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ACTIVE SYNTHESIS USING THE DVCCS/DVCVS

MICHAL BIALKO AND WIESLAW SIENKO
Politechnika Gdanska, Poland

ROBERT W. NEWCOMB*
University of Maryland, U.S.A.

SUMMARY

In the paper basic properties of a new versatile linear active element acting simultaneously as a Differential Voltage Controlled Current Source (DVCCS) and Differential Voltage Controlled Voltage Source (DVCVS) are described. This element called a DVCCS/DVCVS can be used for generation of all linear nondynamic elements and also can be applied directly in active RC synthesis models. The general synthesis model with one DVCCS/DVCVS element and two particular synthesis models derived from the general one are described. All synthesis models presented here allow for cascading the second degree sections without additional buffers.

INTRODUCTION

It is known from experience that in integrated circuit synthesis and realizations it is most convenient to introduce technologically homogeneous building blocks called generating elements. These elements are basic components from which the networks can be constructed theoretically or practically, through interconnections. In a previous work[1] it was shown that in conjunction with the capacitor, the differential voltage controlled current source (DVCCS) forms a sufficient set of generating elements to realize all linear finite networks.

However, using only the DVCCS as a nondynamic generating element leads in some cases to relatively complicated equivalences. For example, to realize a differential voltage controlled voltage source (DVCVS) it is necessary to use three DVCCS's. Thus from the practical point of view it is more convenient to introduce a building block which acts simultaneously as a DVCCS and as a DVCVS element. Such a block, called the DVCCS/DVCVS device was also proposed in Reference 1. In this paper it is shown that the DVCCS/DVCVS element can be used for simple generation of basic sets of active elements which are useful for active RC circuit synthesis. Several synthesis models for the realization of any second degree voltage transfer function are given as well.

ACTIVE ELEMENT GENERATION

The DVCCS/DVCVS is basically a differential input, linear active element with two outputs: one of high impedance and the other of low impedance. The proposed symbol and equivalent circuit of an ideal DVCCS/DVCVS element are shown in Figure 1(a) and 1(b) respectively.

With reference to Figure 1 the DVCCS/DVCVS element can be defined by the relations:

$$I_0 = G(V_- - V_+) \tag{1}$$

$$V_0 = \alpha V_g = \alpha \frac{G}{G_g}(V_+ - V_-), \qquad G_g \to 0 \tag{2}$$

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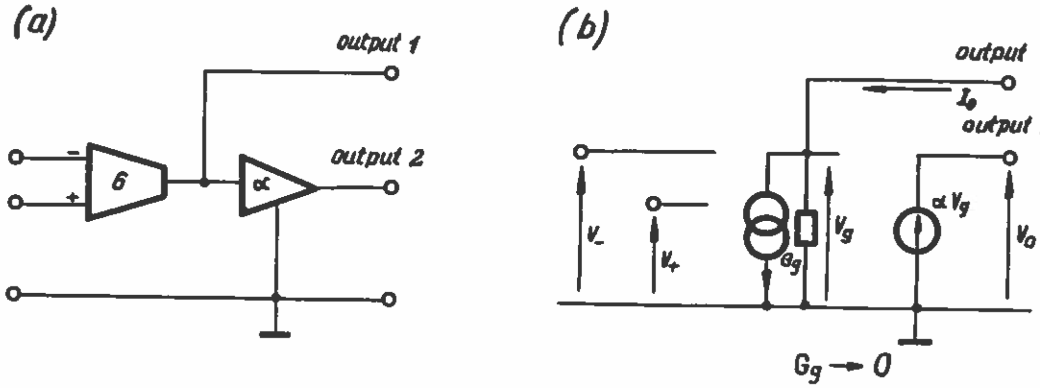

Figure 1. The circuit symbol and equivalent model of the DVCCS/DVCVS

where G and α are assumed to be independent of frequency and input voltage levels. From the representation of Figure 1 it is seen, that the impedances at input and output 1 are infinite and the output 2 impedance is zero. The linearity of the element implies zero voltage levels at both outputs when $(V_- - V_+)$ is zero. Using the element defined above, simple generation of basic sets of other active elements can occur. As an example the realization of a DVCVS is shown in Figure 2. It should be noted that if R_L, in Figure 2, is infinite then the circuit is recognized as an operational amplifier. In this case as shown in Figure 3, the output 1 can be used for frequency compensation of the operational amplifier frequency characteristic.

