=\frac{V_o}{V_i}=\frac{V_o}{\sigma_B}\frac{\sigma_B}{F}\frac{F}{V_i} \tag{2}$$

where σ_B is the stress which acts on the bridge, and F is the force caused by the input voltage V_i. The first term in (2) represents the transfer function of the bridge, the second term is the transfer function of the vibrating beam, and the third term is the transfer function of the input transducer. We will find separately these three terms, thus giving $T(s)$.

As mentioned above, the input transducer of our silicon beam device is of electrostatic type. It is well known [9, pp. 63–66] that the voltage applied between two metal plates separated by a dielectric will produce an electrostatic force given by

$$f=v^2\frac{A\epsilon}{2a^2} \tag{3}$$

where v is the applied voltage, A is the area of the plates, a is the distance between the plates, and ϵ is the permittivity of the dielectric.

In the case of silicon beam devices the dielectric will normally be air, so that the permittivity will be $\epsilon_0=8.85\times 10^{-12}$ F/m. If the voltage v is given by

$$v=E_o+V_i\cos\omega t \tag{4}$$

and $E_0\gg V_i$ the signal component of the force will be

$$F\cos\omega t=\frac{AE_0\epsilon_0}{2}V_i\cos\omega t$$

or

$$\frac{F}{V_i}=\frac{AE_0\epsilon_0}{a^2} \tag{5}$$

which represents the last term in (2).

The middle term in (2) is derived by finding the resonant frequencies and equations of motion of a silicon beam resonator. The derivation, presented in the Appendix, is based on the well-known theory of vibrating beams [10, ch. 4]. It is shown that the fundamental resonant frequency is determined by geometric dimensions and physical properties of the material of which the beam is made and is given by

$$f_1=0.16165\,(E/\rho)^{1/2}T/L^2 \tag{6}$$

where T is the thickness, and L is the length of the beam, ρ is the density, and E is the Young modulus. The middle term of (2) is shown to be

$$\begin{aligned}\frac{\sigma_B}{F}&=\frac{\sigma(x_B)}{F}\\&=\frac{-3.5ET}{(s^2+\omega_1^2+(1/Q)\omega_1 s)\rho WT(L-L_1)L^2\pi\beta_1}\\&\quad\cdot C(x_B)\left(0.707\left(\sinh\pi\beta_1\frac{L_1}{L}-\sin\pi\beta_1\frac{L_1}{L}\right)\right.\\&\quad\left.-0.518\left(\cosh\pi\beta_1\frac{L_1}{L}+\cos\pi\beta_1\frac{L_1}{L}\right)\right)\end{aligned} \tag{7}$$

where x_B is the coordinate of the center of the bridge (see Fig. 1(b)), β_1 is the coefficient given by (A-4) and $C(x)$ is given by

$$\begin{aligned}C(x)=0.707&\left(\cosh\pi\beta_1\frac{x}{L}+\cos\pi\beta_1\frac{x}{L}\right)\\&-0.518\left(\sinh\pi\beta_1\frac{x}{L}+\sin\pi\beta_1\frac{x}{L}\right).\end{aligned} \tag{8}$$

From (8) we can see that $C(x)$ is maximal for $x=0$ and zero for $x=L$. This means that σ_B/F for the output will be greatest if the output bridge is positioned close to $x=0$.

The output transducer used for the silicon beam device is a piezoresistive bridge placed on the top side of the beam as shown in Figs. 1 and 2. It converts the vibrations of the beam into an electric signal through the unbalancing action of a vibration on the differently placed arms. As was mentioned before, the output voltage should be zero when the beam does not vibrate. That will be obtained if the bridge is balanced by resistors having the same equilibrium value. If the bridge is fully on the beam, connected as Fig. 1(b) shows, by simple calculation we find on analyzing Fig. 2(c) that

$$V_o = V_B\left(\frac{R_{p2}}{R_{p2}+R_{n2}} - \frac{R_{n1}}{R_{p1}+R_{n1}}\right) \tag{9}$$

where V_B is the dc supply voltage of the bridge, R_{p1} and R_{p2} are resistances of the resistor placed parallel to the length of the beam, and R_{n1} and R_{n2} are for the resistors placed perpendicular to the length of the beam. Using the theory of piezoresistivity [12, p. 3323] the above equation, as is shown in the Appendix yields

$$\frac{V_o}{\sigma_B} = V_B\frac{\pi_{44}}{2} \tag{10}$$

which represents the first term in (2).

