Letters to the Editor

VCO Controlled by One Variable Resistor

MARIJA HRIBŠEK AND ROBERT W. NEWCOMB

***Abstract*—A multiloop active filter circuit of Kerwin is shown to yield a VCO whose oscillation frequency is conveniently controlled with one variable resistor. Using an FET to obtain the voltage variable resistor, experimental results are shown to agree with the theory developed.**

I. Introduction

The voltage controlled oscillator (VCO) has a number of important applications [1], [2], [3, p. 567] among which is its use as the critical feedback element in phase-locked loops [4], [5]. And various means of realization of a VCO have been given [6] among which are some using varactor diode capacitance control of a Clapp oscillator [2, p. 1151], variable capacitance control in a Wien bridge [7], variable diode resistance in an *RC*-phase-shift oscillator [8, p. 844, 856], voltage controlled current sources in integrator-Schmitt-trigger or emitter-coupled multivibrator circuits [4, p. 44], and resistor control in an op-amp *RC*-Wien bridge oscillator [9], [10]. Indeed a classification of *RC* active VCO's containing two charge-controlled devices has been presented in [11] where an integrated VCO controlled by the gate voltage of an FET is given.

Here we show that by using the latter concept of FET gate control within the multiloop feedback structures of Kerwin [12] an alternate means of obtaining VCO's results.

II. Basic Circuits

Two of the multiloop feedback circuits of Kerwin [12, p. 35, 38], those using a minimum number of capacitors, are shown in Fig. 1. Fig. 1(a) is a low-pass filter of transfer function while Fig. 1(b) shows a corresponding bandpass filter of transfer function

$$T_{LP}(p) = \frac{V_0}{V_i} = \frac{K_1 K_2/(a_3 a_4 R^2 C^2)}{p^2 + p\dfrac{1}{RC}\left[\dfrac{1}{a_1} + \dfrac{1}{a_3} + 1 + \dfrac{1}{a_3 a_4}(1 - K_1)\right] + \dfrac{1}{(RC)^2}\dfrac{[a_1 + 1 - K_1 K_2]}{a_1 a_3 a_4}} \tag{1a}$$

$$T_{BP}(p) = \frac{V_0}{V_i} = \frac{pK_1 K_2/(RC[1 - K_1 K_2])}{p^2 + p\dfrac{1}{RC[1 - K_1 K_2]}\left[\dfrac{1}{a_3 a_4} + \dfrac{1}{a_4} + 1 + \dfrac{1}{a_2}(1 - K_1)\right] + \dfrac{1}{(RC)^2}\dfrac{[a_2 + 1]}{a_3 a_4[1 - K_1 K_2]}}. \tag{1b}$$

In both it is seen that by a suitable choice of K_1, independent of K_2, the denominator coefficients of p can be made to be zero. Consequently, with these values of K_1, natural frequencies of the two circuits may be obtained on the $j\omega$ axis. As these are short-circuit natural frequencies, they will be physically realized by

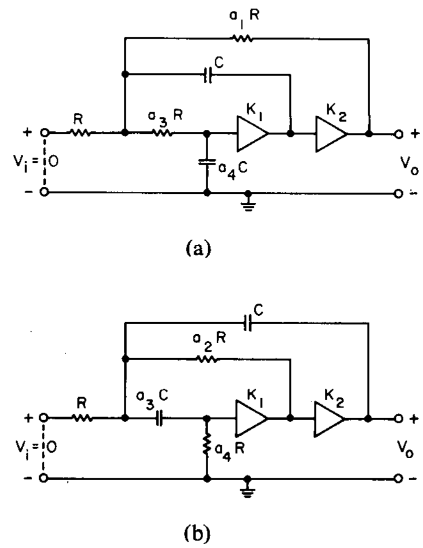

Fig. 1. Multiloop minimal capacitor filters of Kerwin. (a) Low-pass filter. (b) Bandpass Filter.

connecting the input resistor to ground. And in both cases these natural frequencies, ω_0, being the square roots of the denominator constant terms, are independently controllable through K_2 which in Kerwin's case was taken negative [12, p. 33].

