A Note On Lumped-Distributed Synthesis

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Abstract—**Discussion is given on recently published works concerning a problem in lumped-distributed cascade synthesis.**

I. Introduction

RECENTLY there has been considerable interest in the cascade synthesis of resistively terminated uniform transmission lines and lumped passive lossless 2-ports. Recent papers [1], [2] have demonstrated basic errors in the earlier interesting work of Koga [3]. The object of this letter is to show where the proof given in [3] breaks down. Since the ideas behind Koga's technique are relatively straightforward it has seemed worthwhile pinpointing errors in the reasoning in order to perhaps obtain a modified synthesis, even though explicit formulas do exist for some recently presented syntheses [4], [5].

II. Error in Proof

We shall show that the following statement of [3, p. 449] is not necessarily true:

> "...recalling that g_i and h_i and therefore g_{i*} and h_{i*} are relatively prime, we see from (39) and (40) that $k = k_1 k_2 \cdots k_t$ divides both ξ_{t+1} and η_{t+1};..."

In fact, using [3, eq. (39)] one gets

$$k\xi(\lambda,1) = 2^{\nu_1+\nu_2+\cdots+\nu_t} g_1 g_2 \cdots g_t \xi_{t+1}(\lambda,1)$$

$$k\eta(\lambda,1) = 2^{\nu_1+\nu_2+\cdots+\nu_t} h_1 g_2 \cdots g_t \xi_{t+1}(\lambda,1)$$

which does not prove the above assertions since k could divide $g_2 \cdots g_t$. A similar argument holds for [3, eq. (40)]. In fact, it cannot be shown that $k = k_1 k_2 \cdots k_t$ is a constant, and this condition has to be assumed in order for the synthesis problem to be solvable as given in [3]. Unfortunately, the assumption that k is a constant cannot be made at the outset in a simple way, but rather, in each synthesis cycle, k_i has to "come out" as constant; otherwise, active sections may appear in the process.

Therefore, [3, Lemma 7], which is fundamental to the theory, cannot be proven, since it depends on the assumption that $k_i \equiv 1$ and hence some of the ξ_i $(i = 1,2,\ldots, t+1)$ and the g_i $(i = 1,2,\ldots, t+1)$ may not be Hurwitzian, in which case the remaining scattering matrix $s_i = \eta_i/\xi_i$ need not be bounded-real. An additional condition is therefore needed to guarantee that χ_i in [3, eqs. (29) and (30)] are equal to $g_i g_{i*} - h_i h_{i*}$ as at (31).

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III. Discussion of the Counter Example

The counterexample given in [1] and modified in [2] illustrates the above. In the notation of [3], the scattering function corresponding to the counterexample in [1] is

$$S(\lambda,\mu) = \frac{h(\lambda,\mu)}{g(\lambda,\mu)}$$

where

$$g(\lambda,\mu) = (\lambda + 1)(7\lambda + 11)(2\lambda + 5)(\mu + 1)^2 + 2(10\lambda - 1)(1 - \mu^2) - 15(\lambda - 1)^2(1 - \mu)^2$$

$$h(\lambda,\mu) = -3(\lambda + 1)(7\lambda + 11)(1 + \mu)^2 + 4(2 - 7\lambda) \cdot (1 - \mu^2) + 5(5 - 2\lambda)(\lambda - 1)^2(1 - \mu)^2.$$

Following the synthesis procedure in [3], one gets

$$g(\lambda,1)/h(\lambda,1) = g_1(\lambda)/h_1(\lambda),$$

$$h_1(\lambda) = -3, \qquad g_1(\lambda) = 2\lambda + 5$$

$$h_1\xi_1 - g_1\eta_1 = 4(1 - \mu)[(2\lambda - 1)(7\lambda + 11)(1 + \mu) + 5(\lambda - 1)^2(\lambda^2 - 4)(1 - \mu)] = \pm(1 - \mu)\chi_1\eta_2 \qquad \text{by (29)}$$

$$g_{1*}\xi_1 - h_{1*}\eta_1 = 4(1 + \mu)[(\lambda + 1)(7\lambda + 11)(-\lambda^2 + 4) \cdot (1 + \mu) - 5(\lambda - 1)(2\lambda + 1)(1 - \mu)] = (1 + \mu)\chi_1\xi_2 \qquad \text{by (30)}$$

$$g_1 g_{1*} - h_1 h_{1*} = 4(-\lambda^2 + 4) = f_1 f_{1*} = k_1\chi_1 \qquad \text{by (33).}$$

Therefore, comparing these with χ_1 the common factor

$$\chi_1(\lambda) = 1 \qquad k_1 = 4(-\lambda^2 + 4)$$

and

$$\eta_2 = 4[(2\lambda - 1)(7\lambda + 11)(1 + \mu) + 5(\lambda - 1)^2 \cdot (\lambda^2 - 4)(1 - \mu)]$$

$$\xi_2 = 4[(\lambda + 1)(7\lambda + 11)(-\lambda^2 + 4)(1 + \mu) - 5(\lambda - 1) \cdot (2\lambda + 1)(1 - \mu)].$$

From $\eta_2(\lambda,1)/\xi_2(\lambda,1) = g_2(\lambda)/h_2(\lambda)$, one gets

$$g_2(\lambda) = (\lambda + 1)(-\lambda^2 + 4)$$

$$h_2(\lambda) = (2\lambda - 1)$$

$$g_2 g_{2*} - h_2 h_{2*} = -64(\lambda^6 - 9\lambda^4 + 20\lambda^2 - 15)$$

$$h_2\xi_2 - g_2\eta_2 = 160(1 - \mu)(\lambda - 1) \cdot (\lambda^6 - 9\lambda^4 + 20\lambda^2 - 15)$$

$$g_{2*}\xi_2 - h_{2*}\eta_2 = -32(1+\mu)(7\lambda+11) \cdot (\lambda^6 - 9\lambda^4 + 20\lambda^2 - 15).$$

Therefore,

$$\chi_2 = -32(\lambda^6 - 9\lambda^4 + 20\lambda^2 - 15) \quad \text{and} \quad k_2 = 1$$

$$\eta_3 = 5(1-\lambda) \qquad \xi_3 = 7\lambda + 11.$$

Thus, in the first cycle, k_1 does not come out a constant, i.e., $\chi_1(\lambda) \neq g_1 g_{1*} - h_1 h_{1*}$, and hence an active section is encountered since $\xi_2(\lambda,\mu)$ and $g_2(\lambda)$ cannot be made Hurwitzian.

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References

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