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FUNDAMENTALS FOR LIVŠIC'S CHAIN SYNTHESIS

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Abstract

The background development needed for the factorization synthesis of M. S. Livšic for J-lossless matrices is recast and codified in the language of electrical engineers.

" The boat was coming in the dead of night
Clusters of bamboo, rising tide. " [1]

1. INTRODUCTION

In his 1966 book [2] which was recently (1973) translated into English [3], M. S. Livšic presents a theory for the factorization of J-lossless (square) matrices. The theory is general and unified in that it holds for chain, scattering, or transfer scattering matrices (or various combinations of these), any one of which is called a transmission operator and denoted by S in Livšic [3, p. 8]. For any lossless network, S, normalized at infinity to the identity, $S(\infty)=I$, can be represented as

$$S(p) = I + J\Gamma^a[T-pI]^{-1}\Gamma \quad (1)$$

a form of considerable promise for network synthesis since factorization results upon triangularization of the state matrix T.

In the following we recast the ideas of Livšic which lead to this representation of S. In doing this we adhere closely to the notation of Livšic while presenting the time-invariant situation in the complex p-plane.

" The oars shook the starry sky,
A stray bird circled above,
Noiselessly the boat came in the dark. " [1]

2. MAIN RESULTS

The main results we wish to show are [3, p. 25]

$$\psi = [T-pI]^{-1}\Gamma\varphi^- = R\varphi^- \quad (2a)$$

$$\varphi^+ = \varphi^- + \gamma^{-1}\Gamma^a\psi = S\varphi^- \quad (2b)$$

$$T+T^a = -\frac{1}{2}\Gamma[\gamma^{-1}+\gamma^{-a}]\Gamma^a \quad (2c)$$

where φ^- = input (m-vector) [3, p. 25]

φ^+ = output (m-vector) [3, p. 25]

ψ = interior (state, possibly infinite-dimensional) [3, p. 25]

R = input to interior operator [3, p. 42]

S = transmission operator [3, p. 8]

T = interior operator [3, p. 26]

Γ = an input to state space operator [3, p. 25]

and $p = \sigma + j\omega$, superscript a is the adjoint, and $\gamma = BJ$ with $J = J^a$, $J^2 = I$ (with I the identity) and $B = B^a > 0$ (that is, positive definite). The choice $B = I$ is possible giving

$$\gamma = J \quad (3)$$

which on substitution into (2a), (2b) yields (1). We next turn to a derivation of (2).

"The boat went out into the darkness
As the tide kept rising.
The oars again shook heaven and stars." [1]

3. DERIVATION

We begin by considering a lossless time-invariant system with no input (hence called closed). Then for an appropriate choice of (state) variables f,

$$\frac{df}{dt} = Qf \quad , \qquad Q = -Q^a \tag{4}$$

Here $Q = -Q^a$ can be obtained from the losslessness if we take the definition of the latter to mean the instantaneous power is zero as expressed by $\dot{f}^a f = f^a Q^a f = 1/2 f^a (Q+Q^a) f = 0$. Next consider this system as composed of an open system at $x=0$ closed upon coupling channels upon which inputs, φ^-, converge on the open system for $x<0$ and outputs, φ^+, leave for $x>0$; the situation is illustrated in Figure 1. The open system is that for which we wish the description S, while we assume that the lossless coupling channels are described by [3, p. 11]

$$\frac{\partial \varphi(x,t)}{\partial t} = -V \frac{\partial \varphi(x,t)}{\partial x}, \quad x \neq 0 \tag{5a}$$

$$V = BJ, \quad B = B^a > 0, \quad J = J^a, \quad J^2 = I \tag{5b}$$

For example if the coupling channels are lossless transmission lines of capacitance matrices C and inductance matrices L we have [4, p. 64]

$$B = \begin{bmatrix} C^{-1} & 0_k \\ 0_k & L^{-1} \end{bmatrix}, \; J = \begin{bmatrix} 0_k & I_k \\ I_k & 0_k \end{bmatrix}, \; \varphi(x,t) = \begin{bmatrix} v(x,t) \\ i(x,t) \end{bmatrix} \tag{5c}$$

where the subscript $k = m/2$ denotes kxk matrices and $v(x,t)$ and $i(x,t)$ are voltage and current at time t and position x on the transmission lines. When S is the chain or transfer scattering matrices, (5c) is appropriate and one has the freedom of choosing $B = I_m$, say. If S is the scattering matrix then the choice of $\varphi(x,t)$ as an incident voltage for $x<0$ and a reflected voltage for $x>0$, along with $J = I_m$, $B = I_m$, is appropriate [3, p. 14].

Returning to (4) we partition f according to the coupling channels, via $\varphi(x,t)$, and the open system state, $\Psi(x,t)$ as

$$f = \begin{bmatrix} \varphi(x,t) \\ \Psi(t)\delta(x) \end{bmatrix} \tag{6}$$

where δ is the impulse functional. From (5a) we see that the time-space dependence for φ can be taken as $Ix - Vt$, that is we can write

$$\varphi(x,t) = \Phi(Ix - Vt) \tag{7a}$$

Thus we take

$$\varphi(x,t) = \Phi^-(Ix-Vt)u(-x) + \varphi(0,t)\delta(x) + \Phi^+(Ix-Vt)u(x) \tag{7b}$$

where u is the unit-step function. Using this to evaluate both sides of (5a) we obtain

$$\frac{\partial \varphi}{\partial t} = -V \frac{\partial \Phi^-(Ix-Vt)}{\partial x} u(-x) + \frac{\partial \varphi(0,t)}{\partial t}\delta(x) - V \frac{\partial \Phi^+}{\partial x}(Ix-Vt)u(\ x) \tag{8a}$$