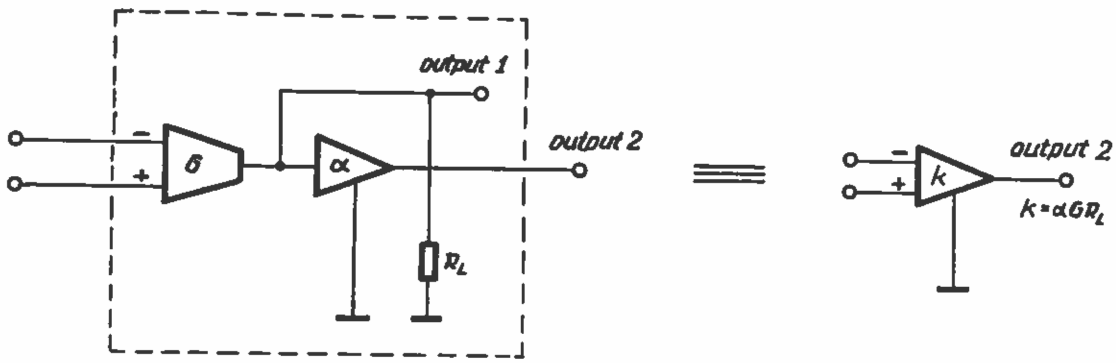

Figure 2. The realization of the DVCVS using the DVCCS/DVCVS element

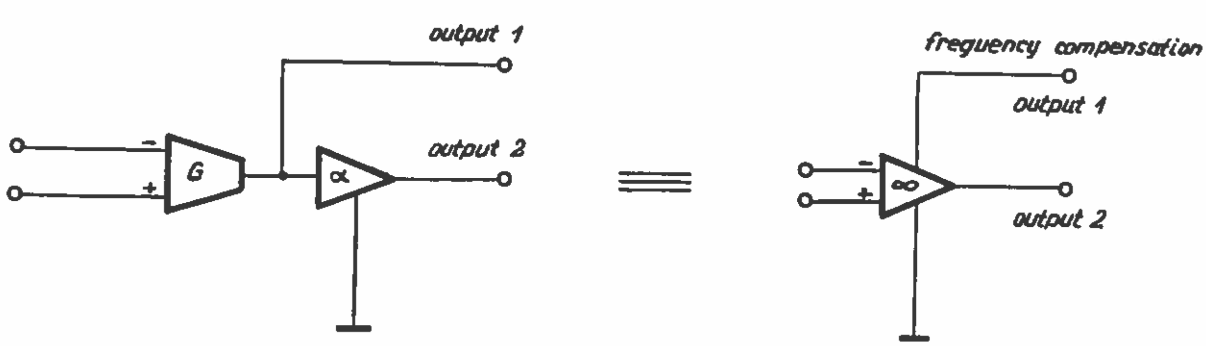

Figure 3. The realization of an operational amplifier using the DVCCS/DVCVS

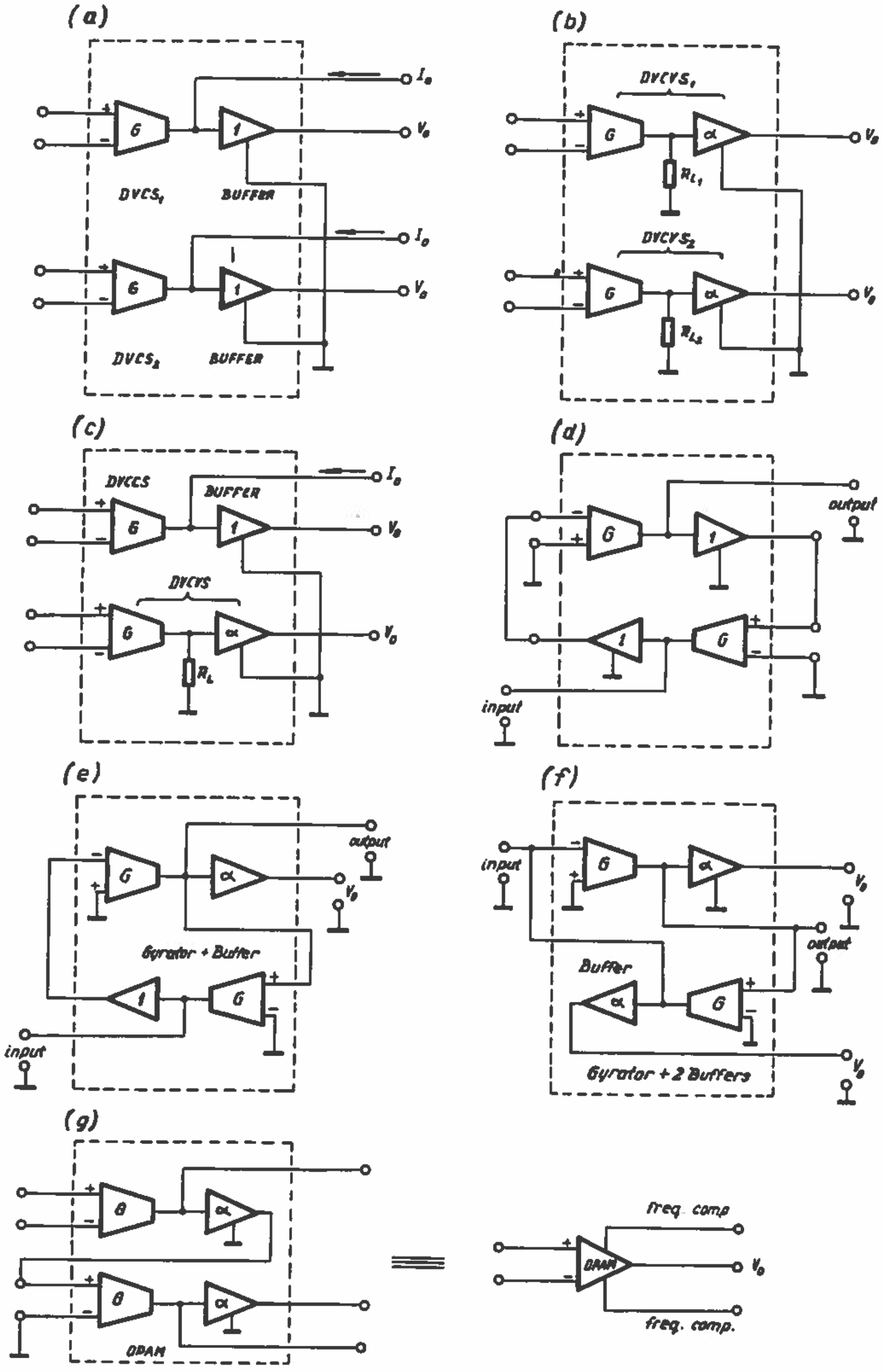

Figure 4. Generation of some basic active elements using the DVCCS/DVCVS

A versatile building block can be obtained placing for example two integrated DVCCS/DVCVS elements into one encapsulation. Such a block can fulfill the following functions:

(a) two DVCCS with two buffers
(b) two DVCVS
(c) DVCCS with buffer and DVCVS
(d) gyrator
(e) gyrator and buffer
(f) gyrator with two buffers
(g) very high gain operational amplifier

These are illustrated in Figure 4(a)–4(g) respectively. A cascade connection of two DVCCS/DVCVS elements gives a very high gain operational amplifier as shown in Figure 4(g). The points of high impedance can be used for compensation of frequency characteristic of the amplifier. From the above listed realizations it is clear that of special importance is the simplicity of generation of a gyrator, for known linear applications, and a conventional operational amplifier, as well as the introduction of a gyrator with two low output impedance buffers.

The structure of the DVCCS/DVCVS element obtained as a result of searching the smallest set of elements for generation of all finite linear circuits,[1] is similar to the structure of second generation operational amplifier like, for example LM 101.[7] However, the output of current source of the operational amplifier is not at zero d.c. level and it is utilized only for the frequency characteristic compensation. The possibility of connection of the external elements to the output 1 of the DVCCS/DVCVS makes it more versatile than the conventional operational amplifier. For example, the practical realization of the DVCVS having finite gain, as shown in Figure 2, is rather difficult using only one conventional op. amp. In engineering practice the structures of the integrated DVCCS/DVCVS elements can be based on existing structures of controlled current sources and conventional operational amplifiers with the possibility of precise determination of the transconductance G value by means of proper selection of external elements.[8,9]

ACTIVE RC SYNTHESIS MODELS USING THE DVCCS/DVCVS ELEMENT

For synthesis purposes the DVCCS/DVCVS element can be used in many ways among which are:
(a) individually as controlled sources
(b) as an element for generation of gyrators

In this section, a synthesis model for realization of any stable, second degree voltage transfer function is described using the DVCCS/DVCVS element as controlled sources. The structure of the proposed synthesis

