Linear Dynamics for Computer-Aided Nonlinear Network Analysis

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Abstract: This paper gives two main results for nonlinear time-invariant networks: 1) The fact that all dynamics can be placed in unit linear capacitors and 2) a computer oriented analysis method for the resultant resistive structure. This first results through the introduction of a nonlinear resistive 3-port while the second relies upon topological means. Taken together the results give an effective digital computer method for setting up the canonical state-variable equations of time-invariant nonlinear networks.

"and yet, through all this tangled complexity and sometimes confusion, it is impossible 'not to fall ultimately, as into a heresy, into unheard-of-simplicity'" [1, p. 46].

I. Introduction

It is well-recognized [2, p. 59] that the analysis of nonlinear networks is most often carried out in terms of the canonical set of first order state-variable equations describing the network. Such state-variable equations are of the form

$$\begin{bmatrix} \dot{\underset{\sim}{x}} \\ \underset{\sim}{y} \end{bmatrix} = \underline{f}\left(\begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{u} \end{bmatrix}\right) \qquad (1)$$

where $\underset{\sim}{x}$ is the state vector, $\underset{\sim}{y}$ the output and $\underset{\sim}{u}$ the input to the network; $\underline{f}(\cdot)$ is a, perhaps nonlinear, transformation which reflects the laws and interconnections of the network elements involved. Since in

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fact one of the heaviest uses of digital computers lies in the area of nonlinear network analysis [3, p. 75], it seems expedient to have available simple and general means of setting up these state-variable equations applicable to computer-aided analysis. And of course by extension such suitable methods can lead into the productive area of design.

To be sure there are several available techniques for setting up the state-variable equations for nonlinear networks [2, p. 64] [4, p. 196]. Here we give an alternative which in general is more applicable to obtaining the canonical equations on a computer; also it may be considered simpler to apply or to give more conceptual insight since it reduces the real analysis to that of a purely resistive network. Once this, or any other method, has been used to obtain the state-variable equations, solutions can be pursued following standard computer routines and techniques [5, pp. 1539, 1545].

In setting up state-variable equations one of the main problems is the proper isolation of derivative determining elements, these latter being called dynamical elements. In previous work [6] it has been shown how time-variable dynamical elements can be replaced by time invariant ones seen through time-variable transformers with the result conveniently leading to stability results [7] and passive characterizations [8]. For nonlinear networks it has also been shown [9] that the introduction of the mutator, the reflector, and the scalor allow for the generation of arbitrary nonlinear structures by appropriate resistive loading, while another study has investigated nonlinear resistive building blocks [10]. Here we show that any finite time-invariant nonlinear network can be considered as far as its ports are concerned as a nonlinear resistive structure loaded in linear capacitors (somewhat the converse of the result of [9]). This result is obtained by an equivalence which replaces nonlinear dynamical elements by linear ones loading a nonlinear resistive network.

The final analysis then reduces to the generally easier problem of analysis of resistive networks. From such an analysis, which can be programmed following the topologically oriented scheme discussed in Section III the canonical state-variable equations are relatively easily set up, when they exist, in a form convenient for digital computer analysis.

II. Equivalences

We first investigate an equivalence for nonlinear capacitors which gives the basic idea from which the method stems. This is followed by a duality consideration which allows for actual computer calculations on many of the resistive structures resulting upon capacitor extractions.

A. Capacitor Equivalence

If i, q, and v are the current through and the charge and voltage on a time-invariant (one-port) capacitor then, assuming appropriate differentiability,

$$i = \frac{dq(v)}{dt} = \frac{dq(v)}{dv}\cdot\frac{dv}{dt}. \qquad (2)$$

If next we consider the nonlinear 2-port resistive network described by the general description

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & \frac{dq(v_1)}{dv_1} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad (3)$$

Then we find upon loading (at port 2) in a unit capacitor and identifying $i=i_1$, $v=v_1$, that Eq. (2) is verified at the input. The process is illustrated in Figure 1.

