A SCATTERING MATRIX PROGRAM FOR HIGH FREQUENCY CIRCUIT ANALYSIS*

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Summary

A time-shared program, SPEEDY, is discussed. The program is for small signal analysis of high frequency active circuits. It is based directly upon 2-port interconnections through the scattering matrix, thus eliminating continuous conversions of parameters of previous routines. Program advantages are speed of calculation, due to the generality of S parameters and the limitation to 2-ports; the convenience of accepting measured S parameter data, and the consequent avoidance of transistor equivalent circuits. The program includes two unique subroutines: one that handles all four possible combinations of parallel and series 2-port interconnections, and one that converts 2-port common emitter transistor parameters to common base or common collector configurations.

"Mi letra, hasto mis sentimientos, dependen de la pluma con que escribo." [1, p. 92]

Introduction

With the advent of equipment for measuring directly the scattering matrix of a 2-port, as the HP 8410 Network Analyzer System, there is an advantage to performing network analysis directly in terms of scattering, S, parameters. This is especially true for transistor circuits where, for example, calculations based upon equivalent circuits can often be inaccurate.

The present program is based upon interconnections of 2-ports calculated directly in terms of S parameters. It therefore avoids the often time-consuming conversions to other parameters (as Y's for parallel connections). Too, its adherence to connections only of 2-ports gives it an added advantage in calculation speed, hence its name "SPEEDY".

Program Background

It turns out that any series-parallel combination of ports of 2-ports can be described by one scattering matrix formula. Thus, if S_A and S_B are the, 2 x 2, scattering matrices of two 2-ports to so be connected then the resultant scattering matrix S is [2]

$$S = \begin{bmatrix} a & o \\ o & b \end{bmatrix} \cdot h\left\{\begin{bmatrix} a & o \\ o & b \end{bmatrix} \cdot S_A, \begin{bmatrix} a & o \\ o & b \end{bmatrix} \cdot S_B\right\} \qquad (1.1)$$

with the matrix function h(.,.) defined by, 1_2 being the 2 x 2 identity matrix,

$$h\{S_1, S_2\} = (31_2 + S_1)^{-1}[(S_1 - 1_2) + 4(1_2 + S_1)S_2\{(31_2+S_1) - (S_1-1_2)S_2\}^{-1}(1_2 + S_1)] \qquad (1.2)$$

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The scalars a and b are chosen from the following table

Connection	a	b
parallel-parallel	1	1
series-series	-1	-1
series-parallel	-1	1
parallel-series	1	-1

Likewise, if two 2-ports of scattering matrices S_A, followed by, S_B are in cascade, then the resultant scattering matrix S is

$$S = \begin{bmatrix} S_{11_A} + S_{12_A} S_{11_A} (1 - S_{22_A} S_{11_B})^{-1} S_{21_A} & S_{12_A} (1 - S_{22_A} S_{11_B})^{-1} S_{12_B} \\ S_{21_B} (1 - S_{22_A} S_{11_B})^{-1} S_{21_A} & S_{21_B} (1 - S_{22_A} S_{11_B})^{-1} S_{22_B} S_{12_B} + S_{22_B} \end{bmatrix} \quad (1.3)$$

Note that when 2-ports are cascaded only scalars need to be inverted so the calculations for Eq. (1.3) are relatively simple.

Finally, we mention that the scattering matrix $S = [S_{ij}]$ of a 3-terminal network satisfies

$$\sum_{i=1}^{3} S_{ij} = \sum_{j=1}^{3} S_{ij} = 1 \quad (1.4)$$

Consequently, the scattering matrix of a transistor treated, for example, as a common collector 2-port can be found from that measured in the common emmitter configuration quite readily through the use of Eq. (1.4).

Program Description

The program implements the four equations of the last section; a flow chart is shown in Fig. 1. Data is handled as follows:

a) Input Information Required

Transistors

For the active devices, the program requires only the measured S or Y parameters in the CE configuration. CB and CC parameters are computed when needed from Eq. (1.4).

Passive Elements

Resistors, capacitors, and inductors - actual value.

Transmission lines - characteristic impedance, length, relative velocity.

Waveguides - characteristic impedance, length, width.

Transformers - turns-ratio.

Also, if desired, any of the passive elements can be described by their measured two-port S or Y parameters.

Circuit Information

All information as to the type of circuit element or as to the type of circuit connection is given in an easily understandable abbreviated form. R stands for resistance, L for inductance, S for series connection, etc. For example, SRLS stands for a series RL circuit connected in series, SRLP for the same connected in parallel.

For fast operation, the main program is saved in compiled form and all data are read in both real and alpha variable form.

b) Output

Standard printout includes the frequency, the overall polar S parameters for the network, and the stability factor if the circuit contains active elements. Optional output can include interstage mismatch losses, stability circle locations, etc.

c) Subroutines

Since any series or parallel combination of passive one-ports can be completely described by its reflection coefficient, setting up the S matrices for these passive elements can always be done by using function statements rather than subroutines. Subroutines are used to compute the two-port representations and for the interconnections of the two-ports, as described by Eqs. (1.1) - (1.3).

Conclusions

The nature of a program, SPEEDY, has been outlined. In actuality the program was written for time-shared operation. It has been used both with conversational and batch processed time sharing to demonstrate the speed of the program. As an example, a 2-stage RF amplifier was analyzed in the conversational time-shared mode of a SIGMA 7 computer. For ten different frequencies, it took slightly less than 1 second CPU time at 15¢/second. This cost, of course, can be considerably reduced by using batch-processing. The program is saved in the compiled form, thus the user does not have to pay for compilation every time the program is run.