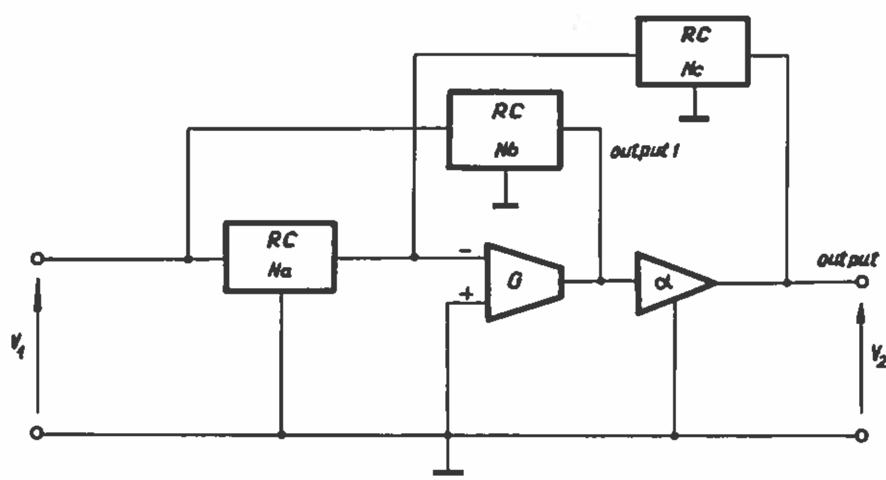

Figure 5. Structure of the synthesis model with one DVCCS/DVCVS

model is shown in Figure 5. The model is composed of one DVCCS/DVCVS and three passive RC unbalanced two-ports. Routine analysis yields the following expression for the voltage transfer function of the circuit shown in Figure 5:

$$K(s) = \frac{V_2(s)}{V_1(s)} = \frac{\dfrac{\alpha G y_{21a}}{y_{11c}+y_{22a}} - \alpha y_{21b}}{y_{22b} - \dfrac{\alpha G y_{12c}}{y_{11c}+y_{22a}}} \tag{3}$$

where:
y_{ikl}, $l = a, b, c$, are admittance matrix elements of the two-ports N_a, N_b, N_c respectively, and α and G are real.

A synthesis procedure of a prescribed transfer function $K(s)$ using the formula (3) can be rather difficult since it requires the synthesis of three RC two-ports for which too many free parameters are available. To simplify the synthesis procedure two special cases are discussed.

Model 1

It is assumed here that the two-ports N_a and N_b have forms of inverted el's, and the two-port N_c is formed by a floating one-port as shown in Figure 6. Such assumptions replace the two-port synthesis by the simpler one of one-ports Y_i, $i = 1, 2 \ldots 5$.

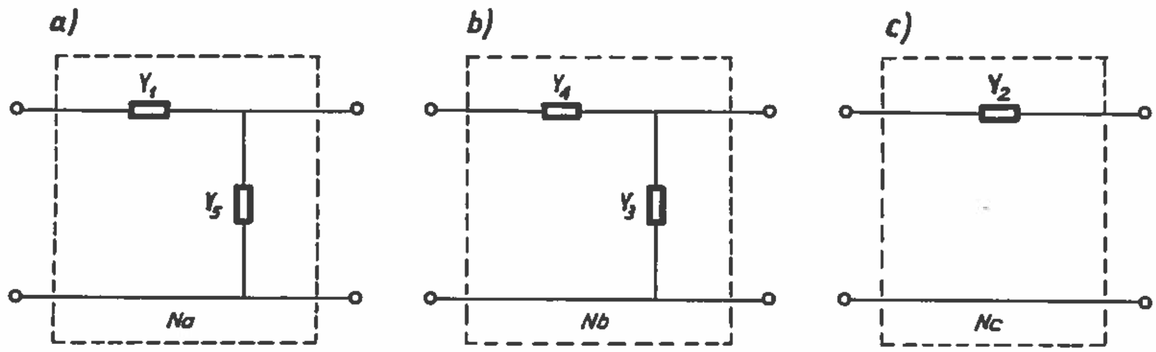

Figure 6. Assumed structures for passive two-ports N_a, N_b, N_c

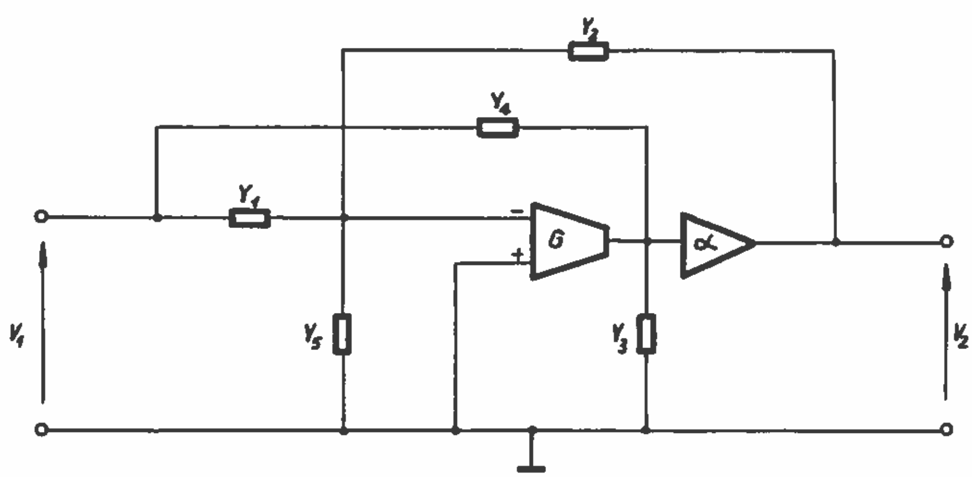

Figure 7. Synthesis model of Figure 5 with two-ports N_a, N_b, N_c as in Figure 6

With these assumptions the synthesis model of Figure 5 takes the form shown in Figure 7. Now the expression for the voltage transfer function for the circuit of Figure 7 is:

$$K(s) = \frac{V_2(s)}{V_1(s)} = \frac{\alpha\left(Y_4 - \dfrac{GY_1}{Y_1+Y_2+Y_5}\right)}{Y_3+Y_4+\dfrac{\alpha GY_2}{Y_1+Y_2+Y_5}} \tag{4}$$

If one assumes that the admittance Y_2 is real:

$$Y_2 = G_2 \tag{5}$$

then the function (4) becomes similar to the form of the formula obtained by Holmes in his gyrator model[2]:

$$K(s) = \frac{N(s)}{D(s)} = \alpha\frac{Y_4 - \dfrac{GY_1}{Y_1+G_2+Y_5}}{Y_3+Y_4+\dfrac{\alpha GG_2}{Y_1+Y_5+G_2}} \tag{6}$$

It should be noted that the proposed model is more convenient than Holmes' model because of the possibility of the cascade connection of several networks without the additional buffers. Thus, the practical realizations of higher order transfer functions become simpler.

Since for the cascade realization of the gyrator filter at least three conventional operational amplifiers should be used, the relation (6) shows, that a filter with similar properties can be realised using only one DVCCS/DVCVS element.