In order to construct a device with the general description (3) it is convenient to consider a more universal 3-port called a VACCCS (voltage adjustable current controlled current source). The VACCCS is defined by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \hat{v}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ a(v_3) & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \tag{4}$$

where $a(\cdot)$ is the voltage dependent current gain. A representation of the VACCCS is given in Figure 2a) while an interconnection of its ports, as shown in Figure 2b), gives a construction of the resistive 2-port of Eq. (3).

In summary a nonlinear capacitor can be replaced by a VACCCS constructed resistive 2-port loaded in a unit capacitor. By making a matrix expansion similar to Eq. (2) the result extends to coupled and multiport capacitors.

B. Conversion to Dual Variables

To make the method to be described widely applicable it is convenient to note that a gyrator, described by

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{5}$$

and symbolized in Figure 3a), can be used to convert a load network N into its dual N^d as illustrated in Figure 3b). In the dual conversion process it is of particular importance to note the gyration conductance sign and magnitude, $g = +1$, adapted for nonlinear situations. As an illustration to make this clear, if $-i_2 = f(v_2)$ then, from Eq. (5), $v_1 = (1/g)f(i_1/g)$ which yields the dual if $g = +1$ but not in general if $g = -1$.

Since the dual of a capacitor is an inductor, Figure 4a) shows an application of Figure 3b) which allows all dynamical elements to be assumed to be capacitors. Taken in conjunction with the results of Section IIA we conclude that any dynamical time-invariant element can be represented through unit (uncoupled) capacitors loading a nonlinear and nondynamical (e.g., resistive) network. In fact for later purposes it is useful to observe that the dual transformation of Figure 3b) can be used to obtain a current controlled resistor from a voltage controlled one, as shown in Figure 4b).

With these preliminaries we can turn to the actual establishment of the state variable equations.

III. Capacitor Extractions - State-Variable Equations

Given a network constructed of a finite number of (nonlinear, time-invariant) circuit elements we can make the equivalences of the last section to remove all dynamical elements as unit capacitors. This yields a resistive network loaded in unit capacitors, as shown in Figure 5a), which is equivalent at its ports to the original configuration.

For concreteness of the treatment let it be assumed that what is of interest is voltage transfer from a set of first n ports to a second set of m ports; the external ports can then be partitioned as shown in Figure 5b). We will further assume that the hybrid equations

$$\underset{\sim}{H}\left(\begin{bmatrix} \underset{\sim}{v}_1 \\ \underset{\sim}{i}_2 \\ \underset{\sim}{v}_3 \end{bmatrix}\right) = \begin{bmatrix} \underset{\sim}{i}_1 \\ \underset{\sim}{v}_2 \\ \underset{\sim}{i}_3 \end{bmatrix} \tag{6}$$

exist. Sufficient conditions for the existence of the nonlinear hybrid transformation $\underset{\sim}{H}(\cdot)$ are available [11, Thm. 4]. Along these lines, if "the Jacobian matrix of $\underset{\sim}{H}(\cdot)$ is uniformly positive definite" then all other hybrid descriptions exist [11, Thm. 3]; for example, the admittance transformation may always be obtained from any other hybrid transformation under the uniformly positive definite assumption.

Setting $\underset{\sim}{x} = \underset{\sim}{v}_3$ for the state k-vector of Figure 5b), with $\underset{\sim}{1}_k$ denoting the k x k identity, we find the state-variable equations as

$$\begin{bmatrix} \dot{\underset{\sim}{x}} \\ \\ \underset{\sim}{v}_2 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{0} & -\underset{\sim}{1}_k \\ \\ \underset{\sim}{0} & \underset{\sim}{1}_m & \underset{\sim}{0} \end{bmatrix} \underset{\sim}{H}\left(\begin{bmatrix} \underset{\sim}{v}_1 \\ \underset{\sim}{0} \\ \underset{\sim}{x} \end{bmatrix}\right) \qquad (7)$$

That is, the state-variable equations result from Eq. (6) by ignoring the first n rows, identifying capacitor voltages as the state, and setting the final m port constraints $\underset{\sim}{i}_2 = \underset{\sim}{0}$. We have also transposed the final m and k rows, after multiplication of the latter by minus one, in order to place the equations in the canonical form of Eq. (1) [12, p.40].

