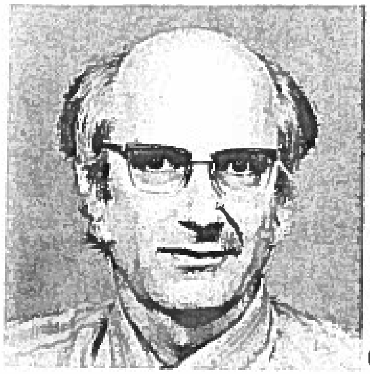

OPERATOR THEORY OF NETWORKS: A Short Exposition *

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"El que te busque en la vida
que estás viviendo, no sabe
más que alusiones de ti." [1, p. 22]

I. Motivation

Besides the very human desires for meaningful interactions one might seek further rationalization for the present interest in and development of the operator theory of networks in the subject matter itself. Toward this the topic title is very descriptive, the field being that which applies and develops the (engineering) network theoretical aspects of the (mathematical) area of operator theory. Within this framework there has recently developed a close working liason between a number of electrical engineers and mathematicians which holds, I believe, unusual promise for the future. Thus, engineers searching for techniques to apply toward more precisely designed networks, as integrated or microwave circuits, have found a compatible mathematical base, while the mathematicians searching for relevancy have found a suitable physical field to further stimulate operator theory. Indeed engineers have developed ideas parallel to those of the mathematicians though most often with a different physical base and not at the same level of generality. Undoubtedly it was a realization of this, coupled with most favorable personality complexes on both sides, that fostered the recent cross-fertilization which researchers have been steadily advancing.

Here an attempt will be made at a brief description of the field in an attempt to concisely point out important works, developments, and ideas, especially for someone interested in pursuing new topics or uses.

"El tiempo se contaba
apenas por minutos:
un minuto era un siglo
una vida, un amor." [1, p. 84]

II. Past

Almost any topic which has been studied in circuit theory and mathematics impinges upon the operator theory of networks. However the earliest most relevant work would probably be that of Heaviside [2] whose use of differential operators led in a significant way to the more recent developments of operational culculi [3] [4] and the theory of distributions [5] [6]. But this and related works are more of historical than practical significance to the field under discussion. Of more fundamentality is the work of Cauer, much of which is catalogues in the English translation of one of his books [7]. Two topics pursued by Cauer are of most interest. The first relates boundary values of positive-real functions to the functions themselves; this topic was more recently pursued by the mathematician-engineer combination of Beltrami and Wohlers in their beautiful work [8] which gives an excellent background for present undertakings in terms of distributions. The second topic from Cauer, and of more interest structurally and practically to our examination, concerns the transformation to canonical forms of various operators for synthesis and design purposes, as practically needed for certain communication networks. After some time it was realized, primarily by the practicing engineer Belevitch who gave initial synthese [9], that the scattering matrix was the basic operator of importance to practical and theoretical network design. Consequently, the foundation work of the more theoretical engineers Youla, Castriota and Carlin [10] appeared giving a treatment of the properties of scattering matrices as operators on Hilbert space. In a slightly different framework the mathematician Dolph developed a theory [11] concerning the properties of passive operators in Hilbert space. Work on synthesis in n-dimensional space continued that on reciprocal synthesis of the engineer Bayard in whose book [13] is also summarized the ultimate nonreciprocal n-port synthesis and equivalence theory of Oono and Yasura [14].

Of course numerous other contributions were made, but the field stood in early 1966 essentially as codified in [15]. Briefly, the properties of passive time-invarient n-port networks were well understood, their behavior as Hilbert space maps being realized as important; basic synthesis of finite (lumped) networks of this class were complete.

"Que alegría, vivir
sintiéndose vivido" [1, p. 35]

III. Present

Recent developments are best looked at, I believe, as beginning in early 1966. These developments are mostly toward extensions to larger classes of networks (e.g., time-variable, lumped-distributed, nonlinear) through the use of deeper mathematics (e.g., differential algebra, characteristic operator functions, Banach algebras). And although other factors certainly played significant roles in setting the present status of the field, in my mind the most influential one was the leave of the Argentinian mathematician A. Gonzàlez-Domínguez to visit the University of California in mid 1967. This let to his work [16] on infinite-dimensional cascades through product integral factorizations but more importantly opened a receptive door of discussion between mathematicians and engineers, a door into which the young Belgian engineer-mathematician P. Dewilde exactly fit.