Using the results of the previous calculations we are now able to find the transfer function of the single silicon beam device. If the beam device is constructed as illustrated in Fig. 1, substituting (5), (7), (8) and (10) into (2) we get

$$T(s) = \frac{V_o}{V_i} = \frac{K}{s^2+s(\omega_1/Q)+\omega_1^2} = \frac{K}{s^2+2\zeta\omega_1 s+\omega_1^2} \tag{11}$$

where

$$K = -0.91\,V_B E_0 \epsilon_0 \pi_{44} L^2 a^{-2} T^2 \omega_1^2 B C(x_B) \tag{12}$$

$$B = 0.707\left(\sinh\frac{1.875L_1}{L} - \sin\frac{1.875L_1}{L}\right) - 0.518\left(\cosh\frac{1.875L_1}{L} + \cos\frac{1.875L_1}{L}\right) \tag{13}$$

and $C(x)$ is given by (8).

As can be seen from (11), the transfer function of the vibrating silicon beam has the standard second-order form in s [7, p. 119], [14, p. 349]. At the first resonance, $s=j\omega_1$ the overall voltage gain will be

$$\left.\frac{V_o}{V_i}\right|_{s=j\omega_1} = j0.91\pi_{44}\frac{V_B E_0 L^2}{a^2T^2}\epsilon_0 B C(x_B) Q. \tag{14}$$

Equation (14) shows the fundamental factors which determine the gain of the silicon beam device. The j factor indicates that at resonance the phase shift between input and output voltages is 90°. It can be also seen that the gain is strongly dependent on the input bias voltage E_0 and the bridge bias V_B, the gain being bigger if these voltages are higher. The maximum permissible value of E_0 is determined from the equilibrium between the dc component of the electrostatic force and the mechanical restoring force of the beam. This maximum value of E_0 will be a function of the dimensions of the beam and the spacing between the beam and the metal plate. Using the same procedure as in [7, p. 120] we get for the maximum value of E_0:

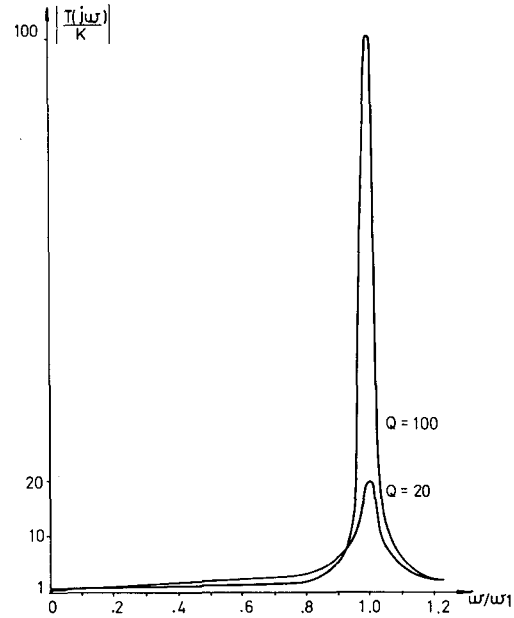

Fig. 3. Frequency response of the second-order low-pass filters.

$$E_{0m} = \left[\frac{2}{27}\frac{a^3T^3E}{L^3(L-L_1)\epsilon_0}\right]^{1/2} \tag{15}$$

where a is the distance between the beam and the metal ground plate when E_0 is zero.

Equation (11) is a typical low-pass transfer function, but for high Q it can be used for bandpass filtering as can be seen from Fig. 3. We will show now that a vibrating silicon beam has very high Q.

As is well known the Q factor is a function of losses. In the case of the vibrating silicon beam we have two kinds of losses: internal losses in silicon and external losses due to friction with air. The first kind of loss is a function of temperature, grain size, frequency, amplitude of the vibration, and elastic modules of silicon [15, p. 314]. The effect of the internal losses can be represented by the logarithmic decrement δ given by [15, p. 333].

$$\delta = \frac{\pi\theta E\gamma^2}{\rho c}\,\frac{\omega/\omega_1}{1+(\omega/\omega_1)^2} \tag{16}$$

where

- ω_1 resonant frequency of the beam
- θ temperature in °C
- E Young's modulus, for silicon, $E=11.26\times10^{11}$ dyn/cm^2
- γ thermal expansion coefficient for silicon, $\gamma=2.5\times10^{-6}$/°C
- ρ density for silicon, $\rho=2.32$ g/cm^3
- c specific heat for silicon $c=0.123\times4.184\times10^7$ dyn/°C mol.