As in the linear case [13], the problem of setting up the state-variable equations is reduced to that of finding a hybrid description of a resistive network. Using topological means the analysis to determine this hybrid description is relatively easily formulated [14, p.51] and in fact in a form suitable for programming on a computer, though presently available routines for this type of one element kind network are time consuming [5, p.1544]. Here we describe one possible technique which is particularly suitable for digital computation determination of $\underset{\sim}{H}(\cdot)$.

In many instances internal components can be assumed to have a resistance (current controlled) description, perhaps by the use of gyrator determined transformations as in Figure 4b). Further the insertion of m unit gyrators at the second set of ports (output) allows $\underset{\sim}{H}(\cdot)$ to be considered an admittance as Eq. (6) shows (the unit gyrators set $\underset{\sim}{i}_2 = \hat{\underset{\sim}{v}}_2$, $\underset{\sim}{v}_2 = \hat{\underset{\sim}{i}}_2$ or $\underset{\sim}{H}(\hat{\underset{\sim}{v}}) = \hat{\underset{\sim}{i}}$ if we also set $\underset{\sim}{v}_1 = \hat{\underset{\sim}{v}}_1$, $\underset{\sim}{v}_3 = \hat{\underset{\sim}{v}}_3$, $\underset{\sim}{i}_1 = \hat{\underset{\sim}{i}}_1$, $\underset{\sim}{i}_3 = \hat{\underset{\sim}{i}}_3$ and collect these vectors in $\hat{\underset{\sim}{v}}$ and $\hat{\underset{\sim}{i}}$). Thus, for analysis purposes we wish to assume voltage sources $\hat{\underset{\sim}{v}}$ applied to the (n+m+k) ports of the network determining $\underset{\sim}{H}(\cdot)$. Letting $\underset{\sim}{e}$ be the vector of total branch voltages, as shown for any branch in Figure 6, $\underset{\sim}{i}_b$ and $\underset{\sim}{v}_b$ be the branch element current and voltage and $\underset{\sim}{v}_s$ the branch source voltages (polarity as in Figure 6) we obtain

$$\underset{\sim}{e} = \underset{\sim}{v}_b - \underset{\sim}{v}_s = \underset{\sim}{\mathfrak{R}}(\underset{\sim}{i}_b) - \begin{bmatrix} \underset{\sim}{0} \\ \hat{\underset{\sim}{v}} \end{bmatrix} \tag{8}$$

Here $\underset{\sim}{\mathfrak{R}}(\cdot)$ is a nonlinear (branch by branch) resistance transformation defined by the (current controlled) circuit components, while we assume the applied voltage source branches to be numbered last. By a proper numbering and application of Kirchhoff's laws we can obtain, with $\underset{\sim}{\mathcal{T}}$ the tie-set matrix [15, p.29]

$$\underset{\sim}{\mathcal{T}}\,\underset{\sim}{e} = \underset{\sim}{0} \qquad \text{(Kirchhoff Voltage Law)} \tag{9a}$$

$$\underset{\sim}{i}_b = \tilde{\underset{\sim}{\mathcal{T}}}\,\underset{\sim}{i}_\ell \qquad \text{(Kirchhoff Current Law)} \tag{9b}$$

Here $\underset{\sim}{i}_\ell$ is the set of link currents and the tilde denotes matrix transposition; by assuming the links for final numbers the tie set matrix has its last ℓ(= number of links) columns as the identity [15, p.30]

$$\underset{\sim}{\mathcal{T}} = [\underset{\sim}{T} \,\vdots\, \underset{\sim}{1}_\ell] \tag{9c}$$

