where P_T/N is the total signal/noise power ratio.

The probability of error curves in Fig. 1 suggests that the minima are fairly broad and there is very little change in the average error rates for $0{\cdot}3 \leqslant P_{dpsk}/P_T \leqslant 0{\cdot}7$. This feature may perhaps be used to increase the effectiveness of this system. Fig. 2 shows the probability of error against total

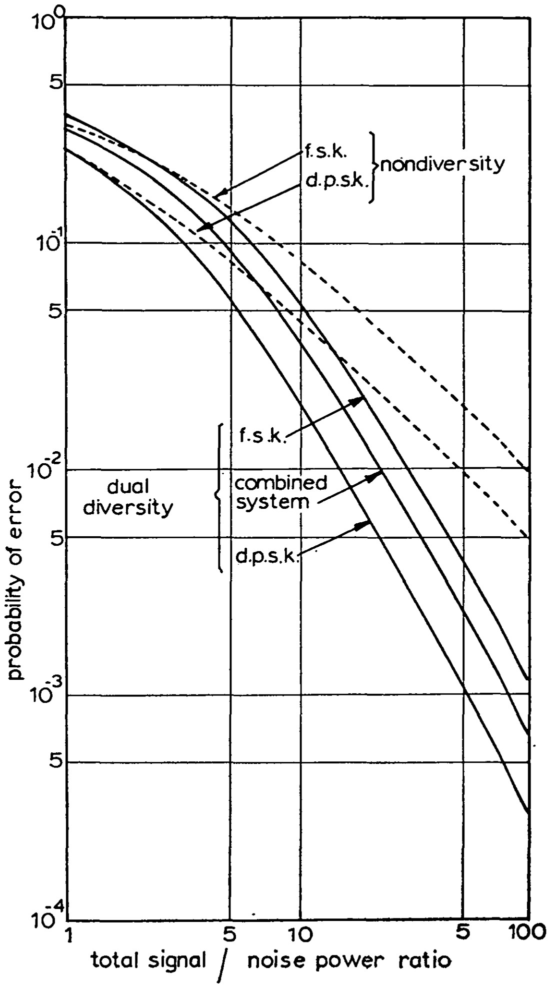

Fig. 2 *Probability of error*

signal/noise ratio. In this Figure, curves for the conventional nondiversity and dual-diversity f.s.k. and d.p.s.k. systems are also shown. It is noticed that the performance characteristics of the proposed system fall between those of the other two systems. Compared with the dual-diversity f.s.k. system, the proposed system requires 1·25 dB less power for the same average errors.

Thus, on the basis of average error rates, the proposed system does not provide any spectacular advantages. However, it is expected that it may have advantages as far as the distribution of errors over a certain time interval is concerned. It is known that, for f.s.k. and d.p.s.k. systems, errors occur in bunches or as bursts. There are two important parameters generally used to describe the burst statistics: (i) the distribution of the duration of bursts, and (ii) the distribution of the interval between bursts. Knowledge of the burst statistics for the proposed system would be very useful for comparing its performance with the conventional dual-diversity f.s.k. or d.p.s.k. system. In the absence of any experimental results, it may be said that the burst statistics for the proposed system may be similar to the other systems, or, if they differ, that the differences may lie in either direction. However, it appears that it should be possible to regulate the burst statistics for the proposed system to some extent by adjusting the power levels in the f.s.k. and d.p.s.k. channels resulting in a more favourable performance. Furthermore, an increase in the order of diversity may be advantageous.

In this letter, we have presented a short description of a new concept called 'diversity of modulation' which has been utilised to describe a new system. This system has a potential for modifying the error-bursts statistics resulting in an improvement in the effectiveness of the error-control coding. Using the new concept, a large number of other communication systems can be considered. For example, we may con-consider diversity of modulation using coherent f.s.k. systems with p.s.k. systems (either self-synchronised or with a transmitted reference) or coherent f.s.k. system with noncoherent f.s.k. system etc. These systems should be candidates for further study.

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19th January 1970

Reference

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HAKIM-THEORY TRANSFER-FUNCTION SENSITIVITY*

Indexing terms: Sensitivity analysis, Active circuits

It is shown that, along with the pole-position sensitivity, the transfer-function sensitivity (and, similarly, the Q and w_n sensitivities) can be arbitrarily assigned using the Hakim theory for 2nd-degree transfer functions.

