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NONLINEAR NETWORK STATE-VARIABLE

EQUATIONS THROUGH LINEAR DYNAMICS

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Abstract

This paper presents a method of setting up state-variable equations for nonlinear networks. It proceeds from the fact that for many nonlinear circuits the dynamics can be placed in unit linear capacitors which are then coupled through nonlinear resistive structures. The results are of interest for the design of nonlinear circuits as well as for computer-aided analysis.

"and yet, through all this tangled complexity and sometimes confusion, it is impossible 'not to fall ultimately, as into a heresy, into unheard-of-simplicity'" [1, p. 46].

1. INTRODUCTION

It is well-recognized [2, p. 59] that the analysis of nonlinear networks is most often carried out in terms of the canonical set of first order state-variable equations describing the network. Such state-variable equations are of the form

$$\begin{bmatrix} \dot{\underline{x}} \\ \underline{y} \end{bmatrix} = \underline{F}\left(\begin{bmatrix} \underline{x} \\ \underline{u} \end{bmatrix}\right) \qquad (1)$$

where $\underline{x}$ is the state vector, $\underline{y}$ the output and $\underline{u}$ the input to the network; $\underline{F}(.)$ is a, perhaps nonlinear, transformation which reflects the laws and interconnections of the network elements involved. Since in fact one of the heaviest uses of digital computers lies in the area of nonlinear network analysis [3, p. 75], it seems expedient to have available simple and general means of setting up these state-variable equations applicable to computer-aided analysis. And of course by extension such suitable methods can lead into the productive area of design.

To be sure there are several available techniques for setting up the state-variable equations for nonlinear networks [2, p. 64] [4, p. 196]. Here we give an alternative which in general is more applicable to obtaining the canonical equations on a computer; also it may be considered simpler to apply or to give more conceptual insight, since it reduces the real analysis to that of a purely resistive network. Once this, or any other method, has been used to obtain the state-variable equations, solutions can be pursued following standard computer routines and techniques [5, pp. 1539, 1545].

In setting up state-variable equations one of the main problems is the proper isolation of derivative determining elements, these latter being called dynamical elements. In previous work [6] it has been shown how nonlinear capacitors can be replaced by linear unit capacitors loading a nonlinear resistive structure. This is the idea we use here. Its application leads to an analysis which reduces to the generally easier problem of analysis of resistive networks. From such an analysis, which can be programmed following the topologically oriented scheme discussed in Section III, the canonical state-variable equations are relatively easily set up, when they exist, in a form convenient for digital computer analysis.

II. EQUIVALENCES

We first review the equivalence for nonlinear capacitors which gives the basic idea from which the method stems. This is followed by a duality consideration which allows all controlled resistors to be current controlled.

A. Capacitor Equivalence [6]

If i, q, and v are the current through and the charge and voltage on a time-invariant (one-port) capacitor then, assuming appropriate differentiability,

$$i = \frac{dq(v)}{dt} = \frac{dq(v)}{dv} \cdot \frac{dv}{dt} \qquad (2)$$

If next we consider the nonlinear 2-port resistive network described by the general description

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & \frac{dq(v)}{dv} \end{bmatrix} \begin{bmatrix} i \\ i_2 \end{bmatrix} \qquad (3)$$

then we find upon loading (at port 2) in a unit capacitor that Eq. (2) is verified. The process is illustrated in Fig. 1 and can be readily extended to coupled capacitors controlled by several voltages [6]. For the topological analysis which follows it is also worth recalling that the nonlinear resistive coupling network described by Eq. (3) can be realized by a voltage adjustable current controlled current source [6].

B. Conversion to Dual Variables

To make the method to be described widely applicable it is convenient to note that a gyrator, described by

$$i_1 = gv_2 \quad , \quad i_2 = -gv_1 \qquad (4)$$

and symbolized in Fig. 2a), can be used to convert a load network N into its dual N^d as illustrated in Fig. 2b). In the dual conversion process it is of particular importance to note the gyration conductance sign and magnitude, g=+1, required for nonlinear situations. As an illustration to make this clear, if $-i_2 = f(v_2)$ then, from Eq. (4), $v_1 = (1/g)f(i_1/g)$ which yields the dual if g = +1 but not in general if g = -1.