The latter's work, as yet not journal published, begun in electrical engineering at Stanford [17] and polished in postdoctoral research in mathematics at Berkeley [18], positions the present status of the operator theory of cascade synthesis. This latter is based upon invariant subspace theory [19] which is used to set up a state-space type of technique which allows cascade synthesis of time-invariant networks of the most general nature, those described by roomy scattering matrices. Professor Gonzàlez-Domínguez was also the first to bring attention in the West to the Russian work of M. Livsic [20].

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Some of the other significantly influential factors to be mentioned here, with relevant references, are:

1) the research teams of the mid 1960's through early 1970's and their interactions, at Cornell (N. DeClaris [21], R. Saeks [22], [23]), Harvard (J. Baras [24], R. Brockett [25] [26]), Stanford (B.D. Anderson (especially) [27] [28] [29], P. Dewilde [30], B.L. Ho [31], R. Kalman [32], E. Kamen [33], R. Newcomb [34], D.A. Spaulding [35]), and Stony Brook (E.J. Beltrami [36], V. Doležal [37] [38], R.G. Douglas [39], J.W. Helton [40] [41] [42], A. Zemanian [43] [44] [45]).

2) the Stony Brook meeting on "The Applications of Generalized Functions" in September 1966 [46].

3) the arrival of N. Levan [47] [48] [49] at UCLA via Stanford in the summer of 1966.

4) the informal Maryland symposium of October, 1972, on "Operator Theory of Networks" [50]

One of the basic ideas in much of this work is that of embedding a scattering operator in a unitary dilation [51]. The mathematics for the operation in Hilbert space comes from the work of Halmos [52] [53, p. 57], with extensions [54] as yet unexplored in network theory, while physically the given scattering operator results by loading a network for the unitary operator in resistors following ideas originally put forth by Belevitch [9]. Although the concept can be generalized from Hilbert spaces to C*-algebras [55], still there are causality [22] and finiteness problems which seem better solved by the use of differential field techniques [34]. Another idea [41] [49] for synthesis of Hilbert space operators is to use the characteristic operator functions of the mathematicians Sz-Nagy and Foias [56] to create state-space [57] type realizations. This technique, in contrast to that of unitary dilations, introduces a new physical concepts associated with the use of a class of infinite dimensional state spaces, for example for finite networks. Consequently, when coupled with ideas on distributed networks, as for example those of Wohlers [58], synthesis techniques are anticipated for microwave networks; and related to microwave networks, where each of the infinite number of modes of propagation can be associated with a port, is the theory of Hilbert ports [59] [60] which is finding its natural setting within the operator theory of networks.

When specialized to finite time-varying networks the scattering operator can take the form of a distributional kernel whose properties are well-studied in the passive case [61]. These properties have been used by Spaulding [35] and Anderson [27] to achieve synthesis of various kernels by expansions produced upon them. Since, however, a complete synthesis in engineering terms still seems open, it is my view that a return to the original operator calculus ideas of Heaviside would prove fruitful, especially when looked at through the ideas of modern algebraic operators. Consequently [34] represents the present state of this aspect of the operator theory of networks; this work extends that of Saeks [62] by considering Galois type extensions within a theory of operators in differential field theory [63].

It should be mentioned that some authors very successfully are turning to time-invariant, infinite-dimensional Hilbert space operators as elements on Hardy spaces. For example, Baras [64] with Brockett [65] and Fuhrmann [66] proceed along this line to obtain state-space realizations while Gibbs [67] characterizes step-up networks in this manner.

Finally I must say that in this day and age it is almost impossible to completely catalog results in a rapidly developing field, as the operator theory of networks, so I hope the shortcomings and omissions of this write-up will be readily recognized. I would mention though that technical details for concepts discussed here are covered in [68] while a collection of background papers is being made available [69]. Further engineering oriented background material can be gained from [70] [71] while a different mathematical approach stems from the scattering theory of Lax and Phillips [72].