It will be shown that the synthesis model of Figure 7 enables the simple realization of any stable second degree transfer function.

Model 2

Another simplification of a general model of Figure 5 will occur if we assume that the prescribed transfer function $K(s)$ has no positive real zeros. In this case the two-port N_a can be removed, $y_{11a} = y_{12a} = y_{22a} = 0$ and the synthesis model is as shown in Figure 8.

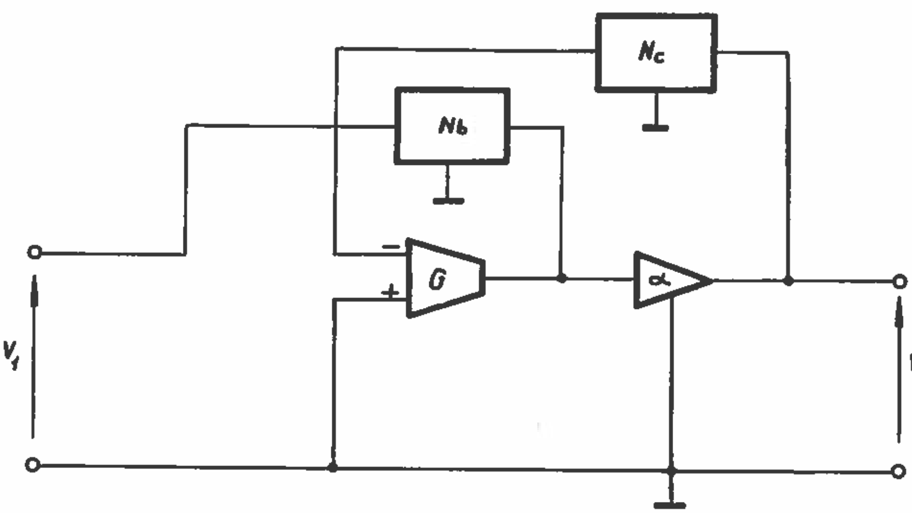

Figure 8. Simplified synthesis model

The voltage transfer function for this model is obtained directly from the expression (3):

$$K(s) = \frac{V_2(s)}{V_1(s)} = \frac{-(\mu y_{21b}/G)}{y_{22b}-(\mu y_{12c}/y_{11c})} \tag{7}$$

where:

$$\mu = \alpha G > 0 \tag{8}$$

The form of the function (7) allows us to apply the synthesis procedure given by Hakim.[4] Thus, the possibility of pole sensitivity minimization of the prescribed transfer function using complex phantom zeroes exists.

SYNTHESIS PROCEDURE

It is known that from a sensitivity point of view it is usually preferable to realize a higher degree transfer function by cascading second degree stages.

Because the proposed models allow cascade connection of several subnetworks without additional buffers, the synthesis procedure for a prescribed transfer function $K(s)$ consists first of all of a factorization into the form:

$$K(s) = \frac{N(s)}{D(s)} = \prod_i K_i(s) = \prod_i \frac{N_i(s)}{D_i(s)} \tag{9}$$

where: polynomials $N_i(s)$ and $D_i(s)$ are real,

$$\delta N_i(s), \delta D_i(s) \leqslant 2 \tag{10}$$

($\delta N_i(s)$, $\delta D_i(s)$ are the degrees of $N_i(s)$ and $D_i(s)$ respectively).

If $\delta D_i(s) = 2$, $D_i(s)$ is a Hurwitz polynomial of the form:

$$s^2+b_1 s+b_2 \tag{11}$$

with $b_1, b_2 > 0$.

Next a synthesis of the second degree terms $K_i(s)$ using the structures, shown in Figure 7 or Figure 8, is performed.

Synthesis procedure for model 1

Under assumptions (10) and (11), using simultaneously the RC: −RC and RC:RL decompositions, every function $K_i(s)$ can be presented in the following form:[3]

$$K_i(s) = \frac{N_i(s)}{D_i(s)} = \frac{N_i(s)/Q(s)}{D_i(s)/Q(s)} = \frac{W_{RC}^1 - W_{RC}^2}{Z_{RL}+Z_{RC}} \tag{12}$$

where: $Q(s)$ is a monic polynomial having only distinct negative real roots, that is

$$Q(s) = s+\sigma_0, \sigma_0 > 0 \tag{13}$$

or

$$Q(s) = (s+\sigma_1)(s+\sigma_2), \qquad \sigma_1 > \sigma_2 > 0 \tag{14}$$

W_{RC}^i, $i = 1, 2$ is an RC driving-point immitance, Z_{RC}, Z_{RL} are RC and RL driving-point impedances respectively.

Comparison of the relationship (12) and (4), under the assumption that $Y_2 = G_2$ and α and G are positive, shows that the function $K_i(s)$ can be realized as a voltage transfer function using the synthesis model of Figure 7 where:

$$\alpha Y_4 = W_{RC}^1 \tag{15}$$

$$\frac{\alpha G Y_1}{Y_1+G_2+Y_5} = W_{RC}^2 \tag{16}$$

$$Y_3+Y_4 = Z_{RL} \tag{17}$$

$$\frac{\alpha G G_2}{Y_1+G_2+Y_5} = Z_{RC} \tag{18}$$

From the solution of equations (15)–(18) the admittances Y_i, $i = 1\ldots5$ of the passive RC one-ports are obtained.

Second degree transfer function synthesis using model I

Let the given function $K_i(s)$ have the form:

$$K_i(s) = \frac{a_0s^2+a_1s+a_2}{s^2+b_1s+b_2} \tag{19}$$

where:

$$b_1, b_2 > 0$$

and $a_0 \geqslant 0$, that is, in general a realization to within a sign of the prescribed function is possible.

In the form of equations (13) and (14) polynomial $Q(s)$ can be selected to give passive RC immitance functions on the right of equations (15)–(18).

Case I: $Q(s) = s+\sigma_0$

For a degree one $Q(s)$, (13), the function $K_i(s)$ can be expressed as:

$$K_i(s) = \frac{\left(a_0s+\frac{a_2}{\sigma_0}+g_1\right)-\left[\frac{(a_0\sigma_0^2+a_2)/\sigma_0-a_1}{s+\sigma_0}s+g_1\right]}{(s+b_1+\sigma_0)+\left(\frac{\sigma_0^2-b_1\sigma_0+b_2}{s+\sigma_0}\right)}$$

$$= \frac{W_{RC}^1-W_{RC}^2}{Z_{RL}+Z_{RC}} \tag{20}$$

In this case the solution of the set of equations (15)–(18) is given in Table I.

The general synthesis model for realization of the transfer function (19) is shown in Figure 9. It should be noted that in some cases the structure of Figure 9 can use fewer elements than shown through suitable choice of values α, g_1 and G. The realizations for second degree transfer functions of various special cases can be directly obtained from the relationships of Table I.