The conditions for oscillation and the oscillation frequencies are then seen to be

low-pass circuit:

$$K_1 = 1 + a_3 a_4\left[1 + \frac{1}{a_1} + \frac{1}{a_3}\right] \tag{2a}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC}\left[\frac{a_1 + (1 - K_1 K_2)}{a_1 a_3 a_4}\right]^{1/2} \tag{2b}$$

bandpass circuit:

$$K_1 = 1 + a_2\left[1 + \frac{1}{a_4} + \frac{1}{a_3 a_4}\right] \tag{3a}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC}\left[\frac{1 + a_2}{a_3 a_4(1 - K_1 K_2)}\right]^{1/2}. \tag{3b}$$

Fig. 2 gives plots of normalized frequency, $\omega_0 RC$, versus the second stage gain for various values of the parameters a_1, a_2, a_3, a_4.

Manuscript received March 14, 1975; revised October 17, 1975. This research was supported in part by the Air Force Office of Scientific Research under Grant AFOSR 70-1910 and in part by NSF Grant ENG 75-03227.

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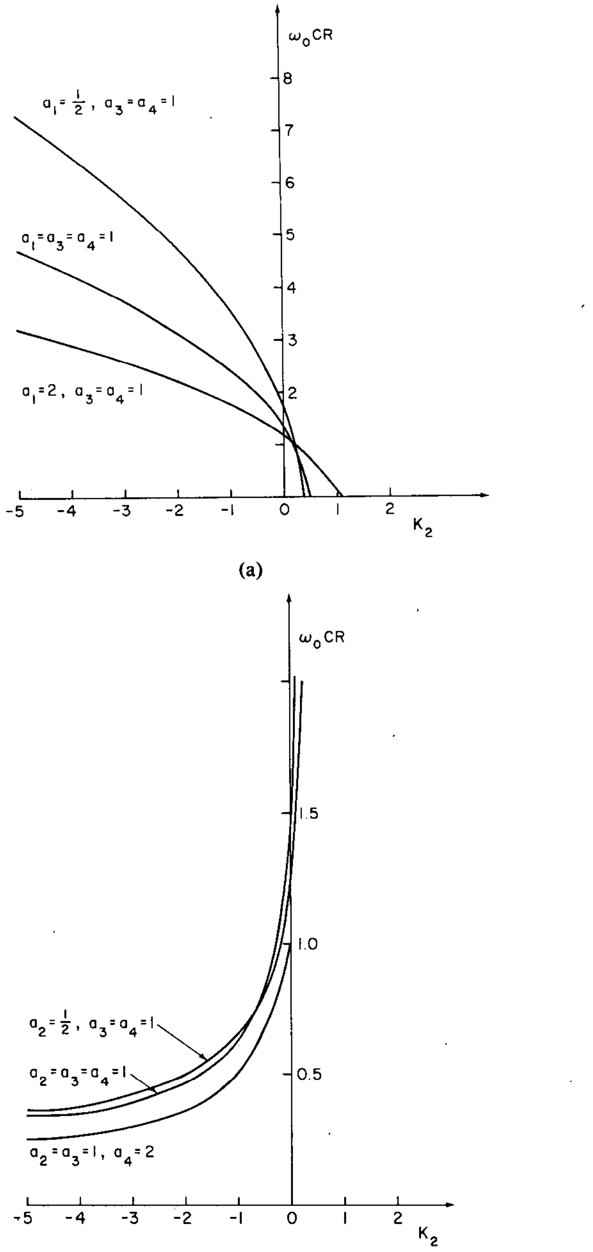


Fig. 2. Oscillation frequency versus K_2. (a) Low-pass circuit. (b) Bandpass Circuit.

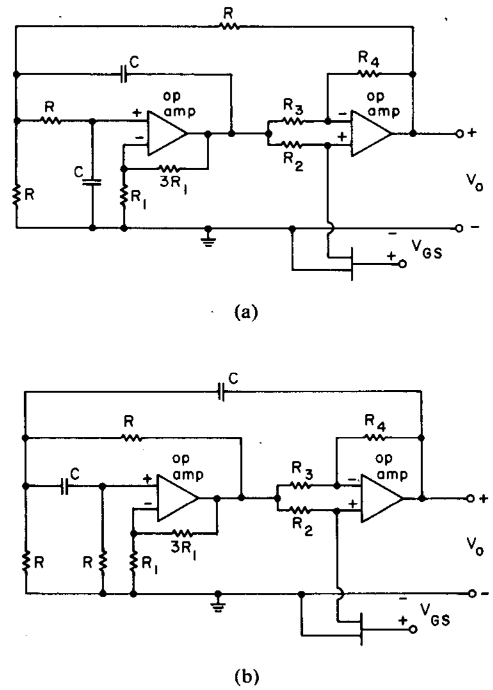


Fig. 3. Typical VCO circuits. (a) Low-pass circuits.