If we also (reasonably) assume that all sources, $\hat{v}_j$, are independent [that is, there are no more sources than links], Eqs. (9) applied to Eq. (8) yield

$$\begin{bmatrix} \underset{\sim}{0} \\ \hat{\underset{\sim}{v}} \end{bmatrix} = \underset{\sim}{\mathcal{T}}\,\underset{\sim}{\mathfrak{R}}(\tilde{\underset{\sim}{\mathcal{T}}}\,\underset{\sim}{i}_\ell) = \hat{\underset{\sim}{\mathfrak{R}}}(\underset{\sim}{i}_\ell) \tag{10}$$

which serves to define the transformation $\hat{\underset{\sim}{\mathfrak{R}}}(\cdot)$, $\hat{\underset{\sim}{\mathfrak{R}}}(\cdot) = \underset{\sim}{\mathcal{T}}\,\underset{\sim}{\mathfrak{R}}(\tilde{\underset{\sim}{\mathcal{T}}}\cdot)$, from link currents into link voltages. Now the port currents $\hat{\underset{\sim}{i}}$ are the final entries of $\underset{\sim}{i}_\ell$ so we can represent $\underset{\sim}{\mathcal{H}}(\cdot)$, which is numerically equal to the admittance under calculation, as the final portion of the transformation inverse to $\hat{\underset{\sim}{\mathfrak{R}}}(\cdot)$; symbolically

$$\hat{\underset{\sim}{i}} = [\hat{\underset{\sim}{\mathfrak{R}}}^{-1}(\begin{bmatrix} \underset{\sim}{0} \\ \hat{\underset{\sim}{v}} \end{bmatrix})]_2 = \underset{\sim}{\mathcal{H}}(\hat{\underset{\sim}{v}}) \tag{11}$$

where the subscript 2 denotes the "lower" portion.

For digital calculation $\mathfrak{J}$, $\underline{\mathfrak{R}}(\cdot)$ and $\hat{\underline{\mathfrak{R}}}(\cdot)$ are relatively easily formulated while the inverse $\hat{\underline{\mathfrak{R}}}^{-1}(\cdot)$, is the most difficult step; but it has been pointed out that the method of Broyden is available and very convenient [16, p. 1821].

IV. Examples

To indicate the procedures we investigate two examples.

First, to illustrate the linear capacitor extraction scheme we consider the equivalent circuit of a tunnel diode as shown in Figure 7a) [17, p. 1916]. The diode conductance $G_D(\cdot)$ represents a voltage-controlled nonlinear time-invariant purely resistive element given by the current-voltage characteristic $i_d = G_d(v_d)$. As explained under Section IIB), through the introduction of the gyrator, this voltage-controlled element can be converted to its current-controlled dual, $v_D = G_d(i_D)$, as needed for the branch by branch resistance transformation $\mathfrak{R}(\cdot)$, while in the same way the linear series inductor, L_s, can be converted to its capacitor dual. The equivalence of the nonlinear capacitor [17, p. 1916], described by $q(v_d) = C_d(v_d)\cdot v_d = Kv_d/(V_j - v_d)^N$ with K,N,V_j constants, is obtained as a current-controlled nonlinear resistive 2-port loaded in a unit linear capacitor through the use of Figures 1,2,3b). The final tunnel diode equivalent appropriate for the analysis method is then shown in Figure 7b).