$$-V\frac{\partial \varphi}{\partial x} = -V\Big[\frac{\partial \Phi^-}{\partial x}(Ix-Vt)u(-x) - \Phi^-(Ix-Vt)\delta(x) + \varphi(0,t)\delta'(x) + \frac{\partial \Phi^+(Ix-Vt)}{\partial x}u(x) + \Phi^+(Ix-Vt)\delta(x)\Big] \tag{8b}$$

which gives the correction factor for (5a) to be valid at $x=0$, that is

$$\frac{\partial \varphi}{\partial t} = -V\frac{\partial \varphi}{\partial x} + \Big[-V\Phi^-(Ix-Vt) + V\Phi^+(Ix-Vt) - \frac{\partial \varphi(0,t)}{\partial t}\Big]\delta(x) + V\varphi(0,t)\delta'(x) \tag{8c}$$

We wish to equate (8c) with the first of (4) when partitioned as for f at (6)

$$\frac{\partial \varphi}{\partial t} = Q_{11}\varphi + Q_{12}\Psi(t)\delta(x) \qquad (9)$$

For this we make the choice (which fixes the $S(\infty)$ normalization)

$$Q_{12}(x)\Psi(t)\delta(x) = \left[V\Phi^{+}(Ix - Vt) - V\Phi^{-}(Ix - Vt)\right]\delta(x) \qquad (10a)$$

so that $Q_{11}(x)\varphi$ is the remainder of the right of (8c). Thus we have, on using (7a) in (10),

$$Q_{12}(0)\Psi(t) = V\left[\Phi^{+}(-Vt) - \Phi^{-}(-Vt)\right] \qquad (10b)$$

On taking Laplace transforms, $L[\,.\,]$, with

$\varphi^{+} = L[\Phi^{+}(-Vt)]$, $\varphi^{-} = L[\Phi^{-}(-Vt)]$, $\psi = L[\Psi(t)]$

we have from (10b)

$$\varphi^{+} = \varphi^{-} + V^{-1}Q_{12}(0)\psi \qquad (11a)$$

which is the desired equation (2b) when

$$\Gamma^{a} = Q_{12}(0) \qquad (11b)$$

is identified. Since $Q = -Q^{a}$ we have

$$\Gamma = Q^{a}_{12}(0) = -Q_{21}(0) \qquad (11c)$$

Then we turn to the second terms of (4) when partitioned according to (6). Using (7b)

$$\frac{\partial \Psi(t)}{\partial t}\delta(x) = Q_{21}(x)\left[\Phi^{-}(Ix - Vt)u(-x) + \varphi(0,t)\delta(x) + \Phi^{+}(Ix - Vt)u(x)\right] + Q_{22}(x)\Psi(t)\delta(x) \qquad (12a)$$

or, on identifying $\delta(x)$ terms and using (11c),

$$\frac{\partial \Psi(t)}{\partial t} = -\Gamma\,\varphi(0,t) + Q_{22}(0)\Psi(t) \qquad (12b)$$

If we again make the reasonable choice

$$\varphi(0,t) = \frac{1}{2}\left[\Phi^{+}(-Vt) + \Phi^{-}(-Vt)\right] \qquad (13)$$

we get on taking Laplace transforms

$$[pI - Q_{22}(0)]\psi = -\frac{1}{2}\Gamma(\varphi^{+} + \varphi^{-}) \qquad (14a)$$

or on using (11a) and (11c),

$$\left[pI - Q_{22}(0) + \frac{1}{2}\Gamma V^{-1}\Gamma^{a}\right]\psi = -\Gamma\varphi^{-} \qquad (14b)$$

which is (2a) with

$$T = Q_{22}(0) - \frac{1}{2}\Gamma V^{-1}\Gamma^{a} \qquad (14c)$$

Since Q is skew-adjoint, $Q_{22} = -Q^{a}_{22}$ and (14c) yields (2c) immediately.

"The boat was now safely moored to a tree." [1]

4. DISCUSSION

We have given a derivation of the representation (1) of the transmission operator S of a lossless network, losslessness being incorporated from the beginning at (4). The derivation has pointed out two choices at our disposal, that for Q_{12} at (10a) which fixes the normalization of $S(\infty) = I$ and that at (13) for which other choices would merely transform the state.

The definition of losslessness is that given by $\dot{f}^{a} f = 0$ which in terms of S becomes [3, p. 30]

$$J - S^{a}(j\omega)\,J\,S(j\omega) = 0_{m} \qquad (15)$$

along with analyticity and conjugacy constraints in Re $p > 0$ for passivity and reality. Any S satisfying (15) is called J-lossless and, consequently, the theory developed here is for J-lossless matrices.

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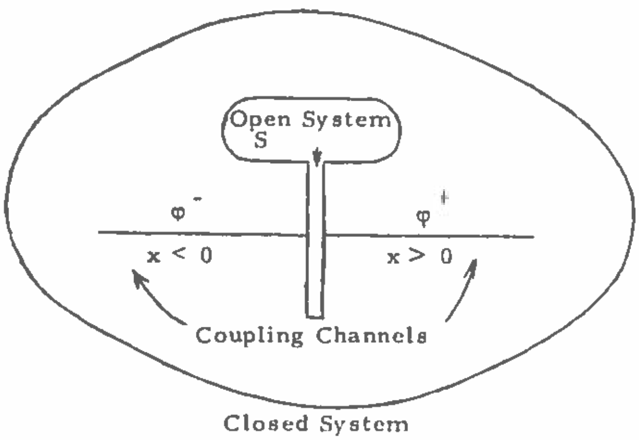

Figure 1
Open System Closed on Coupling Channels

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