$$\omega_0 = \frac{1}{RC}\sqrt{2-4K_2}$$

(b) Bandpass circuit

$$\omega_0 = \frac{1}{RC}\sqrt{\frac{2}{1-4K_2}}$$

III. VCO

As K_2 can be realized to depend only upon a single variable resistor, the oscillation frequency can be varied by the single resistor. And since the single resistor can be realized by an FET working in the range below pinchoff, both of the circuits of Fig. 1 are seen to conveniently yield VCO's. Typical realizations are shown in Fig. 3 for the choice $a_1 = a_2 = a_3 = a_4 = 1$ for which $K_1 = 4$.

For the circuits shown the dependence of K_2 upon the FET drain to source resistance R_{DS} is given by

$$K_2 = -\frac{1}{R_3}\frac{R_2R_4 - R_3R_{DS}}{R_2 + R_{DS}}. \tag{4}$$

In turn, the drain to source resistance, at zero drain to source voltage, is given by [13, pp. 144–148], [14, p. 54]

$$R_{DS} = \frac{1}{\beta[V_{GS} + V_0]} \tag{5}$$

where V_0 is the pinchoff voltage for depletion devices and the negative of the threshold voltage for enhancement mode devices; β is a constant dependent on the properties of the channel. Substituting (5) into (4) then into (2) and (3) yields the dependence of the VCO oscillation frequency f_0 upon the controlling voltage V_{GS}. For example, for the circuits of Fig. 3 we find

low-pass circuit:

$$\omega_0 RC = \sqrt{2}\left(1 + 2\,\frac{\beta R_2R_4(V_{GS} + V_0) - R_3}{\beta R_2R_3(V_{GS} + V_0) + R_3}\right)^{1/2} \tag{6a}$$

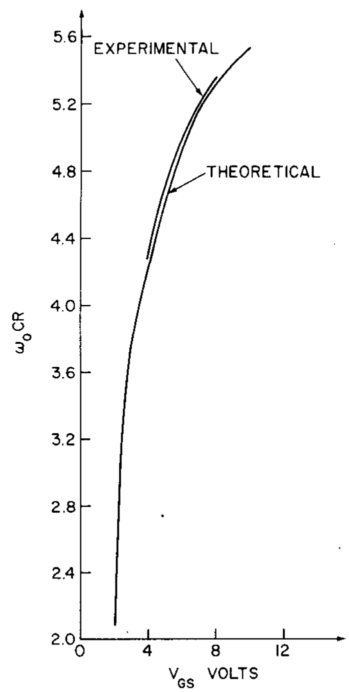
bandpass circuit:

$$\omega_0 RC = \sqrt{2}\left(1 + 4\,\frac{\beta R_2R_4(V_{GS} + V_0) - R_3}{\beta R_2R_3(V_{GS} + V_0) + R_3}\right)^{-1/2}. \tag{6b}$$

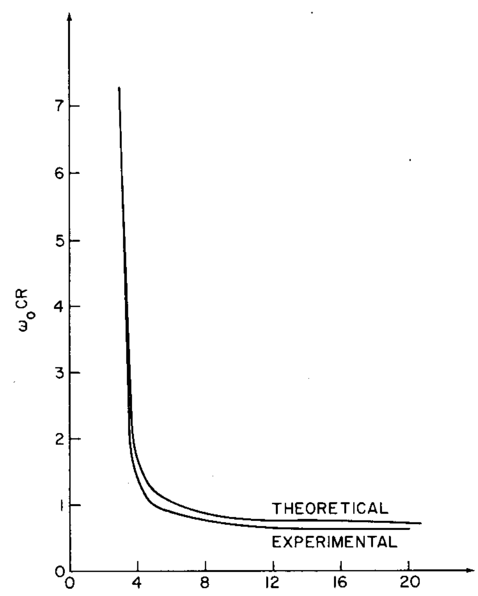
From this we observe that a choice of R_4 large negates the voltage control and R_4 small can lead to imaginary frequencies (exponential instabilities).