The Q factor, ignoring external losses can be calculated now as

$$Q = \frac{\pi}{\delta}. \tag{17}$$

Note that this shows δ to be 2π times damping factor ζ of a second order system as seen from substituting (17) into (11).

Substituting numerical values in (17) we get that the Q factor for silicon at room temperature (20°C) at resonance is around 6000. The actual Q factor will be smaller due to external losses which are impossible to predict, but still Q will be high, certainly over 600.

In order to illustrate the values of the transfer functions we can expect, we will calculate the functions for two different single beam filters, each for two different positions, x_{B1} and x_{B2} of the resistive bridges.

Assuming $V_B = 20$ V, $E_0 = 20$ V, $Q = 1000$ and the dimensions of the first beam (see Fig. 1),

$$L = 0.1375 \text{ cm} \qquad w = 0.0525 \text{ M}$$
$$L_1 = 0.0800 \text{ cm} \qquad T = 10^{-3} \text{cm}$$
$$x_{B1} = 0.0625 \text{ cm} \qquad a = 10^{-3} \text{cm}$$
$$x_{B2} = 0.0150 \text{ cm}$$

we calculate (from (6))

$$f_1 = 5.95 \text{ kHz}$$

and from (11)

$$T(s) = -\frac{5.08 \times 10^5}{s^2 + 37.4s + 1.4 \times 10^9}$$
$$T(j\omega_1) = j0.363$$

if the coordinate of the center of the bridge is at x_{B1} and

$$T(s) = -\frac{1.05 \times 10^6}{s^2 + 37.4s + 1.4 \times 10^9}$$
$$T(j\omega_1) = j0.75$$

if the coordinate of the center of the bridge is at x_{B2}.

If we change the length of the beam to $L = 0.1$ cm leaving everything else unchanged we get

$$f_1 = 11.25 \text{ kHz}$$

and

$$T(s) = -\frac{2.67 \times 10^5}{s^2 + 70.6s + 5 \times 10^9}$$
$$T(j\omega_1) = j0.0534$$

for x_{B1} and

$$T(s) = -\frac{10.3 \times 10^5}{s^2 + 70.6s + 5 \times 10^9}$$
$$T(j\omega_1) = j0.206 \quad \text{for} \quad x_{B2}.$$

III. Construction Procedure

It has been shown in the previous section that the silicon beam structure is actually a high-Q low-pass filter which can be used for bandpass purposes. In this section we will discuss the possibility of practical construction of these silicon-beam devices.

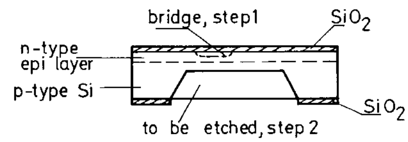

Fig. 4. Deep anisotropic etching from the back.

The whole theory of the silicon beam filters was based on the fact that we have formed silicon beams. The technique which enables us to form silicon beams is anisotropic etching of silicon [16]–[19] which has been used in biomedical pressure transducers [20], [21], and MOS and CMOS transistor technology [22], [23]. The only specific problem in silicon beam construction as it relates to anisotropic etching is that it is performed on both sides of the wafer. This brings up the rather important problem of alignment of masks on different sides of the wafer. This problem is solved either by using a specially designed tool or probably more practically by utilizing once more the selective silicon etching for etching marker-holes through the wafer.

According to [17] we can construct silicon beams of either (100) or (110) surface oriented wafers, but because of better sensitivity for the piezoresistive bridge we will use those of (100) orientation. For this orientation a p-type resistive bridge will give the highest output voltage [13]. This necessitates beginning with a (100) surface oriented wafer with an n-type epitaxial layer.

A possible construction procedure consists of 6 or 7 steps. The first step is the formation of a p-type piezoresistive bridge at the desired position. The resistors can be either diffused or ion-implanted. The second step is for a deep anisotropic etching on the back side of the wafer, as shown in Fig. 4. In the next three steps contact windows are opened, aluminum is deposited and selectively etched to form the desired connections. In the last two steps aluminum is deposited on the back side, and by anisotropic etching the beam is formed. In the case when only one device per package is desired, aluminum deposition on the back side may be deleted because the metal layer on the bottom of any IC package in which the device may be placed, could possibly be used as the bottom plate of the input transducer.