It should be noted, that in discussed transfer function realization, the modulus of the pole sensitivity for variations of the gain $\mu = \alpha G$ of the active element depends on the value of the coefficient σ_0. According to the theorem of Calahan,[3] the pole sensitivity modulus reaches its minimum value for $\sigma_0 = b_1/2$.

The pole-locus of s_1, for optimal Calahan decomposition of $D(s)$ is given in Figure 10, where $D(s) = s^2+b_1s+b_2$ and $D(s_1) = 0$.

The Q and ω_0 values of a pole of the transfer function given by (19), realized using the structure shown in Figure 9 are:

$$Q = \frac{\{(C_1+C_5)(C_3+C_4)[(G_2+G_5+G_1)(G_3+G_4)+\alpha GG_2]\}^{\frac{1}{2}}}{(C_3+C_4)(G_1+G_2+G_5)+(C_1+C_5)(G_3+G_4)} \tag{21}$$

$$\omega_0 = \left[\frac{(G_1+G_2+G_5)(G_3+G_4)+\alpha GG_2}{(C_1+C_5)(C_3+C_4)}\right]^{\frac{1}{2}} \tag{22}$$

for $Q \gg 1$.

Table I.

Element of the model Figure 9	Prescribed function: $K_t(s) = \dfrac{a_0s^2+a_1s+a_2}{s^2+b_1s+b_2}$ $b_1, b_2 > 0, \quad a_0 \geqslant 0,$
$Y_1 = C_1s+G_1$	$C_1 = \dfrac{G_2[(a_0\sigma_0^2+a_2)/\sigma_0-a_1+g_1]}{\sigma_0^2-b_1\sigma_0+b_2}; \quad G_1 = \dfrac{g_1\sigma_0}{\sigma_0^2-b_1\sigma_0+b_2}$
$Y_2 = G_2$	$0 < G_2 < \infty$
$Y_3 = C_3s+G_3$	$C_3 = \left(1-\dfrac{a_0}{\alpha}\right); \quad G_3 = b_1-\sigma_0-\dfrac{g_1+a_2/\sigma_0}{\alpha}$
$Y_4 = C_4s+G_4$	$C_4 = \dfrac{a_0}{\alpha}; \quad G_4 = \dfrac{g_1+a_2/\sigma_0}{\alpha}$
$Y_5 = C_5s+G_5$	$C_5 = G_2\dfrac{\alpha G-(a_0\sigma_0^2+a_2-a_1\sigma_0)/\sigma_0-g_1}{\sigma_0^2-b_1\sigma_0+b_2}$ $G_5 = G_2\dfrac{\alpha G\sigma_0-(\sigma_0^2-b_1\sigma_0+b_2+g_1\sigma_0)}{\sigma_0^2-b_1\sigma_0+b_2}$
α	$\alpha \geqslant \max\left[a_0, \dfrac{a_2+g_1\sigma_0}{(b_1-\sigma_0)\sigma_0}\right]$
G	$G \geqslant \max\left[\dfrac{a_0\sigma_0^2+a_2}{\alpha\sigma_0}+\dfrac{g_1-a_1}{\alpha}; \quad \dfrac{\sigma_0^2-b_1\sigma_0+b_2+g_1\sigma_0}{\alpha\sigma_0}\right]$
g_1	$g_1 \geqslant \max\left[0, -\dfrac{a_2}{\sigma_0}, \dfrac{-a_0\sigma_0^2+a_1\sigma_0-a_2}{\sigma_0}\right]$
σ_0	$0 < \sigma_0 < b_1$ when $b_1^2 < 4b_2$ $0 < \sigma_0 < (b_1-\sqrt{(b_1^2-4b_2)})/2$ or $b_1+\sqrt{(b_1^2-4b_2)}/2 < \sigma_0 < b_1$ when $b_1^2 > 4b_2$

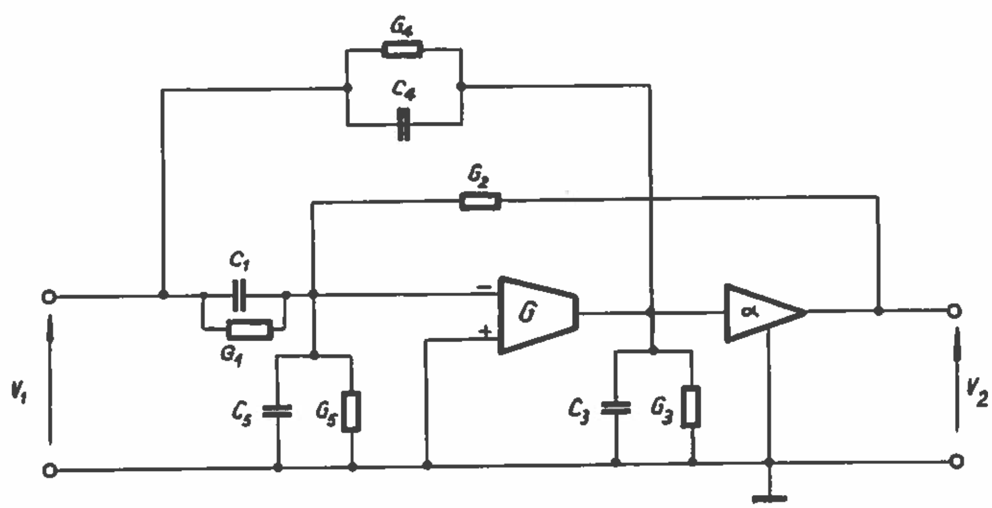

Figure 9. Realization of the transfer function (19) using model 1

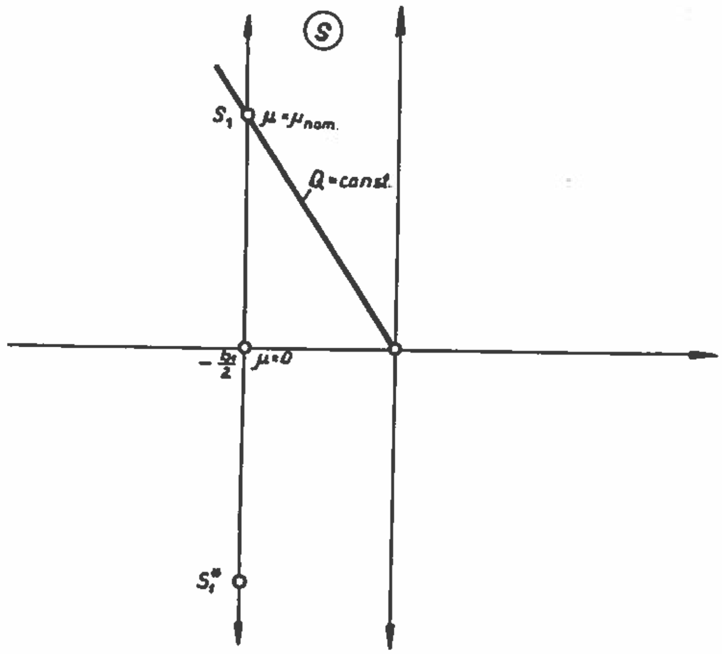

Figure 10. Pole-locus of s_1 for optimal Calahan decomposition

The Q-sensitivity and ω_0-sensitivity to active element variations are:

$$S_{\alpha}^{Q} = S_{G}^{Q} = S_{\alpha}^{\omega_0} = S_{G}^{\omega_0} = \frac{\alpha G G_2}{2[(G_1+G_2+G_5)(G_3+G_4)+\alpha G G_2]} \tag{23}$$

and are smaller than 1/2 independently of Q.