Hakim[1] has introduced an interesting and useful synthesis method which allows the design of a given 2nd-degree transfer function with an active *RC* circuit to achieve an arbitrarily prescribed pole-position sensitivity. In Reference 2 this is discussed with regard to integrated circuits, but although a low-passband transfer-function sensitivity was obtained (Reference 2, p. 206), the full significance was not realised, in view of the development of a general expression only for an impracticable ($k = K$) case (Reference 2, p. 129) (for which, also, a factor $1-k$ was inadvertently dropped). Here the general results are presented, using the development of Reference 2, p. 126, which shows that the transfer-function sensitivity, though amplified by a factor of Q, can be arbitrarily prescribed with the pole-position sensitivity.

The configuration considered is that of Fig. 1, where, in

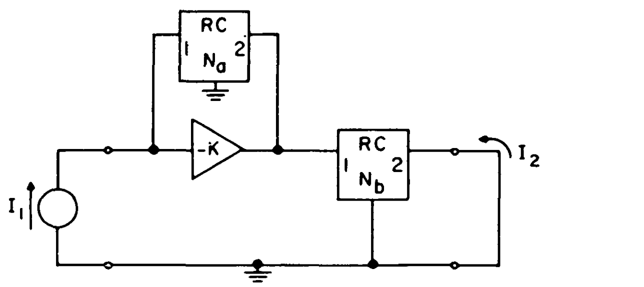

Fig. 1 *Hakim current-transfer-function-synthesis configuration*
K = voltage gain, $K > 0$

terms of the admittance matrices and amplifier voltage gain K we have, on specification as a stable 2nd-degree transfer function,

$$\frac{I_2}{I_1} = \frac{-Ky_{21b}}{y_{11a}-Ky_{12a}} = \frac{b_3 p^2 + b_2 p + b_1}{p^2 + 2\zeta\omega_n p + \omega_n{}^2} \tag{1a}$$

The design procedure introduces a 1st-degree polynomial into the numerator and denominator of the right-hand side of eqn. 1*a*, while decomposing the denominator to get an

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RC realisable y_{11a} out of the process. This yields (Reference 2, p. 128)

$$\frac{I_2}{I_1} = \frac{-K\left\{\dfrac{-1}{K_1(1-k)}\ \dfrac{b_3 p^2+b_2 p+b_1}{p+\sigma_0}\right\}}{\left(\dfrac{p^2+2\zeta\omega_n p+\omega_n^2\gamma}{p+\sigma_0}\right)-K\left\{\dfrac{-k}{K_1(1-k)}\ \dfrac{p^2+2\zeta_0\omega_0 p+\omega_0^2}{p+\sigma_0}\right\}} \quad . \ . \ . \ . \ (1b)$$

where the introduced quantities are determined as follows. Given a prescribed pole-position sensitivity $s_K^{p_j} = jm$, $m > 0$, one determines ζ_0 and ω_0 from (corrected eqn. 4.6.5 ff. of Reference 2, p. 128)

$$\zeta_0 = \zeta \Big/ \left\{1+\frac{2m}{\omega_n}\sqrt{(1-\zeta^2)}\right\}^{\frac{1}{2}} \qquad \omega_0 = \zeta\omega_n/\zeta_0 \ . \ \ . \ (2a)$$

Next, a k in the range (see Fig. 4.6.2 of Reference 2)

$$\frac{\zeta_0^2}{\zeta^2}\left(\frac{1-\zeta^2}{1-\zeta_0^2}\right) < k < \frac{\zeta_0^2}{\zeta^2} \quad . \ . \ . \ . \ . \ . \ (2b)$$

is chosen. This guarantees y_{11a} is RC realisable through the choice of σ_0 and

$$\gamma = \frac{1-k\zeta^2/\zeta_0^2}{1-k} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (2c)$$

Next, a synthesis of y_{11a} for the now prescribed zeros of y_{12a} occurs; this fixes the level constant K_1 through the realisable gain required in the passive transformerless RC circuit. The amplifier gain K is then nominally fixed at $K = K_1$ and one determines that $s_K^{p_j} = jmk/K$ satisfies the required specifications. The differences between k and K are important, though subtle, as, practically, $K \gg k$.

For the sensitivity study, we rewrite eqn. $1b$ as

$$\frac{I_2}{I_1} = \frac{\dfrac{K}{K_1(1-k)}(b_3 p^2+b_2 p+b_1)\Big/\left\{1+K\dfrac{k}{K_1(1-k)}\right\}}{p^2+2\zeta\omega_n p+\omega_n^2\left[\left\{\gamma+K\dfrac{k}{K_1(1-k)}\ \dfrac{\zeta^2}{\zeta_0^2}\right\}\Big/\left\{1+K\dfrac{k}{K_1(1-k)}\right\}\right]}$$

$$= \frac{K(b_3 p^2+b_2 p+b_1)/\{K_1+k(K-K_1)\}}{p^2+2\zeta\omega_n p+\omega_n^2[\{K_1+k(\zeta^2/\zeta_0^2)(K-K_1)\}/\{K_1+k(K-K_1)\}]} \quad . \ . \ . \ . \ (3)$$

