A STATE-VARIABLE AND GYRATOR REALIZATION-COMPARISON*

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"A young man said to me:
I am interested in the problem of reality." [1,p.32]

SUMMARY

Gyrator and state-variable methods are carried out for a degree three Cauer response. Element values will be given with an orientation toward complete integration such that the methods can be properly compared for integrated circuit design.

INTRODUCTION

The advent of integrated circuitry has meant a reevaluation of theoretical synthesis methods as far as their applicability to practical situations is concerned. As a continuation of this reevaluation we here outline the results of applying several suggested methods to a particular transfer function.

Because of their importance to integrated circuit technology, where the separate components have been adequately integrated, we concentrate upon state-variable and gyrator type circuits. As the theories are basically available elsewhere, [2] - [6], this talk will primarily present the results of applying these theories.

We point out, however, that in obtaining the resultant circuits, an effort has been made to minimize the number of capacitors as well as to keep their values, and those of resistances, reasonable for integration with accuracies placed upon ratios rather than absolute values.

DESIGN SPECIFICATIONS

We take a voltage transfer device as specified with a passband of 0 to 7 KHz in which no more than 1 db of attenuation is allowed while above 10KHz an attenuation of at least 20 db is required. Where appropriate we shall take load and source resistances of 600 Ω.

The design tables of Skwirzynski apply [7, pp. 289-325] from which we find that a degree three structure will satisfay the requirements. From Skwirzynski we determine the transfer function for the normalized Cauer type response to be

$$\frac{V_2}{V_s}(p) = H\frac{0.56617p^2+1}{p^3+0.80867p^2+0.87177p+0.36495} \qquad (1a)$$

$$= H\frac{1}{p+0.50766} \times \frac{0.56617p^2+1}{p^2+0.0301014p+0.71898} \qquad (1b)$$

Again from Skwirzynski, normalized inductor tee and capacitor pi circuits are determined which when terminated in unit resistors yield Eq. (1a). To denormalize the elements we raise all resistances by $r_n = 600$, all capacitances by $1/(r_n\omega_B) = 1/(600 \times 5.26 \times 10^4) = 3.16 \times 10^{-8}$ and all inductances by $r_n/\omega_B = 600/(5.26 \times 10^4) = 1.14 \times 10^{-4}$. This yields capacitances in the nano-farad (10^{-9}f) range, which are rather too large for reasonable integration.

DENORMALIZATION OF TEE STRUCTURE

By inserting a gyrator after the input source resistance, and one at the output to cancel it, the inductor tee mentioned above can be turned into the capacitor tee where an extra degree of freedom can be used to alter the element values. Properly choosing this extra parameter allows one to obtain reasonable values for integration. The structure, however, uses one extra capacitor. Nevertheless, the total capacitance is relatively small.

GYRATOR-CASCADE SYNTHESIS

A similar result, but one which uses the minimum number of capacitors, is obtained by following the ideas of [2] using the sequences of calculations outlined in [5, pp. 165-167]. This begins by obtaining a suitable input admittance to synthesize. For this we can use the admittance y(p) of the inductor tee circuit mentioned above. In terms of the parameters and values given by Skwirzynski we find

$$y(p)=\frac{a_2(a_1+[\zeta_1/a_2])p^2+a_2p+1}{a_1a_2(a_1+2[\zeta_1/a_2])p^3+a_2(a_1+[\zeta_1/a_2])p^2+2a_1p+1} \qquad (2a)$$

$$= \frac{2.2154\ p^2+0.8373p+1}{5.4786p^3+2.2154p^2+3.9396p+1} \qquad (2b)$$

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The process then consists of 1) removing a gyrator for capacitance scaling, 2) removing a shunt capacitor for a zero of transmission at infinity, 3) removing another gyrator for further capacitance scaling, 4) removing a nonreciprocal Brune section for a zero of transmission at $j\Omega_{\infty 1}=$ j 1.3289, 5) a final gyrator removal to yield a unit load resistance.

Judicious choice of the free parameters, the equivalent of three gyration conductances, is then made to obtain element values which are reasonable for a complete integration (upon denormalization). The final result has all gyrators conveniently grounded. Nevertheless, it should be observed that a very high degree of accuracy is needed in making the numerical calculations, as well as in maintaining two of the gyration conductances for practical operation of the circuit. Although the element value spread is reasonable, it is higher than ideally desired.

OPERATIONAL AMPLIFIER-CASCADE SYNTHESIS

To cut down on the element value spread we next synthesize following the state-variable synthesis discussed in [4]. For this synthesis we need to further decompose the degree two factor of Eq. (1b) to exhibit the constant (0.56617) at infinity.

$$\frac{V_2}{V_s} = H\frac{1}{p + 0.50766} \times$$

$$\times \left[0.56617 + \frac{-0.17043p + 0.59214}{p^2 + 0.301014p + 0.71898}\right] \qquad (3)$$

Using a negative gain integrator with a 5KΩ resistor and a 50pf capacitor fixes $\lambda = -0.760$ for the integrator scale factor [5, p.69]. Appropriately choosing summer resistances yields the final structure which turns out to be readily integrated. However, it does contain a large number of components, including seven operational amplifiers in which case power dissipation and chip heating are points of concern.