Since the dual of a capacitor is an inductor, Fig. 3a) shows a well-known application of Fig. 2b) which allows all dynamical elements to be assumed to be capacitors. Taken in conjunction with the results of Section IIA we conclude that any dynamical time-invariant element can be represented through unit (uncoupled) capacitors loading a nonlinear and nondynamical (e.g. resistive) network. In fact for later purposes it is useful to make the important observation that the dual transformation of Fig. 2b) can be used to obtain a current controlled resistor from a voltage controlled one, as shown in Fig. 3b).

With these preliminaries we can turn to the actual establishment of the state variable equations.

III. CAPACITOR EXTRACTIONS - STATE-VARIABLE EQUATIONS

Given a network constructed of a finite number of (nonlinear, time-invariant) circuit elements we can make the equivalences of the last section to remove all dynamical elements as unit capacitors. This yields a resistive network loaded in unit capacitors, as shown in Fig. 4a), which is equivalent at its ports to the original configuration.

For concreteness of the treatment let it be assumed that what is of interest is voltage transfer from a set of first n ports with $\underline{v}_1$ as inputs to a second set of m ports with $\underline{v}_2$ as outputs; the external ports can then be partitioned as shown in Fig. 4b). In order to proceed we will further assume that the hybrid equations

$$\mathscr{H}\left(\begin{bmatrix} \underline{v}_1 \\ \underline{i}_2 \\ \underline{v}_3 \end{bmatrix}\right) = \begin{bmatrix} \underline{i}_1 \\ \underline{v}_2 \\ \underline{i}_3 \end{bmatrix} \qquad (5)$$

exist for the resistive subnetwork of Fig. 4b). Sufficient conditions for the existence of the nonlinear hybrid transformation $\mathscr{H}(.)$ are available [7, Thm. 4]. Along these lines, if "the Jacobian matrix of $\mathscr{H}(.)$ is uniformly positive definite" then all other hybrid descriptions exist [7, Thm. 3]; for example, the admittance transformation may always be obtained from any other hybrid transformation under the uniformly positive definite assumption. $\mathscr{H}(.)$ of Eq. (5) can then be used immediately to get the state-variable equations (1) as follows.

Setting $\underline{x} = \underline{v}_3$ for the state k-vector of Fig. 4b), with $\underline{1}_k$ denoting the k x k identity, we find the state-variable equations as

$$\begin{bmatrix} \dot{\underline{x}} \\ \\ \underline{v}_2 \end{bmatrix} = \begin{bmatrix} \underline{0} & \underline{0} & -\underline{1}_k \\ \\ \underline{0} & \underline{1}_m & \underline{0} \end{bmatrix} \mathscr{H}\left(\begin{bmatrix} \underline{v}_1 \\ \underline{0} \\ \underline{x} \end{bmatrix}\right) \qquad (6)$$

That is, the state-variable equations result from Eq. (5) by ignoring the first n rows, identifying capacitor voltages as the state, and setting the final m port constraints $\underline{i}_2 = \underline{0}$. We have also transposed the final m and k rows, after multiplication of the latter by minus one, in order to place the equations in the canonical form of Eq. (1) [8, p. 40]. We conclude that if any hybrid transformation exists for the resistive coupling network we can very simply obtain the state-variable equations.

As in the linear case [9], the problem of setting up the state-variable equations is now reduced to that of finding a hybrid description of a resistive network. Using topological means the analysis to determine this hybrid is relatively easily formulated [10, p. 51] and in fact in a form suitable for programming on a computer, though presently available routines for this type of one element kind network are time consuming [5, p. 1544]. Here we describe one possible technique which is particularly suitable for digital computation determination of $\mathscr{Y}(.)$.

By the use of gyrators as described at the end of Section II, and the simple extension of the idea to multiports [11], the internal components can be assumed to have current controlled (resistance) descriptions. Further the insertion of m unit gyrators at the second set of ports (output) allows $\mathscr{Y}(.)$ to be considered an admittance as Eq. (5) shows (the unit gyrators set $\underline{i}_2 = \hat{\underline{v}}_2$, $\underline{v}_2 = \hat{\underline{i}}_2$ or $\mathscr{Y}(\hat{\underline{v}}) = \hat{\underline{i}}$ if we also set $\underline{v}_1 = \hat{\underline{v}}_1$, $\underline{v}_3 = \hat{\underline{v}}_3$, $\underline{i}_1 = \hat{\underline{i}}_1$, $\underline{i}_3 = \hat{\underline{i}}_3$ and collect these vectors in $\hat{\underline{v}}$ and $\hat{\underline{i}}$). Thus, for analysis purposes we wish to assume voltage sources $\hat{\underline{v}}$ applied to the (n+ m+ k) ports of the network determining $\mathscr{Y}(.)$.