"¡ Mañana! Qué palabra
toda vibrante, tensa
de alma" [1, p. 16]

IV. Future

Because the reservoir of mathematical techniques has barely been tapped and because there are a number of significant engineering problems requiring solutions, I feel the future looks particularly promising for the further development of the operator theory of networks. This is especially true in the area of engineering applications toward design. For example, there presently are no rigorous synthesis methods for microwave networks, of the quality available for finite networks, while the material on hand points to such syntheses within the operator theory framework. Likewise the practice of active lumped-distributed networks using tapered RC lines is of considerable importance to the theory of integrated circuits [73, p. 19] where reductions in size can be achieved through appropriate tapers; however, at present no solid theory of design is available though such should be obtainable by a proper specialization of operator theory. Also of considerable importance to microwave and integrated circuit designs are needed theories of approximation [74] and interconnections [75] [76] which are finding a proper home within operator theory. Undoubtedly solution of practical problems of this nature will require a commitment by practicing engineers and theoretical engineers and mathematicians to listen to each other and discuss needs and ideas. On the part of the latter group I feel that the door is now open and a forum being established [77] while past epxerience [78] shows that diverse personalities can constructively contribute. Of course, for some time creative mathematically inclined engineers and mathematicians will find numerous theoretical topics of interest of a kind that a detailed study of the references will show. But it is also my hope that the practicing engineers will find it profitable too to enter their significant ideas into the field.

In summary, the operator theory of networks is a field in which rather deep concepts of both mathematical and engineering significance can lead to relevant developments. The field is relatively well established now and holds considerable future promise especially where sophisticated engineering concepts are concerned.

"Tú vives siempre en tus actos
Con la punta de tus dedos
pulsas el mundo; es tu música.
Yo no puedo darte más
No soy más que lo que soy" [1, pp. 9, 37]

I wish to thank Professor Saeks for the opportunity to express these views while dedicating the work to Professor C. Desoer whose encouragement for the field has been a source of inspiration.

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OPERATOR THEORY OF SYSTEMS

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I. Introduction

"If one takes the point of view that a system is a black box with inputs and outputs being functions of time then it is natural to model such systems in terms of an operator carrying input time functions into output time functions. The specific mathematical models which are used to describe this operator characterise the three main approaches to systems theory, namely, the state space, the transfer function, and the operator theoretic approach.

The state space approach utilizes differential and/or difference equations and it finds application in the study of linear and nonlinear multivariable systems, whose state can be defined by a finite number of parameters. The transfer function approach utilizes the Fourier, Laplace, or Z transforms as appropriate and is applicable to time invarient linear systems with single input-single output cases working best. In the operator theoretic approach, the system input and output are put in correspondence with the elements of an abstract mathematical set equipped with an axiomatic structure which is adjusted according to the nature of the specific problem under study. Such an approach yields a unification of the continuous and discrete function theories and simultaneously allows one to formulate a unified theory which is valid for time variant, distributive, linear, and nonlinear systems alike. Surprisingly, however, the great potential of the operator theoretic approach is only recently being realized and it seems appropriate to remark on some of the reasons for its belated development.

First, the operator theoretic approach utilizes the tools of functional analysis and as such is less accessible to the engineering community than state variable or transform approaches. Moreover, early efforts to apply classical operator theory typically faltered when optimal controllers proved to be noncausal, feedback systems unstable, or coupling networks nonlossless. Attempts to circumvent these difficulties by adding causality or stability constraints to the problems were hindered by the fact that these time based concepts were not a natural feature of the abstract Hilbert and Banach spaces of the classical operator theory.

Secondly, the system theory community, stimulated by aerospace applications and neighboring automata theory, was enamored by state variable methods and their utility in optimization and stabilization settings. The ramifications of the maximum principal, dynamic programming, Kalman filtering, Liapunov stability etc. rightfully preempted the attention of both applied and theoretical engineers. Functional analysis and the operator theoretic approach played primarily a supporting role to the state variable movement.

A natural consequence of the above factors is the paucity of engineering schools offering curriculum conducive to the operator theoretic view point. Let me hasten to include my own institution among those suffering from this malady. The factors contributing to this situation are far too numerous and complex to consider here. The present flat growth rate, in student enrollments, academic hiring, and funding of engineering schools indicate that this situation is not likely to rapidly change.

Over the past few years some of these difficulties have been alleviated. The technical problems have been solved in part by the emergence of a modified operator theory wherein time related concepts such as causality, stability, and passivity are well defined and readily studied. Although the techniques were developed independently by a number of different authors they are characterized by two consistent trends. First, the time characteristics of a function are characterized by

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