LOW-GAIN, LUMPED-DISTRIBUTED CIRCUITS

The number of amplifiers can be reduced to one using the low-gain structures developed by Kerwin [3]. Of most interest is the lumped-distributed structure for which one needs to further frequency scale to bring the numerator zeros to ±j1. Thus, set $p_n^2 = 0.56617p^2$ or $p = 1.32889p_n$; the degree two factor in Eq. (1b) becomes

$$T_2(p_n) = H_2\frac{p_n^2 + 1}{p_n^2 + 0.223488p + 0.407065} \qquad (4)$$

This has poles at $p_{1,2} = -0.112744 \pm j0.62815$ which when used in the appropriate design chart [5, p.91] yields an amplifier gain K = 1.04 and a normalized shunt capacitance at the amplifier input of C = 0.046. Choosing $r_n = 10^4$ and $\omega_B' = (1.329)(5.26 \times 10^4) = 6.99 \times 10^4$ then fixes the denormalization. The degree one factor of Eq.(1b) can be realized by an RC voltage divider for which a forty kilohm resistance level is chosen to bring down the capacitor value (which is frequency denormalized throught the previous ω_B).

It will be seen that a relatively large spread of element values results but that overall the circuit is obtainable in a relatively compact design. In order to see the size advantage of distributed elements a comparable design [5, p.89] using completely lumped elements was made. From this latter one can see that the use of distributed elements allows almost a three to one savings in capacitor area.

STATE-SPACE GYRATOR SYNTHESIS

Having considered the previous methods it is instructive to investigate a combination of the techniques [6]. This involves using gyrators to realize the state-variable equations through the intermediary of an admittance coupling structure. The latter when synthesized by gyrators and resistors and when loaded in capacitors yields the desired transfer function.

If we rewrite Eq. (1a) with literal coefficients as

$$\frac{V_2}{V_s}(p) = \frac{c_2p^2 + 1}{p^3 + \alpha_3p^2 + \alpha_2p + \alpha_1} \qquad (5a)$$

then we are led directly to the state-space equations

$$\dot{x}_o = A_ox_o + B_ov_1, \quad v_2 = C_ox_o \qquad (5b)$$

with

$$A_o = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 \end{bmatrix}, \; B_o = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; C_o = [1, 0, c_2] \qquad (5c)$$

By the transformation $x_o = Px$ with P resulting from $K = P\tilde{P}$, ~ = transpose, and K satisfying ($\dot{+}$ = direct sum, $0_2 = 2 \times 2$ zero)

$$A_oK + K\tilde{A}_o = -[0_2 \dot{+} 1] \qquad (5d)$$

we can obtain a passive coupling network described by

$$Y_c = \begin{bmatrix} 0 & 0 & \tilde{B}_o\tilde{P}^{-1} \\ 0 & 0 & C_oP \\ -P^{-1}B_o & -\tilde{P}\tilde{C}_o & -P^{-1}A_oP \end{bmatrix} \qquad (5e)$$

On loading this coupling network, which contains gyrators and one passive resistor, in unit capacitors the normalized transfer function results [6], when also a unit gyrator is cascaded at the output port. Since a voltage transfer function is on hand we can arbitrarily impedance scale to get desired element values.

We find for K and P

$$K = \frac{1}{2(\alpha_2\alpha_3 - 1)}\begin{bmatrix} \alpha_3/\alpha_1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & \alpha_2 \end{bmatrix},$$

$$P = \begin{bmatrix} \sqrt{k_{11}} & 0 & 0 \\ 0 & \sqrt{k_{22}} & 0 \\ \frac{-k_{22}}{\sqrt{k_{11}}} & 0 & \sqrt{\frac{k_{11}k_{33}-k_{22}^2}{k_{11}}} \end{bmatrix} \qquad (5f)$$

For the numbers on hand we find

$$P = \begin{bmatrix} 1.806 & 0 & 0 \\ 0 & 1.213 & 0 \\ -0.814 & 0 & 0.781 \end{bmatrix}, \qquad (6a)$$

$$P^{-1} = \begin{bmatrix} 0.5535 & 0 & 0 \\ 0 & 0.825 & 0 \\ 0.5755 & 0 & 1.281 \end{bmatrix}$$

and

$$Y_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.281 \\ 0 & 0 & 1.346 & 0 & 0.442 \\ 0 & -1.346 & 0 & -0.672 & 0 \\ 0 & 0 & 0.672 & 0 & -0.644 \\ -1.281 & -0.442 & 0 & 0.644 & 0.809 \end{bmatrix} \qquad (6b)$$

On scaling to as reasonable numbers as seem possible, having C = 250pf and hence $r_n = 76K\Omega$ (since ω_B is fixed), we obtain a final structure. In this circuit all components [gyrators (6), resistors (1), and capacitors (3)] are grounded while the resistor can be transformed through the input gyrator to become a series source resistance. Too, we observe that the element values are on the large size, though possible, for integration, while the layout, as with all of the previous, can be made with no lead crossings (except perhaps for bias circuitry).

CONCLUSIONS

By considering the design of an actual response which may be desired, several design methods can be compared for their practicality. We see that in general the actual choice of circuit used may well depend upon other factors than become apparent in the circuits synthesized here. For example, the designer's familiarity with distributed structures may well be the factor in choosing the lumped-distributed structure, which is limited in values of Q obtainable when compared to the state-variable synthesis. Or sensitivity, which is always a factor to consider, may well lead one to the passive-type gyrator structures. Alternatively bias circuitry may be of prime importance, and may rule out any of the gyrator structures. Similarly noise behavior may be of prime importance.

Considering all factors,the state-variable approach seems to be most highly recommended for general designs with the gyrator cascade synthesis for precision high degree filters.

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"I said: Really!
Then I saw him turn to glance, surreptitiously,
In the big mirror, at his own fascinating shadow."

[1, p. 32]

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