From this, we check that the transfer function is as desired when $K = K_1$. Transfer-function sensitivity is next found by writing $T = N/D = I_1/I_2$, with N and D numerator and denominator polynomials, and using

$$S_K^T = \frac{K}{T}\frac{\partial T}{\partial K} = S_K^N - S_K^D$$

Thus,

$$S_K^T = \frac{K_1(1-k)}{K_1+k(K-K_1)} - \frac{K/K_1}{p^2+2\zeta\omega_n p+\omega_n^2\left[\dfrac{1+\{(K/K_1)-1\}k(\zeta^2/\zeta_0^2)}{1+\{(K/K_1)-1\}k}\right]} \times \frac{\omega_n^2 k\{(\zeta^2/\zeta_0^2)-1\}}{\left\{1+k\left(\dfrac{K}{K_1}-1\right)\right\}^2} \quad . \ . \ . \ . \ . \ . \ . \ . \ (4a)$$

Letting $K = K_1$, and using eqn. $2a$, we have the key result:

$$S_K^T = (1-k) - \frac{\omega_n^2 k}{p^2+2\zeta\omega_n p+\omega_n^2}\ \frac{2m}{\omega_n}\sqrt{(1-\zeta^2)} \ (4b)$$

Choosing $k = \zeta_0^2/\zeta^2$ from eqn. $2b$ to determine the optimal transfer-function sensitivity, we obtain, using eqn. $2a$,

$$S_K^T = \frac{2m\sqrt{(1-\zeta^2)}\,p}{\omega_n\{1+(2m/\omega_n)\sqrt{(1-\zeta^2)}\}}\left(\frac{p+2\zeta\omega_n}{p^2+2\zeta\omega_n p+\omega_n}\right) \quad (4c)$$

For high Q ($Q = 1/2\zeta$) and small m/ω_n, this yields

$$S_K^T \simeq \frac{2m}{\omega_n}\ \frac{p^2+2\zeta\omega_n p}{p^2+2\zeta\omega_n p+\omega_n} \quad . \ . \ . \ . \ . \ . \ (4d)$$

and we observe

$$S_K^{T(0)} \equiv 0 \qquad S_K^{T(j\omega_n)} = \frac{2m}{\omega_n}(1+jQ) \qquad S_K^{T(\infty)} = \frac{2m}{\omega_n} \quad (4e)$$

Since $|S_K^{T(j\omega_n)}| \simeq 2m_Q/\omega_n$, we see that the transfer-function sensitivity at the band edge can be arbitrarily specified through m, though it is percentage-wise $2Q$ times that for the pole (the explanation being as in Fig. 4.6.3 of Reference 2, ω_n giving a normalisation due to the method of definition of pole-position sensitivity).

From the form of eqn. 3 we immediately see (by a change of frequency scale) that

$$S_K^Q = S_K^{\omega_n} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (5a)$$

The right-hand side of eqn. 3 yields the last of these, for $K = K_1$, as

$$S_K^{\omega_n} = k\left(\frac{\zeta^2}{\zeta_0^2}-1\right) = \frac{2mk}{\omega_n}\sqrt{(1-\zeta^2)} \simeq \frac{2m}{\omega_n} \quad (\text{for high } Q) \quad . \ . \ . \ . \ (5b)$$

In summary, the Hakim theory has all sensitivities treated as being proportional to the arbitrary parameter m, in which case any one of them can be arbitrarily assigned. The result falls in line with a result of Ur and Huang (Reference 3, p. 32) which relates pole–zero sensitivities to transfer-function sensitivities (accounting somewhat, also, for the Q factor of eqn. $4e$) and resolves the somewhat paradoxical situation created by the previous loss of the factor m in a discussion of the transfer-function sensitivity for the (generally impractical) case of $k = K$ (Reference 2, p. 129). It is also worth mentioning that the smaller one chooses m, the more of a constraint eqns. $2a$ and b place on k, with an apparent increase in the spread of element values.

The author is indebted to many collegues and students for discussions on this and other still open problems associated with the Hakim theory and its generalisations, and especially

to Prof. Wagner[4] who pointed out the factors previously dropped.

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X BAND GUNN OSCILLATORS TRIGGERED BY BASEBAND GUNN DIODES

Indexing terms: Gunn oscillators, Amplitude modulation, Trigger circuits

A 155 μm-long Gunn diode produces output pulses of 1·5 V amplitude in resistive loading. These pulses are used to trigger a transistor multivibrator and an X band Gunn oscillator.