The dependence of Q-sensitivities and ω_0-sensitivities given by (23) on σ_0 value is shown in Figure 11.

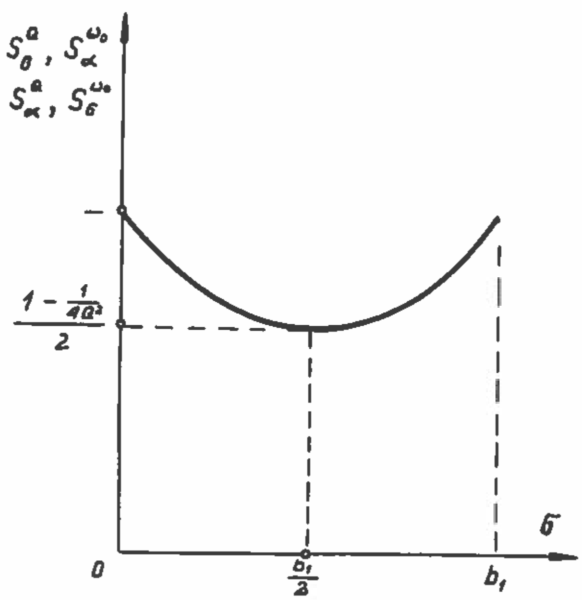

Figure 11. Dependence of sensitivities on σ_0 value

If we take into account the dependence of low-frequency voltage gain αGR_L of the DVCCS/DVCVS element (see Figure 2) on the value of σ_0, we shall find that the choice of $\sigma_0 = b_1/2$ is also optimal, which correspond to the lowest value of the gain αGR_L as shown in Figure 12.

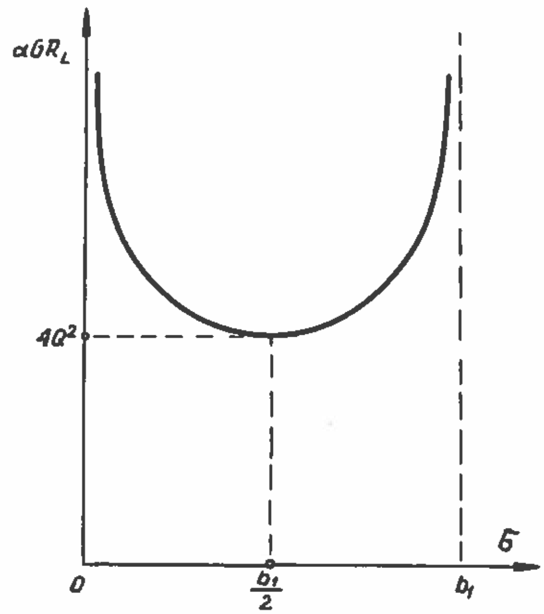

Figure 12. Dependence of low-frequency voltage gain αGR_L of the DVCCS/DVCVS on σ_0 value

It can be shown, that in the structure shown in Figure 9 the Q- and ω_0-passive sensitivities are smaller than 1. It should be noted, that in the structure shown in Figure 9 the capacitance C_3 can fulfill several additional functions:

(a) it can absorb the output capacitance of the current source of the DVCCS/DVCVS element; this gives the possibility to take into account the finite frequency bandwidth of the current source.

(b) it acts as an element compensating the frequency characteristics of the DVCCS/DVCVS element.

Case 2: $Q(s) = (s+\sigma_1)(s+\sigma_2)$

It is known, that the Calahan decomposition of a given polynomial is not unique.[3] In the case of a second degree polynomial, besides the decomposition given by (20) other optimal decompositions exist. For example:

$$D_i(s) = s^2+b_1s+b_2 = b(s+\sigma_2)^2+\frac{\mu}{\mu_{\text{nom}}}a(s+\sigma_1)^2 \tag{24}$$

where:

$$\mu = \text{active parameter}$$

and

$$\sigma_1, \sigma_2 > 0$$

It should be noted, that this decomposition gives an additional degree of flexibility, having two free parameters instead of one. This allows a zero Q-sensitivity realization, for example. The values of the a, b, σ_1, σ_2

can be calculated from the following expressions:

$$a+b = 1 \tag{25}$$

$$1-\frac{b_1^2}{4b_2} < b < 1 \tag{26}$$

$$\sigma_1 = \frac{b_1(1-b)+[b(1-b)(4b_2-b_1^2)]^{\frac{1}{2}}}{2(1-b)} \tag{27}$$

and

$$\sigma_2 = \frac{b_1-2(1-b)\sigma_1}{2b} \tag{28}$$

The root-locus of the polynomial $D_i(s)$ when the parameter μ is varied is shown in Figure 13.

For the polynomial decomposition given by (24), the pole sensitivity for variation of the active parameter μ is

$$S_{\mu}^{s_1}\Big|_{\mu=\mu_{nom}} = \frac{(4b_2-b_1^2)/2[(1-2b)-2j\sqrt{(b(1-b))}}{-2j\sqrt{(4b_2-b_1^2)}} \tag{29}$$

Taking into account the polynomial decomposition given by (24), a prescribed transfer function $K_i(s)$ given by (19) can be re-written in the form:

$$K_i(s) = \frac{N_i(s)}{D_i(s)} = \frac{a_0s^2+a_1s+a_2}{s^2+b_1s+b_2} = \frac{a_0s^2+a_1s+a_2}{a(s+\sigma_1)^2+b(s+\sigma_2)^2} \tag{30}$$

Choosing the polynomial $Q(s)$ given by (14) we obtain:

$$K_i(s) = \frac{\dfrac{a_0s^2+a_1s+a_2}{(s+\sigma_1)(s+\sigma_2)}}{\dfrac{a(s+\sigma_1)^2+b(s+\sigma_2)^2}{(s+\sigma_1)(s+\sigma_2)}}$$

$$= \frac{\dfrac{a_2}{\sigma_1\sigma_2}+g_1+\dfrac{(a_0\sigma_1^2-a_1\sigma_1+a_2)/\sigma_1(\sigma_1-\sigma_2)}{s+\sigma_1}s-\dfrac{(a_0\sigma_2^2-a_1\sigma_2+a_2)/\sigma_2(\sigma_1-\sigma_2)}{s+\sigma_2}s+g_1}{a\dfrac{s+\sigma_1}{s+\sigma_2}+b\dfrac{s+\sigma_2}{s+\sigma_1}}$$

$$= \frac{W_{RC}^1-W_{RC}^2}{Z_{RC}+Z_{RL}} \tag{31}$$

where g_1 is a nonnegative constant.