Proceeding toward a topological analysis, we will assume that the externally applied voltage sources $\hat{\underline{v}}$ occur in the last numbered branches, which are also all assumed to be links (that is, the applied sources are also independent). Letting $\underline{v}_b$ and $\underline{i}_b$ be the total branch voltages and currents, we can isolate the applied sources by writing

$$\underline{v}_b = \mathscr{R}(\underline{i}_b) - \begin{bmatrix} \underline{0} \\ \hat{\underline{v}} \end{bmatrix} \qquad (7)$$

Here $\mathscr{R}(.)$ is a nonlinear (branch by branch, generally nondiagonal) resistance transformation defined by the (current controlled) circuit components. Here the sign on $\hat{\underline{v}}$ is chosen such that current flows out of the positive terminal since $\hat{\underline{v}}$ represents sources.

Denoting the tie-set matrix [12, p. 499] by $\mathscr{T}$ we can apply Kirchhoff's laws to obtain, in the standard manner,

$$\mathscr{T}\underline{v}_b = \underline{0} \quad \text{(Kirchhoff Voltage Law)} \qquad (8a)$$

$$\underline{i}_b = \tilde{\mathscr{T}}\underline{i}_\ell \quad \text{(Kirchhoff Current Law)} \qquad (8b)$$

Here $\underline{i}_\ell$ is the set of link currents and the tilde, ~, denotes matrix transposition; by also assuming the links to be finally numbered branches the tie set matrix has its last ℓ(= number of links) columns- as the identity [13, p. 30]

$$\mathscr{T} = [\,\underline{T} \mid \underline{1}_\ell\,] \qquad (8c)$$

Equations (8) applied to Eq. (7) yield $\underline{0} = \mathscr{T}\underline{v}_b = \mathscr{T}\mathscr{R}(\underline{i}_b) - [\,\underline{T}, \underline{1}_\ell\,]\begin{bmatrix} \underline{0} \\ \hat{\underline{v}} \end{bmatrix} = \mathscr{T}\mathscr{R}(\tilde{\mathscr{T}}\underline{i}_\ell) - \begin{bmatrix} \underline{0} \\ \hat{\underline{v}} \end{bmatrix}$

or

$$\begin{bmatrix} \underline{0} \\ \hat{\underline{v}} \end{bmatrix} = \mathscr{T}\mathscr{R}(\tilde{\mathscr{T}}\underline{i}_\ell) = \hat{\mathscr{R}}(\underline{i}_\ell) \qquad (9a)$$

which serves to define the transformation $\hat{\mathscr{R}}(.)$, $\hat{\mathscr{R}}(.) = \mathscr{T}\mathscr{R}(\tilde{\mathscr{T}}.)$; $\hat{\mathscr{R}}$ transforms link currents into applied link voltages. Now the port currents $\hat{\underline{i}}$ are the final n+ m+ k entries of $\underline{i}_\ell$, that is

$$\hat{\underline{i}} = [\,\underline{0} \mid \underline{1}_{n+m+k}\,]\,\underline{i}_\ell \qquad (9b)$$

But $\mathscr{Y}(.)$ is numerically equal to the admittance under calculation, which is hence found from inverting Eq. (9a) and applying (9b)

$$\hat{\underline{i}} = [\,\underline{0} \mid \underline{1}_{n+m+k}\,]\,\hat{\mathscr{R}}^{-1}\left(\begin{bmatrix} \underline{0} \\ \hat{\underline{v}} \end{bmatrix}\right) \triangleq \mathscr{Y}(\hat{\underline{v}}) \qquad (10)$$

For digital calculation $\mathscr{T}$, $\mathscr{R}(.)$ and $\hat{\mathscr{R}}(.)$ are relatively easily formulated while the inverse $\hat{\mathscr{R}}^{-1}(.)$, is the most difficult step; but it has been pointed out that the method of Broyden is available and very convenient [14, p. 1821].

Having found $\mathscr{Y}(.)$ for the resistive coupling network we now have the state-variable equations as shown at Eq. (6).

IV. EXAMPLES

To indicate the procedures we investigate two examples.

