A Note on Transformerless Synthesis of 3-Port Resistor Networks

The synthesis of dominant, real, constant, admittance matrices as n-port, transformerless, resistor networks is well known [1, p. 367], [2, p. 266]. The purpose of this correspondence is to present the dual to the above method which is valid for 3x3, real, dominant, impedance matrices which when reduced (as defined below) are singular.

In the synthesis of dominant admittance matrices, the matrix is decomposed into submatrices representing one-port conductances and elemental two-port conductance networks as shown in Fig. 1(a) and (b). By connecting the subnetworks in parallel one thus realizes the resistor network.

The dual to this is, under the conditions to be derived, the decomposition of a 3x3 impedance matrix into submatrices representing one-port resistances and elemental two-port resistance networks, as shown in Fig. 1(c) and (d), and the subsequent series connection of the subnetworks.

The impedance matrix that we desire to synthesize is assumed dominant, with possibly nonpositive off-diagonal elements, of the general form

$$Z = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{12} & r_{22} & r_{23} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}. \tag{1}$$

The first step in the procedure is to obtain the reduced impedance matrix Z', which is defined as

$$Z' = \begin{bmatrix} |r_{12}|+|r_{13}| & r_{12} & r_{13} \\ r_{12} & |r_{12}|+|r_{23}| & r_{23} \\ r_{13} & r_{23} & |r_{13}|+|r_{23}| \end{bmatrix}. \tag{2}$$

The reduced impedance matrix represents a network which is constructed only of elemental two-port resistance networks. The original Z matrix represents a network consisting of the Z' network with series connected one-port resistances.

Matrix Z' can be decomposed into three submatrices, each corresponding to an elemental two-port resistance network. The decomposition of Z' is

$$Z' = \begin{bmatrix} |r_{12}| & r_{12} & 0 \\ r_{12} & |r_{12}| & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} |r_{13}| & 0 & r_{13} \\ 0 & 0 & 0 \\ r_{13} & 0 & |r_{13}| \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & |r_{23}| & r_{23} \\ 0 & r_{23} & |r_{23}| \end{bmatrix}. \qquad (3)$$

Since a realization of Z' clearly leads to one for Z we now show that a realization through the decomposition of (3) can be obtained if and only if Z' is singular.

Precisely this result is valid because every realization of Z' by this method results in a wye resistor network, as seen by direct connections, and conversely every wye resistor "3-port" has an impedance matrix of the form given in (2). Singularity of Z' is dependent upon the vanishing of its determinant; we find

$$\det Z' = 2\left\{|r_{12}||r_{13}||r_{23}| + r_{12}r_{13}r_{23}\right\}. \qquad (4)$$

Thus, det Z' = 0 if and only if either an odd number of the terms r_{12}, r_{13}, and r_{23} are negative or any of the terms are zero. Next we observe that for a wye network, as illustrated in Fig. 2, the only possible port orientations are when all ports are oriented in the same direction or when one port is oriented oppositely to the other two. In the first case all of the nonzero coupling elements (off-diagonal elements) of Z' are negative and in the second case only one nonzero pair of coupling elements can be negative. Equation (4) then shows that det Z' = 0, while if det Z' = 0 Fig. 2 (with perhaps a relabeling of ports) will always yield a realization (as seen by cataloging all cases).

As an example of this method the Z matrix from [2, p. 267] will be used.

$$Z = \begin{bmatrix} 7 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}. \qquad (5)$$

The reduced impedance matrix is

$$Z' = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 3 \end{bmatrix} \tag{6}$$

which is singular; hence a synthesis from the decomposition of (5) results. The interconnection of the corresponding subnetworks is shown in Fig. 3. When this type of synthesis was attempted in [2, p. 267] a valid circuit was not obtained because of an improper interconnection of elemental two-port resistance networks. The interconnection step is very critical because there are eight ways of interconnecting the 3 two-port subnetworks of Z' and only two of the ways will yield a network with the correct Z' matrix. As can be seen, the method of interconnection in Fig. 3 results in a valid realization.