As a second example, to illustrate the setting up of the canonical state-variable equations, we consider the simple single-input single-output circuit shown in Figure 8a) where the nonlinear resistive element is as denoted in Figure 4b), and the linear capacitor is assumed of unit capacitance by normalization; the resistor r is assumed linear. After appropriate conversions the circuit's equivalent is shown in Figure 8b), where branch numberings are also indicated. A suitable graph is given in Figure 8c) where darkened branches belong to the chosen tree. From the graph the tie-set matrix is easily determined and the branch by branch impedance results directly from Figure 8b); thus

$$\mathfrak{J} = \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathfrak{R}(\cdot) = \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & f(\cdot) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12a}$$

where f(·) operates only on the variable in its column, the other entries following standard linear matrix algebra rules. Combining gives

$$\hat{\mathfrak{R}}(\cdot) = \mathfrak{J}\mathfrak{R}(\tilde{\mathfrak{J}}\cdot) = \begin{bmatrix} f(\cdot) & 1 & -1 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & r & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12b}$$

from which the inverse is calculated as

$$\hat{\mathfrak{R}}^{-1}(\cdot) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{1}{r} & 0 & -\frac{1}{r} \\ 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & -\frac{1}{r} & -1 & \left\{\frac{1}{r}+f(\cdot)\right\} \end{bmatrix} \tag{12c}$$

As explained in Section III, Eq. (11), the final portion of the transformation inverse to $\hat{\mathfrak{R}}(\cdot)$ is $\mathfrak{H}(\cdot)$. Therefore, since the first two link source voltages are zero,

$$\mathfrak{H}(\cdot) = \begin{bmatrix} \frac{1}{r} & 0 & -\frac{1}{r} \\ 0 & 0 & 1 \\ -\frac{1}{r} & -1 & \left\{\frac{1}{r}+f(\cdot)\right\} \end{bmatrix} \tag{12d}$$

On next performing the operations of Eq. (7) we finally arrive at the canonical equations

$$\dot{x} = -\frac{x}{r} - f(x) + \frac{1}{r}v_1 \tag{12e}$$

$$v_2 = x \tag{12f}$$

These equations can easily be checked by direct inspection of Figure 8a), but note that the procedure we have used gives a systematic method suitable for general analysis.

V. Discussion

By using a capacitor extraction the state-variable equations for a voltage transfer network have been formulated, Eq. (7), through a hybrid description of the nonlinear coupling resistive network. The result is valid for finite time-invariant networks for which the decomposition of Eq. (2) is true. Although the same technique applies to linear time-variable networks its extension to nonlinear time-variable structures does not seem apparent because of an added term $\partial q(v,t)/\partial t$ needed for Eq. (2).

Of interest then is the general result that for finite nonlinear time-invariant networks the dynamics can be assumed placed entirely in unit (linear) capacitors, this result being that upon which the theory of state-variables for nonlinear structures has always rested. Thus, it is no surprise that using this extraction we are able to obtain the canonical state-variable equations, when they exist. For sure these canonical equations need not always exist, as is readily seen by replacing the voltage controlled "resistor" in Figure 8a) by a current controlled one. Nevertheless, when applicable, in contrast to more classical approaches, the method presented reduces in the end to an analysis of purely resistive circuits to which the topological formulation indicated is relatively easy to apply. For sure we have only outlined the ideas for voltage to voltage transfer, Figure 5), but other situations are handled in like manner, generally through the use of some other hybrid description. In the text we have set up the analysis by treating the hybrid matrix as an admittance through appropriate gyrator insertions. In general this is actually the most convenient for setting up all-purpose routines, but in special cases it may be more convenient to omit the extra gyrators by calculating $\underset{\sim}{H}(\cdot)$ as an actual hybrid transformation. To be sure there are situations, as in fact the second example, where the state-variable equations can be found almost by inspection. But the methods used in such instances do not, as yet anyway, seem programmable to handle general structures. Hence the importance of the method discussed, which in fact uses less storage than at first appears necessary, due to the sparsity of the matrix transformation.