IV. Experimental Results

Both circuits were constructed using identical components and an n-channel enhancement FET. The results are shown in Fig. 4 where $\omega_0 RC$ versus V_{GS} is compared with the theoretical values obtained from (6). As can be seen reasonably close agreement was obtained over the operating range. Outside of the experimental ranges shown the oscillations were observed to persist but in a relaxation mode. Too, some of the bigger differences occurring at the higher values of V_{GS} come from the



(a)



(b)

Fig. 4. VCO characteristics:
$R = 10\ \mathrm{k}\Omega$ $C = 0.018\ \mu\mathrm{F}$ $R_1 = 10\ \mathrm{k}\Omega$
$R_2 = 1\ \mathrm{k}\Omega$ $R_3 = 10\ \mathrm{k}\Omega$ $R_4 = 10\ 0\ \mathrm{k}\Omega(\mathrm{LP})$
$= 10\ \mathrm{k}\Omega(\mathrm{BP})$
FET = 2N4351 ($\beta = 3.32 \times 10^{-4}, V_0 = -V_T = 1.3$ V)
op-amps = μA741 (dc bias = ± 15 V).
(a) Low-pass circuit. (b) Bandpass circuit.

fact that calculations at higher V_{GS} are made using the expression for R_{DS} of (5) while secondary effects taking into account parasitic resistances [14, p. 55] and decreases in β due to decreases in the mobility of carriers [14, p. 53] should be considered. The theoretical expressions for f_0, (2b) and (3b), do not take into account the frequency responses of the amplifiers used while in the experiments the high-frequency rolloff of the operational amplifiers was also found to be an important factor in checking the measured and theoretical results.

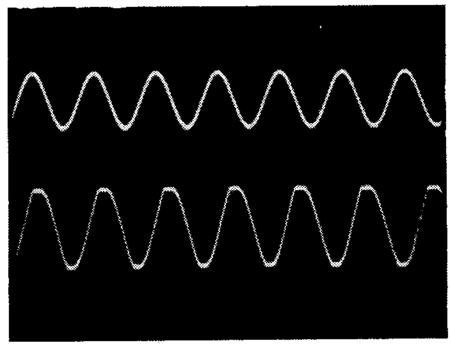


Fig. 5. Typical waveforms—Bandpass circuit; $f_0 = 740$ Hz. Upper trace: 5 V/cm, 1 ms/cm, $K_1 = 3.80$, $K_2 = -0.505$. Lower trace: 10 V/cm, 1 ms/cm, $K_1 = 4.05$, $K_2 = -0.505$.

In Fig. 5 we present typical sinusoidal and nonsinusoidal waveforms obtained for the bandpass realization. The sinusoidal waveform was found not to be distorted when K_1 varied within a range of ± 1.4 percent of its nominal value. Measurements taken with a Tektronix 3L5 Spectrum Analyzer show that the rejection of any spurious responses is greater than 60 dB.

The study of the frequency stability to temperature shows that the oscillation frequency can be insensitive to temperature changes. The relative change of oscillation frequency can be expressed as [15, p. 104]

$$\frac{\Delta\omega_0}{\omega_0} = \sum_{i=1}^{\mu} S_{R_i}^{\omega_0} \frac{\Delta R_i}{R_i} + \sum_{j=1}^{c} S_{C_j}^{\omega_0} \frac{\Delta C_j}{C_j} + \sum_{m=1}^{x} S_{K_m}^{\omega_0} \frac{\Delta K_m}{K_m} \tag{7}$$

where μ is the number of resistors, c is the number of capacitors, and x is the number of amplifiers. For both configurations one finds

$$\sum_{i=1}^{\mu} S_{R_i}^{\omega_0} = \sum_{j=1}^{c} S_{C_j}^{\omega_0} = -1. \tag{8}$$

If all resistors and all capacitors have the same temperature coefficients, i.e.,

$$\frac{\Delta R_i}{R_i} = \frac{\Delta R}{R}$$

and

$$\frac{\Delta C_j}{C_j} = \frac{\Delta C}{C}$$

then

$$\frac{\Delta\omega_0}{\omega_0} = -\left(\frac{\Delta R}{R} + \frac{\Delta C}{C}\right). \tag{9}$$

The ΔK is zero because K depends only upon the ratio of the resistances. From (9) it is easy to see that $\Delta\omega_0$ can be zero if

$$\frac{\Delta R}{R} = -\frac{\Delta C}{C}$$

which is possible to realize in practice with opposite temperature coefficients for the resistors and capacitors.