In high-bit-rate communication systems, Gunn diodes do not show the frequency limitation of junction devices.[1] It should therefore be possible to develop p.c.m. systems with considerably increased information-flow rates with respect to presently available techniques. The new baseband pulses have to be suitable for the modulation of microwave oscillators, if, for example, some frequency or phase-shift keying is to be employed for wideband communication. This letter reports successful amplitude modulation of X band Gunn oscillators by pulse signals from another Gunn diode in resistive loading.

The baseband diode had an interelectrode distance of $l = 155\,\mu m$, the resistivity was 1·806 Ωcm and its low-field resistance was 500 Ω. The diode could only be operated under bias conditions owing to the heat dissipation problems. The bias pulse of 100 Hz repetition frequency and 220 ns duration was first applied to an integrating circuit, so that the bias frequencies were much lower than the domain-signal frequencies. The bias voltage was set at such an amplitude that a succession of domain pulses occurred at the peak value of applied bias. The Gunn diode was earthed at the anode end. The bias voltage was applied via a 50 Ω coaxial cable, whose impedance formed the series load resistance to the diode.[2] The output pulses were then taken from across the diode and separated from the bias signals via a capacitive filter. The resulting Gunn-effect pulses of 1·5 V amplitude and 750 Mbit/s were first applied to the base terminal of a fast monostable transistor multivibrator,* which was successfully triggered by individual domain pulses. This technique provides a method of studying the occurrance of individual Gunn-diode domains without the use of averaging sampling techniques. The monostable multivibrator had a pulse time constant of 10 ms, which could, of course, easily be displayed on an ordinary oscilloscope.

Subsequently the baseband signals were applied to a low-Q factor X band coaxial Gunn-diode oscillator† together with a direct bias voltage of about 5 V. The d.c. bias was adjusted in such a way that, for a particular resonator-tuning-stub position, the diode was operated near the threshold for microwave emission. A small increase in bias voltage then produces a large increase in microwave output power. The output was finally applied to the input of a sampling oscilloscope,‡ which produced a signal trace representing the X band microwave. (Of course, the 10 GHz periods could not be displayed as this was outside the range of the sampling oscilloscope.) This trace showed very low (almost-zero) microwave power, except during production of the baseband pulses. As soon as the 750 Mbit/s pulses were terminated after about 80 ns, the microwave power returned to its original low level. The microwave pulse produced on the oscilloscope was about 13 mV in amplitude. Taking into account several mismatches at transitions employed, we estimate an output power of about 10 μW. This figure is admittedly very low, but could be improved by further work, in particular with a better cavity design and matching. For the coaxial resonator employed with a Q factor of about 50, it is not to be expected that the microwave will in fact follow the exact shape of the baseband pulses. Future experiments are to be performed with lower Q factors (up to resistive loading) in order to determine ultimate modulation speeds.

It was established by a range of tests that the wave shape observed was, in fact, the baseband-modulated X band output. Reducing the microwave diode bias voltage by only a few percent caused the 80 ns trace to disappear. Equally, an increase in this bias voltage produced high continuous emission of microwaves and no further pulse trace was observed. Detuning of the X band cavity with its tuning screw also caused the pulse trace to vanish. In fact, the quality of the performance was very sensitively dependent on tuning-screw position. A reduction in baseband bias voltage of about 10% eliminated any microwave pulse trace, as no baseband domain signals were then produced, as seen on the sampling scope when fed with an inductive current probe monitoring the baseband diode current. The same effect was also observed when the earthing connection of the baseband diode was removed, so that no current was able to flow through the diode.

It should be possible now with the results obtained to set up, for example, a phase-shift-keying system without the use of additional microwave switches. The system would have to contain two microwave Gunn diodes in very low Q factor circuitry, phase locked to each other by a low background output and switched by baseband signals from long Gunn diodes. The phase shifting is obtained by employing a different length of cable for the connection from each of the microwave diodes to a common output terminal.

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* See Ferranti handbook, June 1969, p. 17
† Gunn diodes from Plessey, TEO2B
‡ Tektronix

LOSSLESS MULTIPORTS WITH TERMINATIONS IN SYNTHESIS PROBLEMS

Indexing term: Network synthesis

Use of lossless multiports with terminations in the synthesis of multivariable positive real functions is discussed. An example is included to illustrate the exploitation of the degrees of freedom in multiterminated-lossless-multiport synthesis.

Lossless networks with suitable terminations have been used in various network problems. Darlington showed[1] that, if a real rational function of a single variable can be realised by terminating a lossless 2-port by a resistor, the function is positive real. Similarly, a special case of Koga's recent general results is that any multivariable reactance function,