The capacitor equivalence is easily formulated in terms of the VACCCS for which one would desire a suitable electronic circuit. But the problem of physically constructing a VACCCS is compounded on two accounts. The first involves the desire for a floating short circuit, port 1 of Figure 2a); integrated structures for a VACCCS having all ports with a common ground are relatively easily formulated, but the floating port requirement, though manageable by the use of gyrators, yiêlds unwieldy circuits. Nevertheless any given structure yields a specific gain function a(·), and it does not seem as yet possible to devise a means of obtaining an arbitrary a(·); this is the more comprehensive second difficulty.

As with the previous time-invariant method [13], and as discussed here, the primary advantages of the technique presented seem to be in the computer aided analysis and design fields where the calculation and specification of the resistive portion is comparatively straight-forward. For example, recent steady state analysis using computerized optimization techniques [18] can readily be applied. Nevertheless, future use for the synthesis of nonlinear networks seems quite promising.

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"We do not need theories so much as the experience that is the source of the theory." "And any theory not founded on the nature of being human is a lie and a betrayal of man" [1, pp. 17,53].

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VI Acknowledgments

The authors are indebted for the support of this work to the Air Force Office of Scientific Research under Contract AFOSR F44620-67-C-0001 and the National Science Foundation under Grant NSF GK-237. The participation of the second author has been made possible through the UNESCO Educational Aid Program to the Faculty of Engineering, University of Lagos. Finally the assistance of Mary Anne Poggi in processing the paper is acknowledged.

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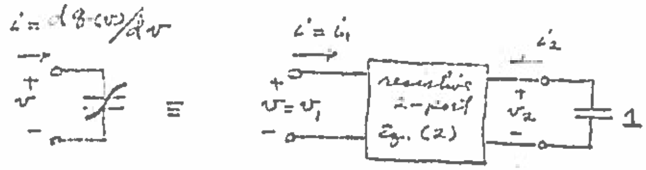
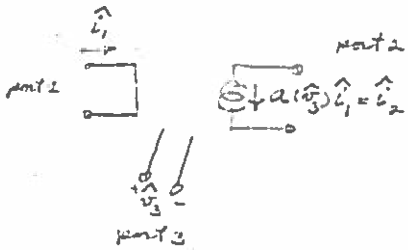
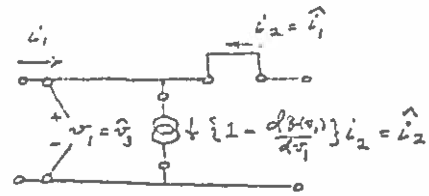


Figure 1
Nonlinear Capacitor Equivalence



a)
VACCCS



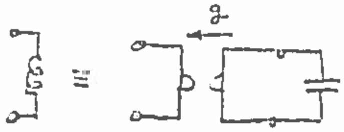
b)
Circuit for Eq. (3)

Figure 2
VACCCS and Its Use

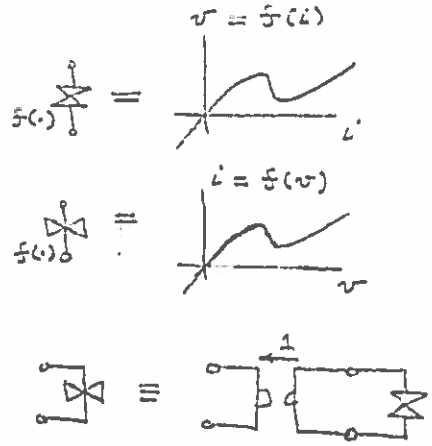
a) *Gyrator*

b) *Dual Transformation*

Figure 3
Gyrator Symbolization and Usage



a)
Inductor Replacement



b)
Transformation of
i = f(v) into v = f(i)