A summation of the advantages of the discussed approach are the following:

1) The measured S parameters of the transistors are used directly in the computations.

2) All parameter conversions are eliminated.

3) All feedback connections are done by a single subroutine.

4) Common base and common collector two-port S parameters can be computed from the common emitter S parameters.

Because of these advantages, the program is operationally fast; it adds to the list of presently available concepts and routines [2] - [6]. The program is set up so that the user need not know any programming.

"Necesito prepararme para todo. Para mí, prepararme significa dejar correr el tiempo indefinidamente" [1, p. 92].

References

1. Moreno, B. Fernández, "La Mariposa y la Viga," Rodolfo Alonso Editor, Buenos Aires, 1968

2. Bodharamik, P., Besser, L., and Newcomb, R., "Two Scattering Matrix Programs for Active Circuit Analysis," in preparation.

3. Murray-Lasso, M. A., Kozemchak, E. B., "Microwave Circuit Design by Digital Computer," IEEE Trans. on M.T.T., August 1969.

4. Green, P. E., "General Purpose Programs for the Frequency Domain Analysis of Microwave Circuits," IEEE Trans. on M.T.T., August 1969

5. Besser, L., "Computerized Wide Band Amplifier Design," 1969 WESCON Technical Papers, Session No. 6-5.

6. Monaco, V. A., and Tiberio, P., "Automatic Scattering Matrix Computation of Microwave Circuits' Alta Frequenza, Vol. 39, No. 2, 1970, pp. 1-6.

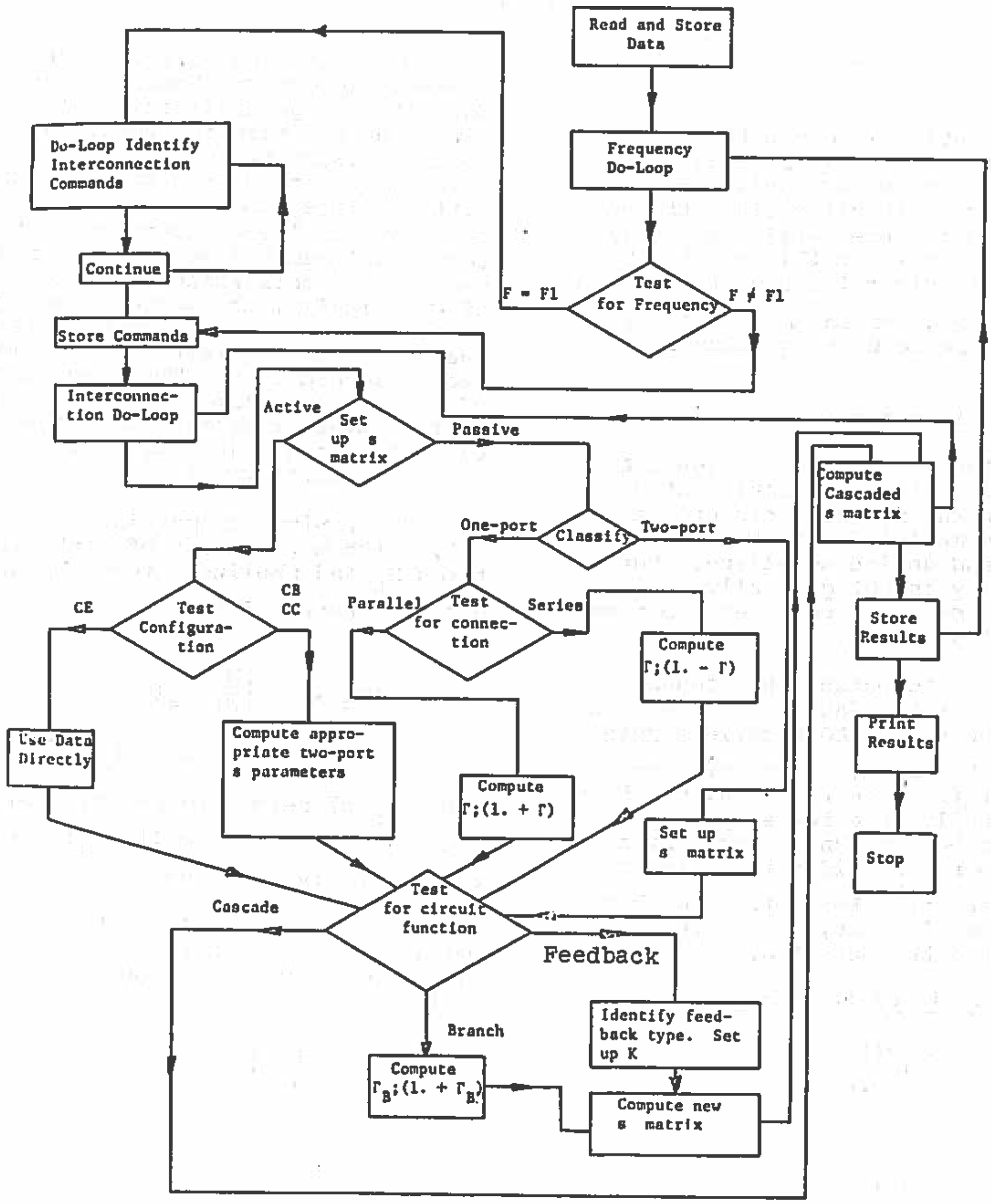

Fig. 1 Flow graph of the SPEEDY analysis routine