IV. Discussion

In this paper we presented a new type of electromechanical filter which can be made using standard integrated circuits technology. It is shown that the transfer function of a simple silicon beam has the standard second order low-pass form with high Q. The resonant frequency is entirely determined by the dimensions of the beam and the physical properties of silicon. The gain at resonance is a function of the ratio of the length and the thickness of the beam. Since the resonant frequency is proportional to the ratio of the thickness to the length, we can conclude that the gain at resonance is higher for low resonant frequencies. This was also proved with the given example.

The knowledge of anisotropic etching of silicon and its piezoresistive properties enabled us to propose the construction procedure of the silicon beam filters. Most of the steps were successfully carried out, but facilities for all steps have not been adequately available to us as yet.

The dimensions of the filter are determined by several factors. The minimum length of the beam is equal to its minimum width, which is determined by the dimensions of the bridge. Since the minimal width of the diffused bridge resistors is about 50 μm [24, p. 74] and the minimal distance between them is about 25 μm, the dimension of the smallest bridge is about 125 μm. Besides, the beam should be wide enough to allow for good connections to the bridge and the applied source. Since the minimal width of a metalization path is 50 μm and the distance between them 25 μm, the minimal width of the beam is found to be 350 μm. This will also be the minimum length of the beam.

The maximum length of the beam is determined by the maximal permissible deflection of the free end of the beam from rest, when the beam is acted upon only by its weight. If we want this deflection to be, say, one tenth of the nominal distance between the plates of the input transducer, then we can calculate that $L_{\max}=5.6$ mm.

The maximum thickness is determined by the condition that the configuration is considered as a beam only if its thickness is much smaller than its length. If we consider maximal thickness as one tenth of the length, then we can calculate the maximal obtainable resonant frequency using $L_{\min}=350$ μm in (6)

$$f_{1\max}=1.125\times10^5\frac{0.1L_{\min}}{L^2_{\min}}=321.42\text{ kHz.}$$

This is slightly higher than the maximum obtainable resonant frequency of the resonant gate transistor [7, p. 117].

As far as the practicality of the structure is concerned we can expect problems due to $1/f$ noise in the piezoresistors and their change with temperature and time. Since this paper represents the theoretical treatment of the device we do not consider these problems here but they will be discussed along with realized filters.

Appendix

I. Derivation of the Middle Term in (2)

The resonant or natural frequencies of a cantilevered silicon beam will be calculated using the well-known theory of vibrating beams [10, Ch. 4]. If no external force is applied, the beam vibrates at its natural frequency which can be calculated from the equations of motion. If the amplitude of vibration is small, the equation of motion of an excited thin lossless beam with uniform cross section is given by [10, p. 166]:

$$-\frac{ek^2}{\rho}\frac{\partial^4 y}{\partial x^4}-\frac{\partial^2 y}{\partial t^2}=q(x,t) \qquad \text{(A-1)}$$

where $\rho=2.32$ g/cm^3 is the density of silicon; $E=11.26\times 10^{11}$ dyn/cm^2 is Young's modulus for silicon; $k=T/\sqrt{12}$ is the radius of gyration of the cross section of the beam, x and y are coordinates as denoted in Fig. 1; and $q(x,t)=F'(x,t)/\rho WT$ where $F'(x,t)$ is the external driving force per unit length (note that we introduce a prime on F to denote "per unit length").

For free vibrations $q(x,t)=0$, and the beam vibrates in simple harmonic motion, so that the solution of (A-1) is of the form

$$Y(x,t)=Y(x)e^{st}, \qquad s=j\omega=j2\pi f. \qquad \text{(A-2)}$$

Now $Y(x)$ satisfies the ordinary differential equation

$$\frac{d^4Y(x)}{dx^4}=\frac{8\pi^2}{L}\beta Y(x) \qquad \text{(A-3)}$$

where β is defined by

$$\beta^4=\frac{4\rho f^2L^4}{\pi^2Ek^2}. \qquad \text{(A-4)}$$

As is well known the general solution of (A-3) is of the form [10, p. 157]

$$Y(x)=a\cosh\left(\pi\beta\frac{x}{L}\right)+b\sinh\left(\pi\beta\frac{x}{L}\right)+c\cos\left(\pi\beta\frac{x}{L}\right)+d\sin\left(\pi\beta\frac{x}{L}\right). \qquad \text{(A-5)}$$

The constants a, b, c, and d can be determined from the boundary conditions, which in this case are given by

$$y(0,t)=0$$
$$\partial y(0,t)/\partial x=0$$
$$F(L,t)=0 \quad\text{or}\quad \frac{\partial^2 y(L,t)}{\partial x^2}=0$$
$$M(L,t)=0 \quad\text{or}\quad \frac{\partial^3 y(L,t)}{\partial x^3}=0 \qquad \text{(A-6)}$$

where F denotes force and M denotes moment.