First, to illustrate the linear capacitor extraction scheme we consider the equivalent circuit of a tunnel diode as shown in Fig. 5a) [15, p. 1916]. The diode conductance $G_D(.)$ represents a voltage-controlled nonlinear time-invariant purely resistive element given by the current-voltage characteristic $i_d = G_d(v_d)$. As explained under Section IIB), through the introduction of the gyrator, this voltage-controlled element can be converted to its current-controlled dual, $v_D = G_d(i_D)$, as needed for the branch by branch resistance transformation $\mathscr{R}(.)$, while in the same way the linear series inductor, L_s, can be converted to its capacitor dual. The equivalence of the non-linear capacitor [15, p. 1916], described by $q(v_d) = C_d(v_d).v_d = Kv_d/(V_j - v_d)^N$ with K, N, V_j

constants, is obtained as a current-controlled nonlinear resistive 2-port loaded in a unit linear capacitor through the use of Fig. 1, [6]. The final tunnel diode equivalent appropriate for the analysis method is then shown in Fig. 5b).

As a second example, to illustrate the setting up of the canonical state-variable equations, we consider the simple single-input single-output circuit shown in Fig. 6a) where the nonlinear resistive element is as denoted in Fig. 3b), and the linear capacitor is assumed of unit capacitance by normalization; the resistor r is assumed linear. After appropriate conversions the circuit's equivalent is shown in Fig. 6b), where branch numberings are also indicated. A suitable graph is given in Fig. 6c) where darkened branches belong to the chosen tree. From the graph the tie-set matrix is easily determined and the branch by branch impedance results directly from Fig. 6b); thus

$$\mathcal{T} = \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11a}$$

$$\mathcal{R}(.) = \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & f(.) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where here, and in all of the following, f(.) operates only on the variable in its column, the other entries following standard linear matrix algebra rules. Combining gives

$$\mathcal{Z}(.) = \mathcal{T}\mathcal{R}(\tilde{\mathcal{T}}.) = \begin{bmatrix} f(.) & 1 & -1 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & r & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{11b}$$

from which the inverse is calculated as

$$\mathcal{Z}^{-1}(.) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{1}{r} & 0 & -\frac{1}{r} \\ 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & -\frac{1}{r} & -1 & \{\frac{1}{r} + f(.)\} \end{bmatrix} \tag{11c}$$

As explained in Section III, Eq. (10), the final portion (last two rows) of the transformation inverse to $\mathcal{Z}(.)$ is $\mathcal{H}(.)$. Therefore, since the first two link source voltages are zero (deleting the first two columns)

$$\mathcal{H}(.) = \begin{bmatrix} \frac{1}{r} & 0 & -\frac{1}{r} \\ 0 & 0 & 1 \\ -\frac{1}{r} & -1 & \{\frac{1}{r} + f(.)\} \end{bmatrix} \tag{11d}$$

On next performing the operations of Eq. (6) we finally arrive at the canonical equations

$$\dot{x} = -\frac{x}{r} - f(x) + \frac{1}{r}v_1 \quad , \quad v_2 = x \tag{11e}$$

These equations can easily be checked by direct inspection of Fig. 6a); but note that the procedure we have used gives a systematic method suitable for general analysis.

V. DISCUSSION

By using a capacitor extraction the state-variable equations for a voltage transfer network have been formulated, Eq. (6), through a hybrid description of the nonlinear coupling resistive network. The result is valid for finite time-invariant networks for which the decomposition of Eq. (2) is true. And as previously shown [6] the result is easily extended to cover coupled capacitors controlled by arbitrary voltages. In some sense the capacitor decomposition is another version of those previously used for time-variable linear capacitors [16], and as a consequence stability results [17] and passive characterizations [18] can probably be obtained in a similar fashion. Indeed the results add to those obtained [19] through the introduction of other nonlinear devices such as the mutator, the reflector, and the scalor, as well as to those results concerned with basic nonlinear resistive building blocks [20] including transistor circuits [21].

Of interest here is the general result that for finite nonlinear time-invariant networks the dynamics can be assumed placed entirely in unit (linear) capacitors, this result being that upon which the theory of state-varables for nonlinear structures appears to have always rested. Thus, it is no surprise that using this extraction we are able to obtain the canonical state-variable equations, when they exist. For sure these canonical equations need not always exist, as is readily seen by replacing the voltage controlled "resistor" in Fig. 6a) by a current controlled one. Nevertheless, when applicable, in contrast to more classical approaches, the method presented reduces in the end to an analysis of purely resistive circuits to which the topological formulation indicated is relatively easy to apply. For sure we have only outlined the ideas for voltage to voltage transfer, Fig. 4), but other situations are handled in like manner, generally through the use of some other hybrid description. In the text we have set up the analysis by treating the hybrid matrix as an admittance through appropriate gyrator insertions. In general this is actually the most convenient for setting up all-purpose computer routines, but in special cases it may be more convenient to omit the extra gyrators by calculating $\mathscr{H}(.)$ as the original hybrid transformation. To be sure there are situations, as in fact the second example, where the state-variable equations can be found almost by inspection. But the methods used in such instances do not, as yet anyway, seem programmable to handle general structures. Hence the importance of the method discussed, which in fact when implemented on a computer could use less storage than at first appears necessary, due to the sparsity of the matrix transformations. Of course, as with all new methods there are many aspects to be further investigated, such as actual computer implementation.