In the 3x3 case there is no need to carry out the interconnection of two-ports in order to realize Z'. After having determined that Z' is singular, the corresponding three-port network can be obtained by inspection. This network will be a wye network as in Fig. 2. The value of each resistor is $R_{ij} = |r_{ij}|$ and the port orientation is determined by the signs of the off-diagonal terms. If r_{ij} is negative then the ith terminal (unprimed terminal of the ith port) and the j'th terminal (primed terminal of the jth port) will be the same. Or, if r_{ij} is positive then the ith terminal and the jth terminal will be the same. In the case where $r_{ij} = 0$ the remaining diagonal terms will determine the port orientation.

We point out that if Z' is not singular a synthesis can also be obtained by converting to $Y' = [Z']^{-1}$; Y' will be paramount and a known synthesis applies [1, p. 372]. Too, other special cases of paramount matrix synthesis have been discussed [3]. Our purpose here is to investigate the construction dual to that for dominant admittances.

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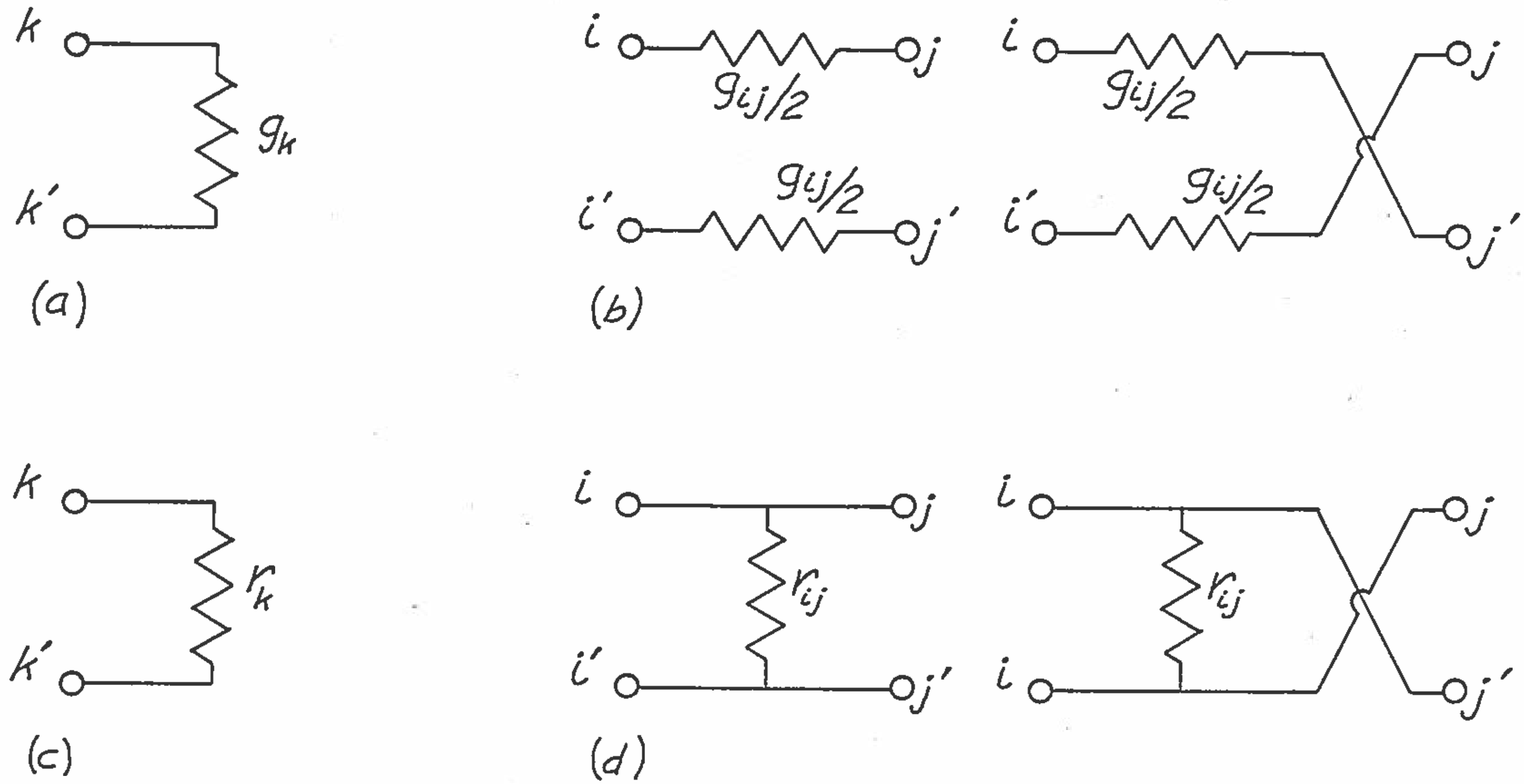