Using these boundary conditions we can also obtain the equation

$$\coth^2(\pi\beta/2)=\tan^2(\pi\beta/2). \qquad \text{(A-7)}$$

The solution of (A-7) represents the allowed values of β which are labelled as β_n, $n=1,2,\cdots$, and their values are given by approximation in Table I.

TABLE I

1	0.597
2	1.494
3	2.5
$n>3$	$\cong(n-1/2)$

Substituting the values of β_n into (A-4) we calculate the allowed natural frequencies as

$$f_n=\frac{\pi\beta_n^2}{2L^2}\frac{(Ek^2)^{1/2}}{\rho^{1/2}}, \qquad n=1,2,\cdots \qquad \text{(A-8)}$$

From (A-8) the fundamental frequency of the beam of thickness T and length L can be calculated as

$$f_1=0.16156(E/\rho)^{1/2}\,T/L^2 \qquad \text{(A-9)}$$

where T and L are in cm, E in dyn/cm^2, and ρ in g/cm^3.

For silicon, since E and ρ are known, f_1 can be expressed in the form

$$f_1 = 1.125\times 10^5\, T/L^2 . \tag{A-10}$$

The solution of (A-3) corresponding to the allowed frequency f_n is called a characteristic function and it is given by [10, p. 159]:

$$Y_n(x) = a_n\left(\cosh\frac{\pi\beta_n x}{L} - \cos\frac{\pi\beta_n x}{L}\right) + b_n\left(\sinh\frac{\pi\beta_n x}{L} - \sin\frac{\pi\beta_n x}{L}\right) \tag{A-11}$$

where

$$-b_n = a_n\frac{\cosh(\pi\beta_n)+\cos(\pi\beta_n)}{\sinh(\pi\beta_n)+\sin(\pi\beta_n)} = a_n\frac{\sinh(\pi\beta_n)-\sin(\pi\beta_n)}{\cosh(\pi\beta_n)+\cos(\pi\beta_n)} \tag{A-12}$$

and a_n is chosen so that $\int_0^L Y^2_n\, dx = L/2$, by analogy with the sine functions for the vibrating string [10, p. 159].

After finding the natural frequencies we can discuss the forced vibrations of the same beam. If the beam is driven by a force $F(x)e^{st}$, following well-known procedures [10, p. 166], we find for the steady-state motion

$$y(x,t) = \sum_{n=1}^{\infty}\frac{g_n Y_n(x)e^{st}}{\omega_n^2+s^2} \tag{A-13}$$

where

$$\omega_n = 2\pi f_n$$

$$g_n = \frac{2}{\rho WTL}\int_0^L F'(x)Y_n(x)\,dx. \tag{A-14}$$

Resonance occurs whenever the frequency f of the driving force is equal to one of the natural frequencies f_n, unless the corresponding g_n is zero.

As we know for damped systems, and according to Morse [10, pp. 97, 105, 166], if the losses in the beam are not neglected the steady state motion will be similarly described with the presence of dissipation terms; if we assume a quality factor Q of the beam which is independent of n, then

$$y(x,t) = \left[\sum_{n=1}^{\infty}\frac{g_n}{\omega_n^2+s^2+\frac{1}{Q}\omega_n s}Y_n(x)\right]e^{st}. \tag{A-15}$$

In the case of a good vibrating silicon beam the amplitude at resonance of the overtones are much smaller than the amplitude of the fundamental frequency. Then the steady state motion can be described with the simple one term approximation:

$$y(x,t) = \frac{g_1}{\omega_1^2+s^2+\frac{1}{Q}\omega_1 s}Y_1(x)e^{st} \tag{A-16}$$

where $Y_1(x)$ is calculated from (A-11) using β_1, a_1, and b_1 [10, p. 159] as

$$Y_1(x) = 0.707\left(\cosh\frac{1.875x}{L} - \cos\frac{1.875x}{L}\right) - 0.518\left(\sinh\frac{1.875x}{L} - \sin\frac{1.875x}{L}\right). \tag{A-17}$$