"We do not need theories so much as the experience that is the source of the theory," "And any theory not founded on the nature of being human is a lie and a betrayal of man" [1, pp. 17, 53].

VI. ACKNOWLEDGMENTS

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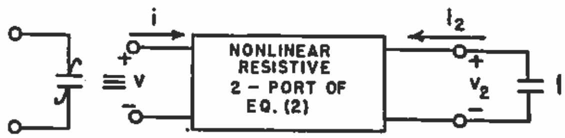
REFERENCES

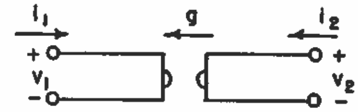
1. Laing, R. D., "The Politics of Experience," Ballantine, New York, 1967.
2. Stern, T. E., "Theory of Nonlinear Networks and Systems," Addison-Wesley, Reading (Mass.), 1965.
3. Calahan, D. A., "Computer-Aided Network Design," Preliminary Edition, McGraw-Hill, New York, 1968.
4. Desoer, C. A., and J. Katzenelson, "Nonlinear RLC Networks," Bell System Technical Journal, Vol. 44, No. 1, January 1965, pp. 161-198.
5. Katzenelson, J., "AEDNET: A Simulator for Nonlinear Networks," Proceedings of the IEEE, Vol. 54, No. 11, November 1966, pp. 1536-1552.
6. Newcomb, R. W., and O. A. Seriki, "Nonlinear Capacitor Simulation," Electronics Letters, Vol. 7, No. 16, August 12, 1971, pp. 452-454.
7. Ohtsuki, T., and H. Watanabe, "State-Variable Analysis of RLC Networks Containing Nonlinear Coupling Elements," IEEE Transactions on Circuit Theory, Vol. CT-16, No. 1, February 1969, pp. 26-38.
8. Zadeh, L. A., and C. A. Desoer, Linear System Theory, the State Space Approach, McGraw-Hill, New York, 1963.
9. Miller, J. A., and R. W. Newcomb, "A Computer-Oriented Technique for Determining State-Variable Equations for Admittance Descriptions," in Aspects of Network and System Theory, N. DeClaris and R. Kalman editors, Holt, Rinehart, and Winston, New York, 1971, pp. 213-222.
10. Stern, T. E., Theory of Nonlinear Networks and Systems, Addison-Wesley, Reading (Mass). 1965.
11. Białko, M., and R. W. Newcomb, "Generation of All Finite Linear Circuits Using the Integrated DVCCS," IEEE Transactions on Circuit Theory, Vol. CT-18, (to appear).
12. Guillemin, E. A., "Introductory Circuit Theory," John Wiley & Sons, New York, 1953.
13. Newcomb, R. W., Network Theory: The State-Space Approach, Libraire Universitaire Uystpruyst, Louvain (Belgium), 1968.
14. Branin, F. H., Jr., and H. H. Wang, "A Fast Reliable Iteration Method for dc Analysis of Nonlinear Networks," Proceedings of the IEEE, Vol. 55, No. 11, November 1967, pp. 1819-1826.
15. Daniel, M. E., "Development of Mathematical Models of Semiconductor Devices for Computer-Aided Circuit Analysis," Proceedings of the IEEE, Vol. 55, No. 11, November 1967, pp. 1913-1920.
16. Anderson, B. D., D. A. Spaulding, and R. W. Newcomb, "Useful Time-Variable Circuit-Element Equivalences," Electronics Letters, Vol. 1, No. 3, May 1965, pp. 56-57.
17. Klamm, C. F., B. D. O. Anderson, and R. W. Newcomb, "Stability of Passive Time-Variable Circuits," Proceedings IEE, Vol. 114, No. 1, January 1967, pp. 71-75.
18. Anderson, B. D. O., and R. W. Newcomb, "Functional Analysis of Linear Passive Networks," International Journal of Engineering Science, accepted for publication. Available as Stanford Electronics Laboratories, Technical Report No. 6559-2, March 1967.
19. Chua, L. O., "Synthesis of New Nonlinear Network Elements," Proceedings of the IEEE, Vol. 56, No. 8, August 1968, pp. 1325-1340.