Fig. 1

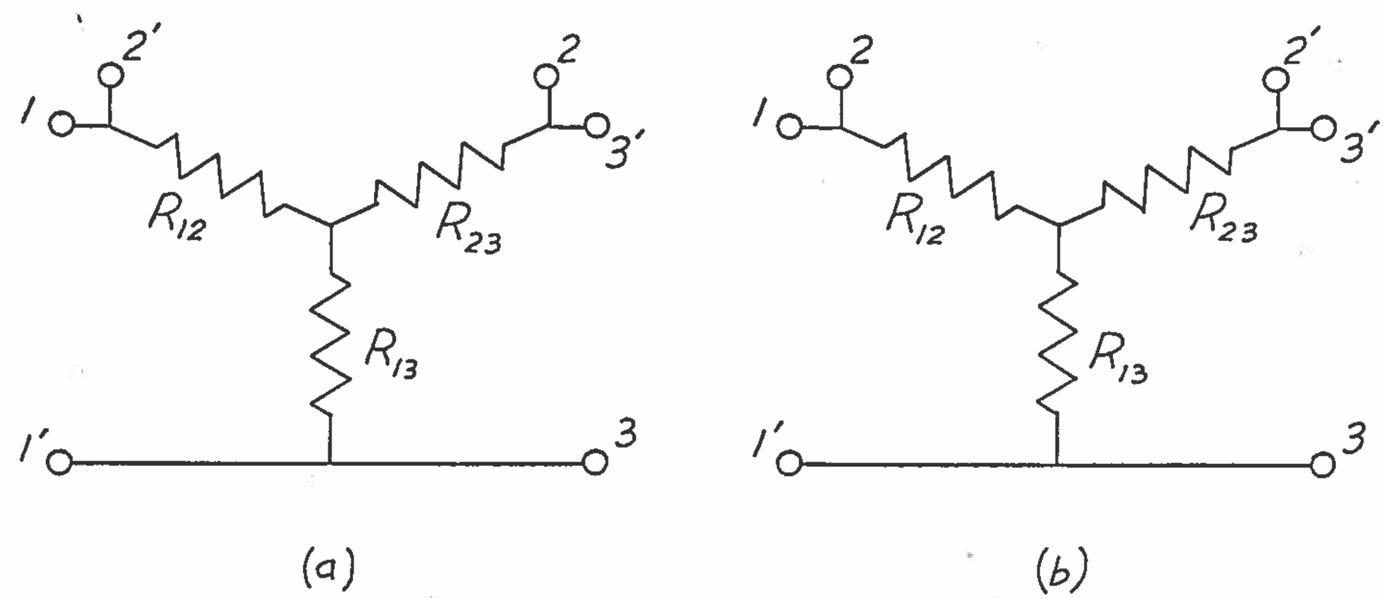
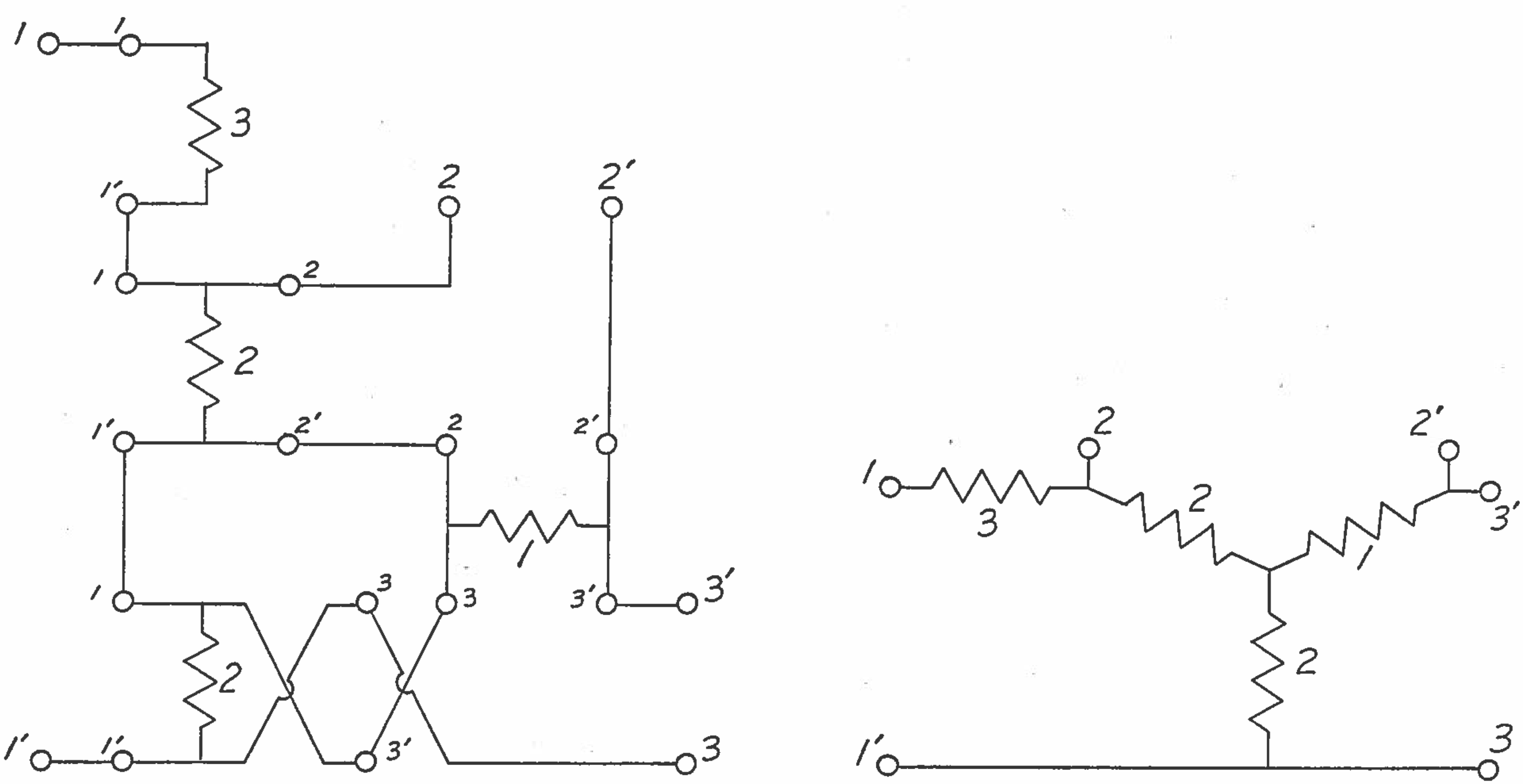


Fig. 2



(a)

(b)

Fig. 3

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FIGURE CAPTIONS:

Fig. 1. (a) One-port conductance. (b) Elemental two-port conductance networks. (c) One-port resistance. (d) Elemental two-port resistance networks.

Fig. 2. (a) Wye network with all ports oriented in same direction. (b) Wye network with one port oriented oppositely to other two ports.

Fig. 3. (a) Realization of Z. (b) Fig. 3(a) redrawn.