Further, for the transfer function calculations to evaluate (2) we need to know the stress $\sigma(x)$ along the length of the beam. The stress can be calculated using the available formula [11, p. 47]:

$$\sigma(x,t) = \frac{TM(x,t)}{2I(x)} \tag{A-18}$$

where $\sigma(x,t)$ is the stress on the surface of the beam at the distance x from the clamped end, T is the thickness of the beam, $I(x)=I=WT^3/12$ is the moment of the inertia of the cross section of the beam, and $M(x,t)$ is the bending moment. Since $M(x,t)$ is given by

$$M(x,t) = -EI\frac{\partial^2 y}{\partial x^2} \tag{A-19}$$

we can express $\sigma(x,t)$ in the form

$$\sigma(x,t) = \sigma(x)e^{st}. \tag{A-20}$$

Using (A-16)–(A-20) we get

$$\sigma(x) = \frac{1.75ETg_1}{(\omega_1^2+s^2+(1/Q)\omega_1 s)L^2}\cdot\left[0.707\left(\cosh\frac{1.875x}{L}+\cos\frac{1.875x}{L}\right) - 0.518\left(\sinh\frac{1.875x}{L}+\sin\frac{1.875x}{L}\right)\right]. \tag{A-21}$$

Since in our case the driving force $F(x)=F$ for $L_1<x<L$ as given by (5) we can calculate g_1 using (A-14)

$$g_1 = -\frac{2F}{\rho WT(L-L_1)\pi\beta_1}\cdot\left[a_1\left(\sinh\pi\beta_1\frac{L_1}{L} - \sin\pi\beta_1\frac{L_1}{L}\right) + b_n\left(\cosh\pi\beta_1\frac{L_1}{L} + \cos\pi\beta_1\frac{L_1}{L}\right)\right]. \tag{A-22}$$

Substituting (A-22) into (A-21) we get the middle term of (2)

$$\frac{\sigma_B}{F} = \frac{C(x_B)}{F} = \frac{-3.5ET}{(s^2+\omega_1^2+(1/Q)\omega_1 s)\rho WT(L-L_1)L^2\pi\beta_1}\cdot C(x)\left[0.707\left(\sinh\pi\beta_1\frac{L_1}{L} - \sin\pi\beta_1\frac{L_1}{L}\right) - 0.518\left(\cosh\pi\beta_1\frac{L_1}{L} + \cos\pi\beta_1\frac{L_1}{L}\right)\right] \tag{A-23}$$

where x_B is the coordinate of the center of the bridge (see Fig. 1(b)) and $C(x)$ is given by

$$C(x)=0.707\left(\cosh \pi\beta_1 \frac{x}{L}+\cos \pi\beta_1 \frac{x}{L}\right) -0.518\left(\sinh \pi\beta_1 \frac{x}{L}+\sin \pi\beta_1 \frac{x}{L}\right). \quad \text{(A-24)}$$

II. Derivation of the Term V_o/σ_B

When the beam does not vibrate all four resistors of the bridge have the same value R_0, and from (9) it is obvious that the output voltage is zero. When the beam vibrates, and since the bridge is small, the resistances R_{ni} and R_{pi} can be expressed as follows:

$$R_{pi}=R_0+\Delta R_p$$
$$R_{ni}=R_0+\Delta R_n. \quad \text{(A-25)}$$

Substituting (A-25) into (9), and dividing by R_0 we get

$$V_o=\frac{V_B(\Delta R_p/R_0-\Delta R_n/R_0)}{2+\Delta R_p/R_0+\Delta R_n/R_0}. \quad \text{(A-26)}$$

Using the theory of piezoresistivity [12, p. 3323], the relative change of the resistance can be expressed as follows:

$$\frac{\Delta R_p}{R_0}=\pi_{lp}\sigma_l+\pi_{tp}\sigma_t+\pi_{sp}\sigma_s$$
$$\frac{\Delta R_n}{R_0}=\pi_{ln}\sigma_t+\pi_{tn}\sigma_l+\pi_{sn}\sigma_s \quad \text{(A-27)}$$

where π_{lp}, π_{tp}, and π_{sp} represent respectively the longitudinal, transverse, and sheer piezoresistive coefficients for R_p, and π_{ln}, π_{tn}, and π_{sn} are the longitudinal, transversal, and sheer piezoresistive coefficients for R_n. The stress components in directions parallel to the length of beam and perpendicular to it are denoted as σ_l and σ_t, and sheer stress is denoted as σ_s. In the case of the vibrating beam $\sigma_t=0$ and $\sigma_s=0$ [11, p. 209], so that (A-27) becomes

$$\frac{\Delta R_p}{R_0}=\pi_{lp}\sigma_l$$
$$\frac{\Delta R_n}{R_0}=\pi_{tn}\sigma_l \quad \text{(A-28)}$$

