REALIZATION OF PASSIVE NON-LINEAR CAPACITORS USING TRADITORS*

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Abstract

This paper presents a method to synthesize passive, non-linear capacitors using traditors loaded with a linear constant positive capacitor. Necessary and sufficient conditions are presented and the synthesis of active non-linear capacitors is discussed.

1. INTRODUCTION

It is well known that the resistor, the capacitor, the inductor, the transformer and the gyrator form a complete set within the domain of constant, passive, linear networks or better saying an over-complete set because the inductor can be realized with a gyrator and a capacitor, and the transformer with two gyrators [1, pp.149, 150].

In the search for a set capable of realizing the synthesis of constant, passive, non-linear networks, Duinker [2] introduced the traditor. This element is non-energic in nature and definable by a system of Lagrange's equations.

The basic type traditor is that for which the Lagrangian is a multi-linear form whose factors all correspond to different coordinates, i.e., $L = A x_1 \ldots x_{n-1} \dot{x}_n$. These traditors are distinguished by their degree n and their type, the degree corresponding to the number of ports of the traditor and the type to a distinct Lagrangian within each degree which specifies the x_i as individual charges, fluxes, currents and voltages in the interrelationship between the ports [3].

In this paper other classes of traditors are presented. These traditors are not of the basic class because their Lagrangians are not multilinear forms corresponding to different coordinates but arbitrary functions of those coordinates multiplied by a velocity, i.e., $L = f(x_1, x_2 \ldots x_n) \dot{x}_n$.

The systems having Lagrangians of this form still have the property that the instantaneous total power delivered to the system is always equal to zero [2].

These traditors are used in the sequel for the realization of passive non-linear capacitors.

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2. ANALYSIS

Let us consider a traditor whose Lagrangian is of the form $L = A F(q_1, q_2) v_3$ where F is a continuous function of q_1 and q_2. As n= 3 this is a third degree traditor and hence is characterized as a 3-port with independent port variables $x_1 = q_1$, $x_2 = q_2$, and the velocity $\dot{x}_3 = v_3$.

Let us connect the velocity port to a constant, linear capacitor C, as shown in Fig. 1.

The equations of the traditor can be found from the defining relationship

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_k} - \frac{\partial L}{\partial x_k} = y_k \tag{1}$$

where, when y_k is a voltage, x_k is a charge and $\dot{x}_k$ a current, and, when y_k is a current, x_k is a flux and $\dot{x}_k$ a voltage [2]

Thus we have, for our degree 3 traditor,

$$v_1 = -A \frac{\partial F(q_1, q_2)}{\partial q_1} v_3 \tag{2A}$$

$$v_2 = -A \frac{\partial F(q_1, q_2)}{\partial q_2} v_3 \tag{2B}$$

$$i_3 = A \frac{d}{dt} F(q_1, q_2) \tag{2C}$$

Next we connect ports 1 and 2 in series as in Fig. 1. Thus, $i_1 = i_2$, and assuming all initial conditions equal, $q_1 = q_2 = q$.

Therefore

$$v = v_1 + v_2 = \left[-A \frac{\partial F(q_1, q_2)}{\partial q_1} - A \frac{\partial F(q_1, q_2)}{\partial q_2} \right] v_3 =$$

$$= -A \frac{d F(q,q)}{dq} v_3 \tag{3}$$

$$i_3 = A \frac{d F(q,q)}{dt} = -C \frac{dv_3}{dt} \tag{4}$$

Integrating this latter

$$A F(q,q) = -C v_3$$

$$v_3 = -\frac{A}{C} F(q,q)$$

which gives

$$v = \frac{A^2}{C} \frac{d F(q,q)}{dq} F(q,q) \tag{5}$$

$$\text{or } v = \frac{1}{C} \frac{d\bar{L}}{dq} \bar{L} \tag{6}$$

$$\text{or } v = \frac{1}{2C} \frac{d\bar{L}^2}{dq} \tag{7}$$

where $L = \bar{L} v_3$

This represents a capacitor $v = f(q)$ where

$$f(q) = \frac{1}{C} \frac{d\bar{L}}{dq} \bar{L} = \frac{1}{2C} \frac{d\bar{L}^2}{dq} \tag{8}$$

In summary, the connection of Fig. 1 with a traditor having for its Lagrangian $L = \bar{L}(q_1, q_2) v_3$ yields a charge-controlled capacitor with the law

$$v = \frac{1}{2C} \frac{d\bar{L}(q,q)^2}{dq}$$

3. SYNTHESIS

3.1. <u>THEOREM</u>: A charge-controlled capacitor $v = f(q)$ is realizable by a cascade connection of a traditor and a linear time-invariant capacitor with capacitance $C > 0$, initially uncharged, if and only if $E(\hat{q}) = \int_0^{\hat{q}} f(q)\,dq$ exists for all real $\hat{q}$ and $E(\hat{q}) \geq 0$.

Proof:

a) <u>If</u>: We assume that $E(\hat{q})$ exists and $E(\hat{q}) \geq 0$.

Then let $\bar{L}(q_1, q_2) = \left[2C \sqrt{E(q_1) E(q_2)} \right]^{\frac{1}{2}}$ (9)

The Lagrangian of the traditor will be $L(q_1, q_2) = \bar{L}(q_1, q_2) v_3$. Using (8) we see that the capacitor $v = f(q)$ is obtained.

b) <u>Only if</u>: Given a traditor loaded with a capacitor, we have from (8)

$$v = f(q) = \frac{1}{2C} \frac{d\bar{L}^2}{dq} \quad \text{or} \quad \frac{d\bar{L}^2}{dq} = 2C f(q)$$

$\bar{L}^2(\hat{q}) = 2C \int_0^{\hat{q}} f(q)\,dq + K$ what proves the existence of $\int_0^{\hat{q}} f(q)\,dq$.

