NETWORK DESIGN WITH TOLERANCE:*

INTERVAL ANALYSIS APPROACH

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Abstract

Interval analysis is introduced into network theory for incorporating element tolerances and digital computer round-off error in analysis and design.

"I stumbled all night long on sand and shell
By a lakeshore where time, unfaced, was dark;
I grazed with my left foot a pinched hotel"
[1, p. 43]

1. INTRODUCTION

Because processing techniques yield circuit elements not precisely but to within certain tolerances, practical network constructions are carried out with circuit elements whose values only approximate exact design values to within a certain degree of accuracy. For example, resistors are often most economically available with 10% tolerances. And although one might expect that responses would remain at the design values within a tolerance equal to the worst element value tolerance, such may often not be the case as is clear if two approximately equal large numbers are subtracted. In almost all cases then of network design the actual effects of imprecise elements are only known indefinitely apriori. Indeed an analysis of the actual network constructed could very well show that the specifications are not met, in which case one is faced with a new design. In order to avoid this latter the theory of interval analysis [2] has been investigated [3] and found to yield a tolerance analysis simultaneous with an original design. Further, machine error in calculating element values is automatically taken into account.

2. INTERVAL ANALYSIS

Interval analysis has been introduced into the field of numerical analysis as a means of evaluating the effects of rounding in digital computation [2]. It proceeds from the set $\mathcal{J}$ of all closed intervals upon which the standard operations are defined. Thus $I \in \mathcal{J}$ if and only if $I = [a, b] = \{x | a \leq x \leq b\}$ for any two real numbers a and b, $a \leq b$; the interval number I is uniquely specified by its two end-points. If * is used to denote any one of the four standard operations

$$* = +, -, \times, / \tag{1a}$$

then we define any of these operations on interval numbers $I_1, I_2 \in \mathcal{J}$ by [2, p. 8]

$$I_1 * I_2 = \{x * y | x \in I_1, y \in I_2\} \quad ; \quad \text{if } * = / \text{ then } 0 \notin I_2 \tag{1b}$$

All of these operations can be specified through the end-points, for example $I_1 + I_2 = [a_1 + a_2, b_1 + b_2]$ if

$I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$. The algebraic structure so obtained is compatible with that of a digital computer, and, like the latter, is not distributive as the example of the next paragraph shows. A distance function d(. , .) can be defined through [2, p. 15]

$$d(I_1, I_2) = \max(|a_1 - a_2|, |b_1 - b_2|) \quad (1c)$$

allowing errors to be evaluated and $\mathcal{J}$ to be considered a metric space. Note that $I = [a, a]$ is allowed so that real numbers are special cases within the theory for which d(. , .) is the normal distance, $d(a_1, a_2) = |a_1 - a_2|$.

Since all rational operations are available one can work with rational functions having interval numbers as coefficients. For this, care must be used to observe the order of operations performed as well as to avoid division by intervals containing zero. Too, different expressions for the same real valued function can lead to different intervals when evaluated over $\mathcal{J}$. For example, $f_1(x) = x^2 - x$ and $f_2(x) = x(x-1)$ yield $f_1([0,1]) = [-1, 1] \neq [-1, 0] = f_2([0,1])$. Thus one usually wishes to use expressions which yield the smallest interval result, f_2 in this case.

3. A NETWORK DESIGN APPLICATION

If one is given specifications which include some tolerance, as is customary, then one should be able to approximate the specifications by rational functions with interval coefficients.

Let us then assume that we are given a degree two rational current transfer function with interval coefficients

$$\frac{i_2}{i_1}(p) = \frac{A_2 p^2 + A_1 p + A_0}{p^2 + B_1 p + B_0}, \quad A_i, B_i \in \mathcal{J} \quad (2)$$

By suitably adapting [3] standard synthesis techniques [4] [5, pp. 126-132] to handle interval numbers one can obtain an Hakim circuit for this transfer function where, however, the element values are interval numbers. As an example, Fig. 1 shows the result for

$$\frac{i_2}{i_1}(p) = \frac{[0.9, 1.0]\,p^2 + [9.0, 9.1]}{p^2 + [1.41, 1.42]\,p + [1.0, 1.1]} \quad (3)$$

assuming that the magnitude of the pole sensitivity is no greater than 0.1. A flowchart for the synthesis process is shown in Fig. 2. A choice of actual element values to lie within the specified ranges will guarantee that the coefficients of i_2/i_1 will lie within the intervals given and hence that the specifications from which the transfer function is derived will be met.

"The way and the way back are long and rough
Where Myrtle twines with Laurel - single glow
Of leaf, your own imponderable stuff."
[1, p. 42]

For Anong.