Because the resistances R_p and R_n are assumed to be placed very close and their length is small, we can assume that σ_l is the same for all four resistances, and equal to σ_B at the center of the bridge.

Substituting (A-28) into (A-26), we get

$$V_o=V_B\frac{\sigma_B(\pi_{lp}-\pi_{tn})}{2+\sigma_B(\pi_{lp}+\pi_{tn})}. \quad \text{(A-29)}$$

The coefficients π_{lp} and π_{tn} depend on the crystal orientation of the beam and the type of silicon. For p-type silicon resistors, if the length of the beam is parallel to the crystalographic axis [110] and orthogonal to the crystalographic axis [1$\bar{1}$0], π_{lp} and π_{tn} are [12, p. 3323]

$$\pi_{lp}=\pi_{44}/2$$
$$\pi_{tn}=-\pi_{44}/2 \quad \text{(A-30)}$$

where π_{44} is the fundamental piezoresistance coefficient. As is known [13] p-type silicon has a much higher value of the coefficient π_{44} than n-type; for example, $\pi_{44}=1.381\times 10^{-10}$cm^2/dyn for p-type silicon with resistivity of about 8 Ω·cm versus $\pi_{44}=-13.6\times 10^{-12}$ cm^2/dyn for n-type with resistivity of about 11 Ω·cm.

Substituting (A-30) into (A-29) we get

$$V_o=V_B\sigma_B\pi_{44}/2. \quad \text{(A-31)}$$

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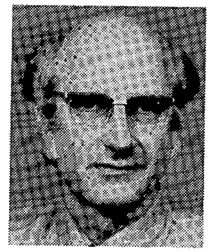

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Optimum LSI Implementation for a Digital Phase-Locked Loop

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Abstract—**A logically selective delay of a parallel output shift register can phase lock the input/output waveforms. The calculated precision of the phase-lock is related to the shift register's bit size and clock and input frequencies. This basic idea is extended to multistage (or multilevel) operations.**

For optimum LSI implementation, a technique is described which minimizes bit size and the delay function logic circuitry for a given precision. The method proves that for high-precision operations the shift register's bit size and the delay selecting logic circuitry are reduced, thus improving operating efficiency.

Also discussed is the design tradeoff between level complexity and circuit size. Finally, extension to synchronization application is considered.

INTRODUCTION

MONOLITHIC phase-locked loops (PLL) of analog design have been treated in the literature [1], [2]. However, large-scale integration (LSI) technology and digital signal processing (DSP) techniques are making digital logic approaches to circuit design very attractive by offering compatibility, reliability, compact circuit size and lower costs. Thus more and more analog circuits have been replaced by digital designs and PLL is no exception.

Most of the work done in digital phase-locked loops (DPLL) has centered on digitizing various analog constituents, such as voltage-controlled oscillator (VCO) and filter. Some designs require a mixed use of analog and digital circuits and some complex circuitry. As a result, the integrated circuit (IC) implementations often call for complex and costly fabrication processes.

The approaches introduced by Pasternack and Whalin [3], and Lee et al. [4] and Horton [5] are readily implementable by means of conventional LSI techniques. However, the circuit size to be implemented for a given phase-locked precision requirement is not accounted for in any of these approaches. The IC design optimization technique, so often meaningful for LSI implementation of circuits and systems, has not been explored in the case of DPLL.

The approach discussed here not only explicitly relates circuit size to the phase-locked precision specification, it also produces an optimum LSI implementation for a particular system configuration. An entirely new digital circuit structure is described and a multistage (or multilevel) structure is synthesized and analyzed for optimum LSI implementation. A general tradeoff criterion for optimum design is then developed to aid designers in selecting circuit size relative to a level of operation.

A discussion follows on the general application in synchronization between an input of frequency f_a and a reference of frequency f_b, which is nominally the same as f_a. Structural modification, bit size requirement formulations, and synchronization precision limitation are illustrated in detail.

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