Finally, we comment that we have chosen Kerwin's circuit because of the independent control of ω_0 with K_2 once K_1 is fixed. This feature we have not located nicely in other active circuits with the same number of amplifiers, while the experiments we have performed have shown the true practicality of this single parameter control of ω_0. It should be mentioned that there are some other biquadratic structures [16], [17] which also have the ability of varying f_0 and Q independently with resistors, but they require three or more amplifiers and the position of the resistors is inconvenient.

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Video Amplifier Design Based on Op Amps

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***Abstract*—A design procedure for video amplifiers is discussed that will 1) determine if a specific op amp can be used to satisfy the design, 2) minimize the number of stages required, and 3) determine individual stage gains required to satisfy the design. A figure of merit, based on device parameters and the specific amplifier requirements, is developed to aid the design procedure.**

The design procedure is directed toward practical application in that it minimizes component cost, eliminates trial and error methods, and is directly applicable to worst-case design.

I. Introduction

In the field of video amplifier design there is a well-known method available for optimizing the overall gain or bandwidth of an amplifier consisting of several noninteracting stages [1].

Manuscript received June 5, 1975; revised October 16, 1975.
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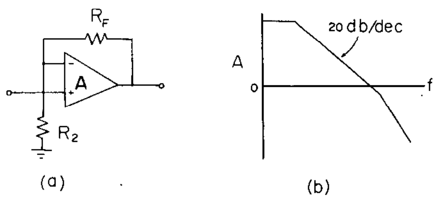

Fig. 1. (a) Noninverting op-amp stage. (b) Open-loop frequency response.

If a given overall gain is required, the overall bandwidth can be optimized by using identical stages with individual stage gains of 1.65. It has also been shown that if overall bandwidth is the specified figure, overall gain can be maximized by setting individual stage bandwidths equal. This result holds for nonidentical stages as well as iterative stages so long as the stages are noninteracting [2].

The goals of a practical design procedure are:

1) to determine if a given device can be used to satisfy the specifications of the amplifier in terms of gain, bandwidth, and impedance levels;
2) to minimize the number of stages required for the design;
3) to determine the individual stage gains and bandwidths required to meet the overall specifications.

While the theory mentioned earlier is helpful in the first step of the design process, it is of little direct value in the remaining steps. The theory is most often directed toward the goal of achieving a required gain along with maximum bandwidth assuming no limitation on the total number of stages. Practical design, due to cost considerations, focuses on achieving the required gain and required bandwidth using the minimum possible number of stages. In the past, the primary method used to reduce stages was trial and error. This is by no means a trivial task. When the number of stages is reduced from the number used to maximize overall bandwidth, the individual stage gains must be adjusted, the individual stage bandwidths will consequently change, and the factor relating overall bandwidth to individual stage bandwidth also changes. The design process based on trial and error methods is highly inefficient. This paper presents a concise design procedure for op-amp stages that requires little time to complete and meets the three goals listed above.

II. The Op Amp

The basic element in this procedure is the op amp compensated for unity gain which is shown in Fig. 1. These devices will exhibit a 20-dB/decade rolloff in gain above the upper corner frequency and will allow gain and bandwidth to be exchanged directly resulting in a constant gain–bandwidth product. This direct exchange of gain and bandwidth is in effect for gains of unity or higher.

Not only do most commercial op amps satisfy these assumptions, the input and output impedance levels of these devices lead to negligible interaction of stages when cascaded. Thus the gain and bandwidth of an individual stage can be adjusted without affecting gains or bandwidth of adjacent amplifier stages.

For the noninverting stage of Fig. 1(a) the gain is given by

$$A_{si} = 1 + R_F/R_2 \tag{1}$$

while the bandwidth can be found from

$$\omega_{si} = GBW/A_{si}. \tag{2}$$