In this case the solutions of the set of equations (15)–(18) are given in the Table II. It should be noted that the solutions may not exist for some negative real zeros positions of the transfer function (30).

The schematic diagram of a circuit realizing the prescribed transfer function $K_i(s)$ is shown in Figure 14.

The above two cases have shown that two realizations of the prescribed transfer function $K_i(s)$ can be obtained depending on the choice of denominator decomposition. For high Q values, both realizations have the same pole-sensitivity values. The circuit of Figure 14 has more elements than the circuit of Figure 9. However, it is interesting to note that in the circuit of Figure 14 the time-constants:

$$R_{12}C_1 = R_{52}C_5 = R_{42}C_4 = R_{32}C_3 = 1/\sigma_1 \tag{32}$$

are equal. This may be of considerable interest in practical integrated circuit realizations.

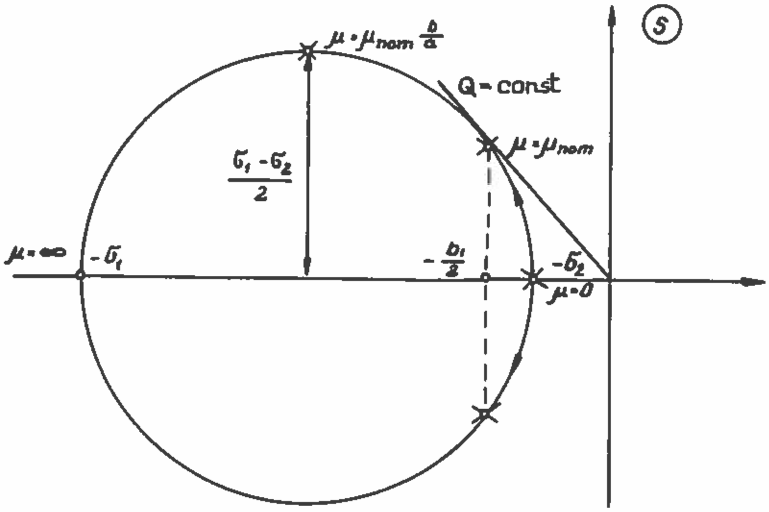

Figure 13. Root-locus of the polynomial $D_i(s)$ for active parameter variations

Table II.

Elements of the model (Figure 14)	Prescribed function: $K_i(s) = \frac{a_0 s^2 + a_1 s + a_2}{s^2 + b_1 s + b_2}$ $b_1, b_2 > 0,\ a_0 \geqslant 0,\ b_1^2 < 4b_2$
$Y_1 = \frac{G_{12}s}{s+(G_{12}/C_1)} + G_{11}$	$G_{12} = G_2\frac{a_0\sigma_2^2 - a_1\sigma_2 + a_2}{a\sigma_2(\sigma_1-\sigma_2)} + \frac{G_2 g_1(\sigma_1-\sigma_2)}{a\sigma_1}$, $C_1 = \frac{G_{12}}{\sigma_1}$, $G_{11} = \frac{G_2 g_1}{a\sigma_1}$
$Y_2 = G_2$	$0 < G_2 < \infty$
$Y_3 = \frac{G_{32}s}{s+(G_{32}/C_3)} + G_{31}$	$G_{31} = \frac{b\sigma_2}{\sigma_1} - G_{41}$, $G_{32} = \frac{b(\sigma_1-\sigma_2)}{\sigma_1} - G_{42}$, $C_3 = \frac{G_{32}}{\sigma_1}$
$Y_4 = \frac{G_{42}s}{s+(G_{42}/C_4)} + G_{41}$	$G_{41} = \frac{a_2}{\alpha\sigma_1\sigma_2} + \frac{g_1}{\alpha}$, $G_{42} = \frac{a_0\sigma_1^2 - a_1\sigma_1 + a_2}{\alpha\sigma_1(\sigma_1-\sigma_2)}$, $C_4 = \frac{G_{42}}{\sigma_1}$
$Y_5 = \frac{G_{52}s}{s+(G_{52}/C_5)} + G_{51}$	$G_{51} = \frac{G_2(\alpha G\sigma_2 - a\sigma_1 - g_1\sigma_2)}{a\sigma_1}$, $C_5 = \frac{G_{52}}{\sigma_1}$, $G_{52} = G_2\frac{\left(\alpha G - \frac{a_0\sigma_2^2 - a_1\sigma_2 + a_2}{\sigma_2(\sigma_1-\sigma_2)} - g_1\right)\sigma_1 - (\alpha G\sigma_2 - g_1\sigma_2)}{a\sigma_1}$
α	$\alpha \geqslant \max\left[+\frac{a_2}{b\sigma_2^2} + \frac{g_1\sigma_1}{b\sigma_2};\ \frac{a_0\sigma_1^2 - a_1\sigma_1 + a_2}{b(\sigma_1-\sigma_2)^2}\right]$
G	$G \geqslant \max\left[\frac{a\sigma_1 + g_1\sigma_2}{\alpha\sigma_2},\ \frac{g_1}{\alpha} + \frac{\sigma_1(a_0\sigma_2^2 - a_1\sigma_2 + a_2)}{\alpha\sigma_2(\sigma_1-\sigma_2)^2}\right]$
σ_1, σ_2, a, b	—are given by relationships (25)—(28)
g_1	$g_1 \geqslant \max\left[0,\ -\frac{a_2}{\sigma_1\sigma_2}\right]$

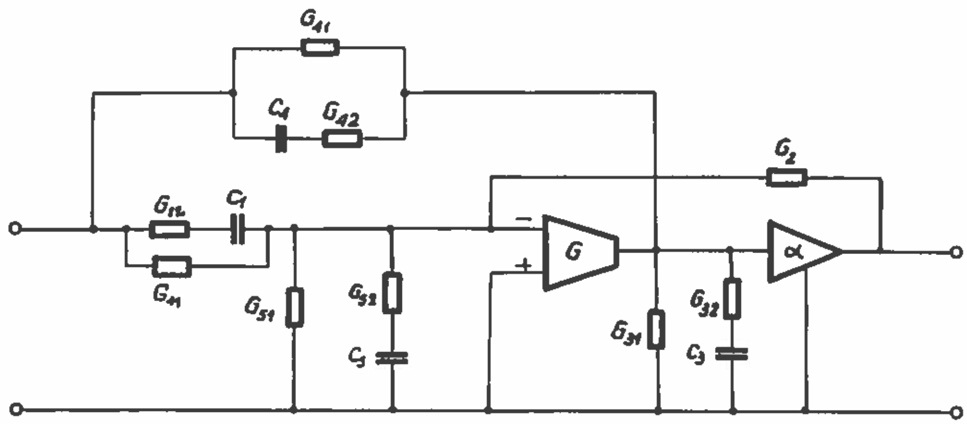

Figure 14. Structure for realization of prescribed function $K_i(s)$ Table II.

