TRANSFER FUNCTION DESIGN WITH INTEGRATED CIRCUITS*

R.W. Newcomb
Stanford Electronics Laboratories
Stanford, California 94305

"Artists who do not go forward go backward" [1, p. 23]

Because of their theoretical generality and their practical utility the theory of state-variables has found an important and special place within the circuit's area [2]. Here we emphasize their uses for linear integrated circuit design and construction [3] though there are equally important applications to computer aided analysis, for example.

The theory is based upon the set of first order (matrix) state-variable equations

$$\dot{x} = A x + B u \quad (1a)$$

$$y = C x + D u \quad (1b)$$

where u is the input (n-vector), y is the output (m-vector) and x is the state (k-vector); x consists of variables associated with dynamical elements, capacitor voltages say for integrated circuits; u and y for circuits would customarily be input and output voltages or currents. If the "realization" matrices A, B, C, D are constant as typically would occur for a time-invariant system, then one can find the transfer function

$$T(s) = D + C[s1_k - A]^{-1}B, \ \mathcal{L}[y] = T(s)\mathcal{L}[u] \quad (2)$$

where 1_k denotes the kxk identity matrix and $\mathcal{L}[\ ]$ denotes the Laplace transform and $s = \sigma + j\omega$ its variable. Likewise, also from (1) or a knowledge of T(s) in the form of (2), one can directly find time responses as

$$y(t) = e^{-At}y(o) + D\,u(t) + \int_o^t Ce^{-A(t-\tau)}B\,u(\tau)d\tau \quad (3)$$

This latter is especially convenient for determining transient responses and variations in transient responses (that is, sensitivity) [4] through the computer, especially when the matrix exp[-At] is evaluated by the method of Liou [5].

For actual design we wish however to recover the state-variable equations (1) from T(s) since (1) can be readily simulated in block diagram form using only integrators and weighted summers, in the standard analog simulation way. Since integrators and weighted summers are readily obtained with operational amplifiers and resistors and capacitors the technique is quite adaptable to integrated circuits. It turns out that when properly carried out the resulting circuits also have very low sensitivities, which lends to the method's practical attractiveness.

Let us consider the practical case of a scalar voltage transfer function T(s), n=m=1, $u=v_1$, $y=v_2$. In order to decrease the sensitivity (by elimination of multiple feedback paths) it is advantageous to decompose T(s) into degree one and two factors, the realizations for which can be cascaded to yield T(s); this also has the advantage of allowing each factor section to be separately impedance scaled to obtain practical element values. In actual fact the factors are conveniently obtained for many important situations from standard tables[6].

One can then give standard degree two sections from which low-pass, band-pass or high-pass responses are obtained by picking signal off at appropriate points [3, p.89], while the degree one sections are easily obtained with only passive components. For illustration let us simply illustrate the theory by considering only the normalized degree-two low-pass transfer function

$$\frac{\mathcal{L}[v_2]}{\mathcal{L}[v_1]} = T(s) = \frac{k}{s^2+2\zeta\omega_n s+\omega_n^2}, \quad (k, \zeta = 1/Q, \omega_n) \text{ constants} \quad (4a)$$

For (1) we can obtain by standard methods [6,p.82]

$$A = \begin{bmatrix} 0 & -\omega_n^2 \\ 1 & -2\zeta\omega_n \end{bmatrix}, \ B = \begin{bmatrix} k \\ 0 \end{bmatrix}, \ C = [0,\ 1],\ D=0 \quad (4b)$$

which is checked by (2) to give (4a). Figure 1 shows a block diagram type implementation of this "realization" which we see will use the minimum possible number of capacitors, 2, for an integrated circuit construction. Also from Fig. 1 we see that the Q can be made variable through the variation of one resistor, as can the ω_n (if we can accept a variation in Q with ω_n); such a resistor variation can of course occur conveniently by electronic means in integrated form through the use of FET devices.

Of particular practical interest for Fig. 1 are the low sensitivities obtained. For example if the gains of the integrator operational amplifiers are both K, which is generally very large, one finds for a typical integrator configuration [7, p.70]

$$S_K^{\omega_n} = \frac{K}{\omega_n} \cdot \frac{\partial \omega_n}{\partial K} = \frac{1}{K+2} \quad (5)$$

Thus, with the transfer function sensitivity, a sensitivity approaching zero can be obtained [7, p.207]. Similarly sensitivities of and to other parameters, as passive element values, are found to be relatively small [3, p.90][8, p.210].

Although there are many advantages to this

* The paper presents work sponsored by the Air Force Office of Scientific Research under Contract F44620-67-C-0001 and the Office of Naval Research under Contract NONR 225(83).

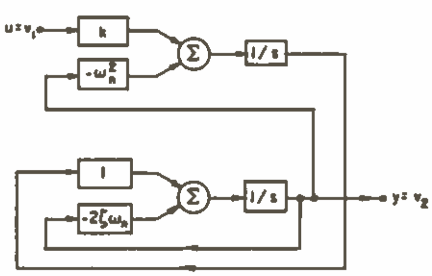
approach the main disadvantages should be mentioned. These are the frequency range, 0 to about 200KH$_z$, and large numbers of components required (with an attendant power consumption).

For multiple input, multiple output systems the state-variable approach comes into its own, whereas it can be also applied to distributed circuits [9] and to the design of filters by the modeling of filter sections, as that of Brune [10], to give alternate but practical results.

"The difficult must become habit, habit easy, and the easy beautiful" [1, p.23].

For J.B.:

"The most important thing is to build the life of the human spirit; therefore the goal of art is spiritual communication with people" [1, p.11].



DEGREE-TWO LOW-PASS DIAGRAM

Figure 1

References

1. Moore, S., "The Stanislavski System," The Viking Press, New York, 1965.

2. Newcomb, R.W., "Network Theory: The State Space Approach," Libraire Universitaire Uystpruyst, Louvain, Belgium, 1968.

3. Kerwin, W.J., L.P. Huelsman, and R.W. Newcomb, "State-Variable Synthesis for Insensitive Integrated Circuit Transfer Functions," IEEE Journal of Solid-State Circuits, Vol. SC-2, No. 3, September 1967, pp. 87-92.

4. Kerlin, T.W., "Sensitivities by the State Variable Approach," Simulation, Vol. 8, No.6, June 1967, pp. 337-345.

5. Liou, M.L., "Evaluation of the Transition Matrix," Proceedings of the IEEE, Vol. 55, No. 2, February 1967, pp. 228-229.

6. Skwirzynski, J.K., "Design Theory and Data for Electrical Filters," D. VanNostrand Co., London, 1965.

7. Newcomb, R.W., "Active Integrated Circuit Synthesis," Prentice Hall, Inc., Englewood Cliffs, 1968.

8. Huelsman, L.P., "Theory and Design of Active RC Circuits," McGraw-Hill Book Co., New York, 1968.

9. Kozemchak, E.B., and R.W. Newcomb, "State-Variable Synthesis of Integrated-Circuit Distributed RC Transfer Functions," Electronics Letters, Vol. 4, No. 15, July 26, 1968, pp. 320-321.

10. Sheahan, D.T., "Inductorless Filters," Stanford Electronics Laboratories, Technical Report No. 6560-15, September 1967.