Figure 4
Specific Dualities

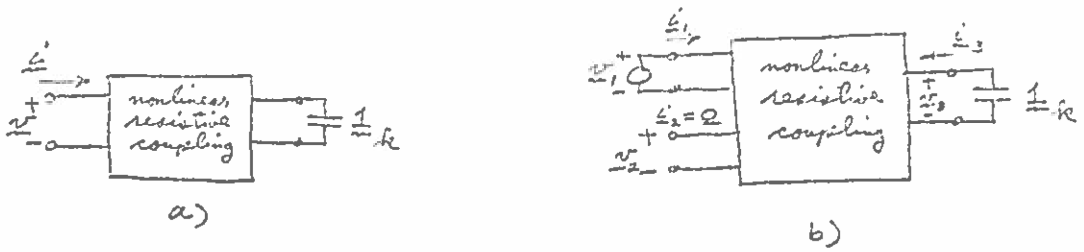


Figure 5
Unit Capacitor Extractions

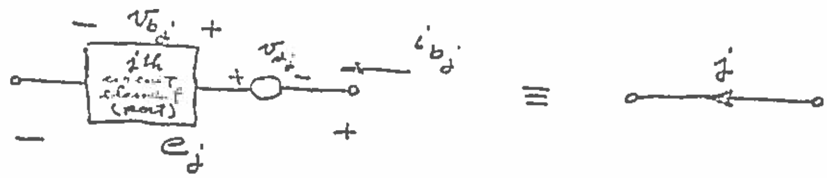
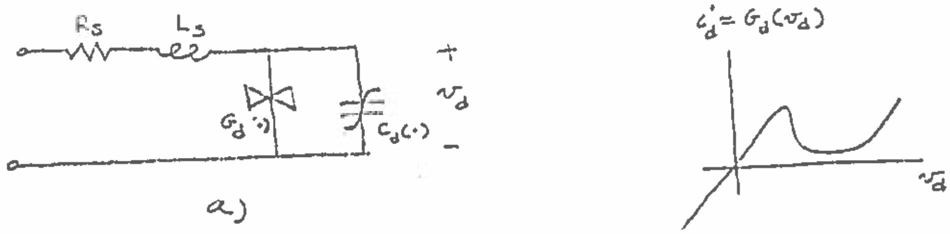
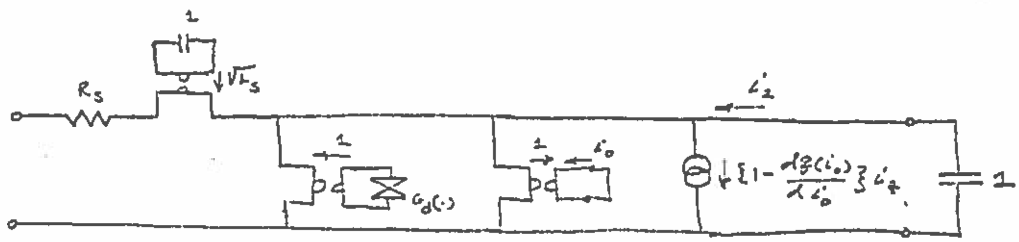


Figure 6
Topological Branch
Conventions



a)

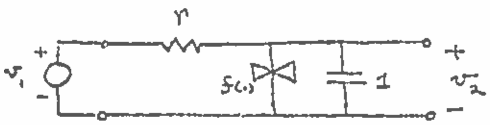
Normal Equivalent



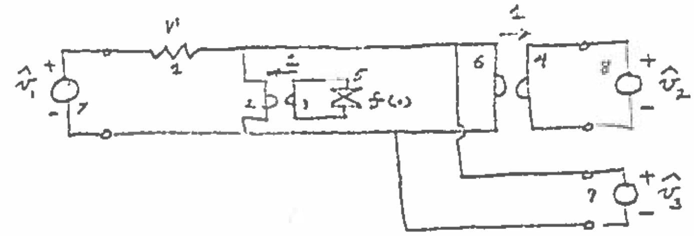
b)

Computer Appropriate Equivalent

Figure 7
Tunnel Diode Equivalent
Transformation



a)
Example Circuit



b)
Resistive Network for analysis

c)
Network Graph

Figure 8
Second Example