It is also worth noting, that the structure shown in Figure 14 allows a realization of zero Q-sensitivity for active parameter variations.

In order to point out this fact it is sufficient to observe that the argument of the pole sensitivity (29), for variation of the active parameter μ, should fulfill the following relationship:

$$\arg S'^{s_1}_{\mu|\mu=\mu_{nom}} = \mathrm{tg}^{-1}\left(-\frac{\sqrt{(4b_2-b_1^2)}}{b_1}\right) \tag{33}$$

The condition (33) is equivalent to the case when the sensitivity vector $S'^{s_1}_{\mu}$ is on the straight line $Q = \text{const}$ passing through the point $\mu = \mu_{nom}$ on the s-plane.

From the relationships (29) and (33) one obtains:

$$\arg S'^{s_1}_{\mu|\mu=\mu_{nom}} = \mathrm{tg}^{-1}\left[\frac{1-2b}{2[b(1-b)]^{\frac{1}{2}}}\right] = \mathrm{tg}^{-1}\left(-\frac{\sqrt{(4b_2-b_1^2)}}{b_1}\right) \tag{34}$$

Hence the optimal value of the parameter b is:

$$b_{opt} = \frac{1}{2}\left[1+\sqrt{\left(1-\frac{b_1^2}{4b_2}\right)}\right] \tag{35}$$

The optimal a, σ_1 and σ_2 values are obtained from the relationship (25), (27) and (28) respectively.

Synthesis procedure using model 2

The synthesis model 2 shown in Figure 8 also allows the cascade realization of a prescribed transfer function $K(s)$. It is assumed, as in previous cases, that the second degree functions $K_i(s)$, obtained as a result of factorization (10) of the function $K(s)$, have the following form:

$$K_i(s) = \frac{N_i(s)}{D_i(s)} = \frac{a_0s^2+a_1s+a_2}{s^2+b_1s+b_2} \tag{36}$$

where

$$b_1, b_2 > 0, \quad b_1^2 < 4b_2$$

and the polynomial $N_i(s)$ has no positive real roots.

The synthesis procedure of the transfer function $K_i(s)$ is based on a decomposition of the polynomial $D_i(s)$ into a sum of two polynomial $D_1(s)$ and $D_2(s)$ which have the following properties:

$$D_i(s) = D_1(s)+kD_2(s) \tag{37}$$

where for some positive k:

1. $D_1(s)$ has only distinct negative real roots
2. $D_2(s)$ has no positive real roots
3. Pole sensitivity $S_\mu^{s_1}$ (where $D_1(s_1) = 0$) assumes some prescribed imaginary number jm, $S_k^{s_1} = jm, m > 0$.

Dividing the numerator and denominator of the function $K_i(s)$ by a polynomial $Q(s)$ given by (14) and taking into account the decomposition (37), the function $K_i(s)$ can be expressed in the following form:

$$K_i(s) = \frac{-\mu \dfrac{-G(a_0 s^2 + a_1 s + a_2)}{\mu_{nom}(1-k)G(s+\sigma_1)(s+\sigma_2)}}{\dfrac{s^2 + b_1 s + b_2 \gamma}{(s+\sigma_1)(s+\sigma_2)} - \mu \dfrac{-k(s^2 + d_1 s + d_2)}{(1-k)(s+\sigma_1)(s+\sigma_2)}} \tag{38}$$

where:

$$d_1 = b_1 \tag{39}$$

$$d_2 = b_2\left[1 + \frac{2m}{\sqrt{b_2}} \Big/ \sqrt{\left(1 - \frac{b_1^2}{4b_2}\right)}\right] \tag{40}$$

$$k \in \left(\frac{4b_2 - b_1^2}{4d_2 - d_1^2}, \frac{b_2}{d_2}\right) \tag{41}$$

$$\gamma = \frac{1 - k(d_2/b_2)}{1-k} \tag{42}$$

Identifying the terms in the relationship (38) with those of the equation (7) one obtains:

$$y_{22b} = \frac{s^2 + b_1 s + b_2 \gamma}{(s+\sigma_1)(s+\sigma_2)} \tag{43}$$

$$y_{21b} = \frac{-G(a_0 s^2 + a_1 s + a_2)}{\mu_{nom}(1-k)(s+\sigma_1)(s+\sigma_2)} \tag{44}$$

$$y_{12c} = \frac{-k(s^2 + d_1 s + d_2)}{\mu_{nom}(1-k)(s+\sigma_3)} \tag{45}$$

$$y_{11c} = \frac{(s+\sigma_1)(s+\sigma_2)}{(s+\sigma_3)} \tag{46}$$

where $\sigma_2 < \sigma_3 < \sigma_1$.

The level constants μ_{nom} and G/μ_{nom} are fixed through the realizable gain required in the passive transformerless RC circuit. The active parameter $\mu = \alpha G$ is nominally fixed at $\mu = \mu_{nom}$ and one checks to see that $S_\mu^{s_1} = jmk/\mu$ satisfies the required specifications, as practically $\mu \gg k$.[4]

In summary one can state, that in this case the main problem in the synthesis of a given transfer function $K_i(s)$ with prescribed pole sensitivity $S_\mu^{s_1}$, concerns the synthesis procedure of passive RC two-ports with parameters given by relations (43)–(46). The two-ports N_b and N_c can be synthesised using one of known methods, for example Guillemin's technique.

DISCUSSION

Basic properties of a new versatile generating element acting simultaneously as a differential voltage controlled current source and differential voltage controlled voltage source are described. This element called a DVCCS/DVCVS can be used directly in active RC synthesis for second degree transfer function realizations or for realization of all other linear nondynamic elements, in which of special importance is the simplicity of realization of a gyrator with low output impedance buffers.

The general synthesis model with one DVCCS/DVCVS element which can be recognized as an active multiloop feedback network is presented. The two particular synthesis models derived from the general one are described. The first model allows the low sensitivity realization of all stable second degree voltage transfer functions. It has properties similar to the gyrator and the operational amplifier synthesis model and at the same time the Y_3 element can be used for frequency compensation of the op. amp. characteristic. The second model allows the realization of high Q voltage second degree transfer functions of very low pole sensitivity. All synthesis models described here allow for cascading the second degree sections without additional buffers.

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