As the connection is passive then

$$\varepsilon(t) = \int_c^t v\,i\,d\delta + \varepsilon(c) = \int_c^t f(q)\,\frac{dq}{d\delta}\,d\delta + \varepsilon(c)$$

$$= \int_{q(c)}^{q(t)} f(q)dq + \varepsilon(c) =$$

Choose $t_o = O$, which implies $c = O$, and $q(c) = \varepsilon(c) = O$ as the capacitor is uncharged at the initial time

$$\varepsilon(t) = \int_0^{\hat{q}} f(q)\,dq \geq O \quad \text{or} \quad E(\hat{q}) \geq O$$

The inequality is a consequence of the passivity of the structure.

3.2 EXAMPLES:

3.2.1 Let $v = q^5$ which is a capacitor with a quintuple characteristic

$$\varepsilon(t) = \int_0^{\hat{q}} q^5\,dq = \frac{q^6}{6} \geq O$$

Therefore this capacitor is realizable using a traditor and a capacitor.

From (9) $\bar{L}^2(q) = 2C\,\frac{q^6}{6}$ and taking

$$C = 1 \qquad \bar{L}^2(q) = \frac{q^6}{3} \qquad \bar{L}(q) = \frac{q^3}{\sqrt{3}}$$

$$L(q) = \frac{1}{\sqrt{3}}\,q^3\,v_3$$

This traditor can be realized using the basic class traditor of degree 4, whose Lagrangian is $L = \frac{1}{\sqrt{3}}\,q_1\,q_2\,q_4\,v_3$ and connecting ports 1, 2 and 4 in series, and loading a unit capacitor at port 3, as seen in Fig. 2.

3.2.2 Let the non-linear capacitor have the following characteristic $v = q\,e^{2q} + q^2\,e^{2q}$

From (9) and choosing $C = 1$

$$\bar{L}^2(q) = \left[e^{2q}\,q^2\right] \qquad L(q) = e^q\,q$$

or

$$L(q) = e^q\,q\,v_3$$

4. CONCLUSIONS

A method is presented to find a traditor that will enable the realization of a time-invariant, passive, non-linear capacitor.

These results could have been obtained using a two port traditor whose Lagrangian would be $L = F(q_1)\,v_2$ where q_1 is the charge at port 1 and v_2 the voltage at the velocity port. The use of the three port traditor facilitates the task of visualizing how to build it as can be seen from example 1.

The realization of a flux-controlled inductor can be similarly obtained using a traditor having a Lagrangian of the form $L = F(\varphi_1, \varphi_2)\,i_3$, and loading the velocity port in a linear inductor, and connecting ports 1 and 2 in parallel as shown in Fig. 3.

This can also be obtained by cascading a gyrator and the non-linear capacitor previously realized.

The method does extend to the realization of a certain class of time-invariant, active, non-linear capacitors as may be seen by loading the traditor on an active linear capacitor $-C$, $C > O$.

This class is characterized by the existence of $E(\hat{q}) = \int_0^{\hat{q}} f(q)\,dq$ and $E(\hat{q}) \leq O$ for all real $\hat{q}$.

In this case choose

$$\bar{L}(q_1, q_2) = \left[+\,2|C|\,\sqrt{E(q_1)\,E(q_2)}\,\right]^{\frac{1}{2}} \qquad (10)$$

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5. BIOGRAPHIES

Carlos Mendonça-e-Moura was born in Porto, Portugal in 1949. He received the Electrical Engineering Diploma from the Universidade do Porto, Porto, Portugal in 1971 and the M.S. degree in Electrical Engineering from the University of Maryland, College Park, Md., in 1974.

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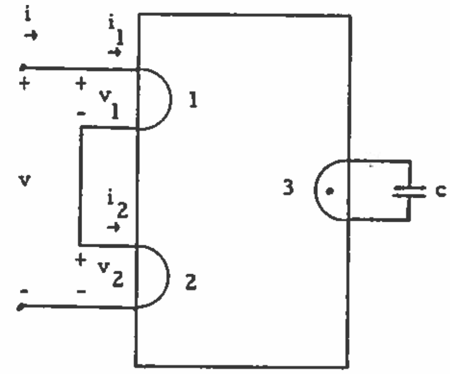

Fig. 1.

Traditor loaded with a capacitor.

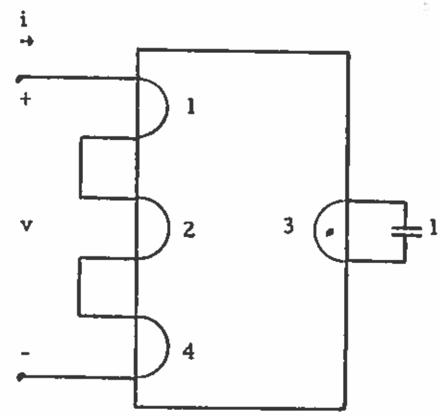

Fig. 2.

Realization of $v = q^5$ capacitor.

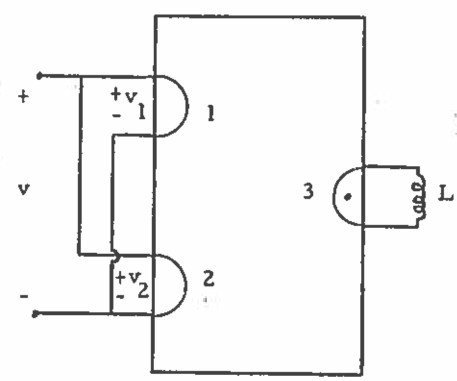

Fig. 3.

Realization of flux-controlled inductor.

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