20. Tellegen, B. D. H., "La recherche pour une série complète d' elements de circuits idéaux non linéaires," Rendiconti Seminario Mathematico e Fisico, Milano, Vol. 25, 1953-1954, pp. 134-144.

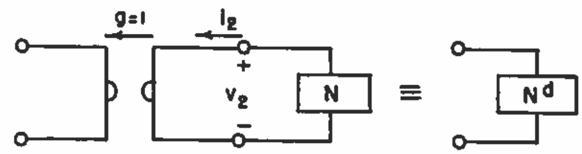
21. Sandberg, I. W., and A. N. Willson, Jr., "Some Network-Theoretic Properties of Nonlinear DC Transistor Networks," BSTJ May-June 1969, pp. 1293-1311.



Nonlinear Capacitor Replacement
Figure 1

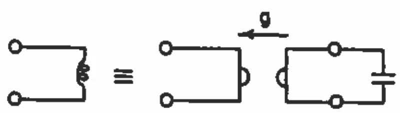


a)
GYRATOR

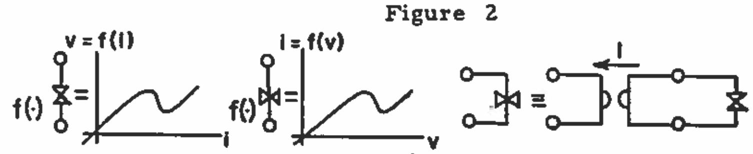


b)
DUAL TRANSFORMATION

Gyrator Symbolism and Usage
Figure 2



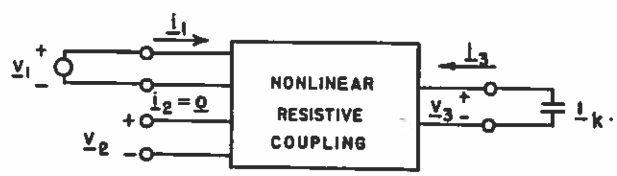
a)
INDUCTOR REPLACEMENT



b)
TRANSFORMATION OF i = f(v) into v = f(i)

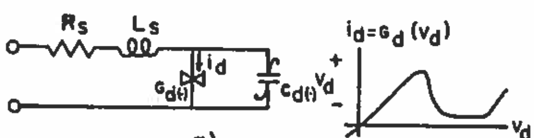
Specific Dualities
Figure 3

a)

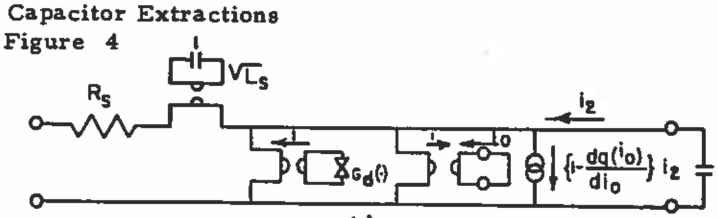


b)

Unit Capacitor Extractions
Figure 4

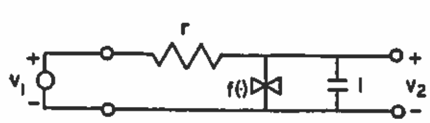


a)
NORMAL EQUIVALENT

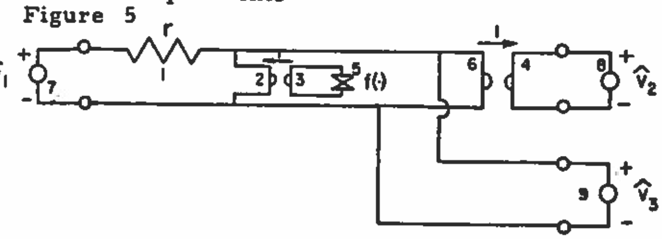


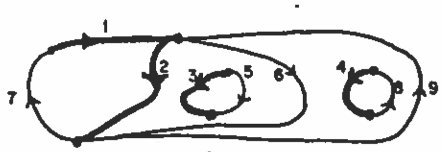
b)
STATE-VARIABLE APPROPIATE EQUIVALENT

Tunnel Diode Equivalents
Figure 5



a)
EXAMPLE CIRCUIT





c)
NETWORK GRAPH

b)
RESISTIVE NETWORK FOR ANALYSIS

Second Example
Figure 6