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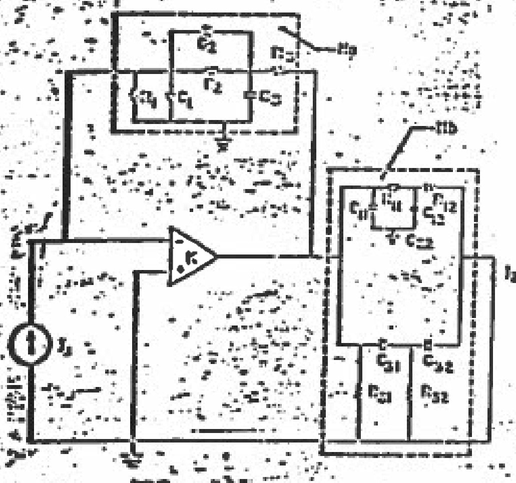

Component	Modified Values				
R1	[	[illegible]	,	[illegible]	]
R2	[	[illegible]	,	[illegible]	]
R3	[	[illegible]	,	[illegible]	]
C1	[	[illegible]	,	[illegible]	]
C2	[	[illegible]	,	[illegible]	]
C3	[	[illegible]	,	[illegible]	]
K	[	[illegible]	,	[illegible]	]
R11	[	[illegible]	,	[illegible]	]
R12	[	[illegible]	,	[illegible]	]
C11	[	[illegible]	,	[illegible]	]
C12	[	[illegible]	,	[illegible]	]
R21	[	[illegible]	,	[illegible]	]
R22	[	[illegible]	,	[illegible]	]
C21	[	[illegible]	,	[illegible]	]
C22	[	[illegible]	,	[illegible]	]

Figure 1. Design Example by Modified Hakim Method

Since all rational [illegible] can work with rational [illegible] numbers as coefficients. For this case must be used to observe the order of operations performed as well as [illegible] division [illegible] intervals containing zero. [illegible] different expressions [illegible] real valued function can lead to different intervals when evaluated. [illegible]

[illegible]

REFERENCES

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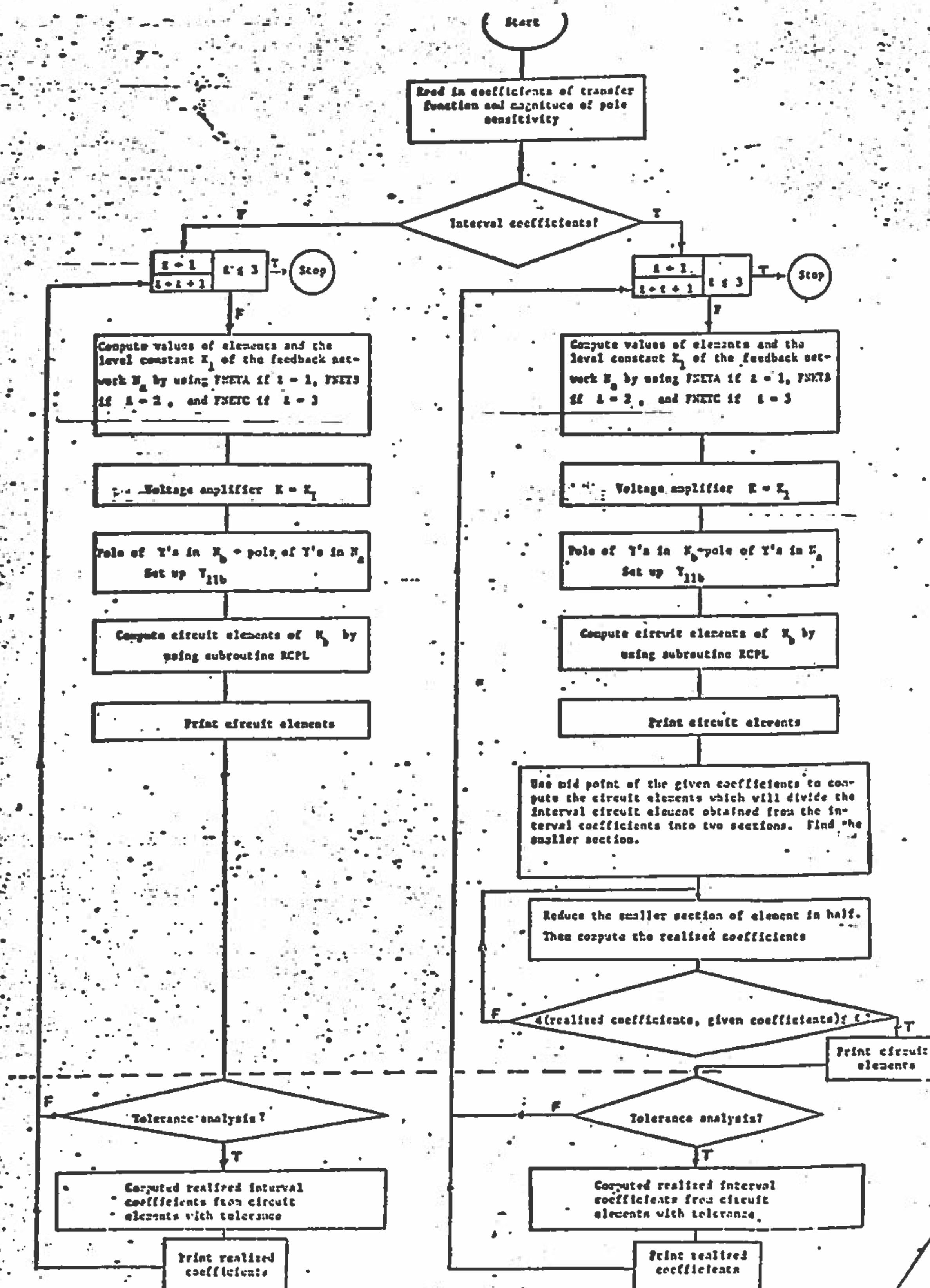

Fig. 2. Flowchart for Modified Hakim Synthesis