Our interest here is in the variation of G with $\Delta\beta/2$. Three conditions can be identified.

1) $\Delta\beta/2=0$. Maximum gain is obtained.

$$G = \cosh^2 gl$$
$$\doteq 1 + (gl)^2 \quad \text{for} \quad (gl)^2 \text{ small}$$

2) $|\Delta\beta/2|=g$. This is the boundary between two distinct regions. When $|\Delta\beta/2|<g$ the solutions are as above, with hyperbolic functions in sl. If $|\Delta\beta/2|>g$ then s is imaginary and the hyperbolic functions are replaced by trigonometrical functions. The result is that for $|\Delta\beta/2|<g$ the signal increases exponentially whereas for $|\Delta\beta/2|>g$ it varies periodically. Thus the condition $|\Delta\beta/2|=g$ defines the limit of exponential gain and has been called the gain threshold. The error that had been made in the past[1] was to assume that, at this condition, there was no excess gain ($G=1$). In fact the gain is not unity but will be given by $G=1+(gl)^2$. Therefore in the limiting case of $(gl)^2$ small there is negligible difference between the gain here and that for the exact match ($\Delta\beta/2=0$). The only conditions for which $G=1$ are as follows:

3) $|s|l=q\pi$. q is an integer. The region within $|s|l<\pi$ is the useful region for gain. For small values of $(gl)^2$

$$|s|l \doteq |\Delta\beta/2| = q\pi.$$

From the above analysis it is seen that the region of exponential gain is limited by the value of g^2 which can be obtained. However, there is still significant gain when $|\Delta\beta/2|>g$, particularly when the maximum gain is small. This latter situation is of importance for optical parametric amplifiers and oscillators.

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[1] We believe that many authors were misled by an incorrect statement by Tien in his classic paper[2] on travelling-wave parametric devices. On p. 1353 of his paper he implied that only the $\cosh^2 sl$ term is needed to describe gain even when $|\Delta\beta/2|\neq 0$. It should be noted that all his other equations are correct and, in particular, our expression for power gain can be obtained from eq. (35) of his paper.

[2] P. K. Tien, "Parametric amplification and frequency mixing in propagating circuits," *J. Appl. Phys.*, vol. 29, pp. 1347–1357, September 1958.

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Adjustable Distributed All-Pass Networks for Time Delay Applications

***Abstract*—A cascade of nonreciprocal all-pass networks, each consisting of a variable gyrator bridged by a transmission line, is used to produce an adjustable ideal delay. The total delay is adjusted in discrete increments by electrically varying the conductances of individual gyrators in the cascade.**

INTRODUCTION

A nonreciprocal all-pass network consisting of a time-invariant gyrator bridged by variable lumped reactance elements was described in an earlier work [1]. Since the all-pass parameter can be adjusted electrically by changing the effective values of the reactance elements, this basic network provides a convenient and direct means of generating a continuously adjustable delay over a limited frequency band.

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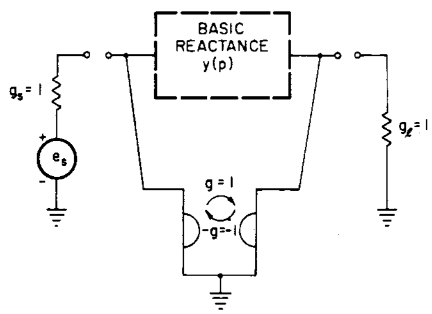

Fig. 1. Nonreciprocal all-pass network.

In an attempt to extend the operating frequency range of the actual delay function for this specific all-pass configuration, the frequency sensitive elements (the lumped reactances) were replaced by broad-band elements (distributed reactances). It was discovered that the modified network could produce ideal delay (a flat delay function extending over an infinite frequency band), but that the effective value of the distributed reactance could not be adjusted electrically without destroying the ideal character of the delay function. Therefore, this basic all-pass network (invariant gyrator, variable reactance) failed to produce adjustable ideal delay, and it was necessary to examine a completely different all-pass configuration (variable gyrator, invariant reactance).

The analysis of this alternate configuration showed that a cascade of such networks does yield an adjustable ideal delay. The total delay through the cascade can be adjusted in discrete increments by electrically varying the individual gyrator conductances, which have the effect of "switching" the corresponding delay section in or out of the cascade. In the following, the key steps in the development of this concept are summarized.

NONRECIPROCAL ALL-PASS STRUCTURE

The all-pass network [1], shown in Fig. 1, was derived from the scattering matrix

$$S(p) = \begin{bmatrix} s_{11}(p) & s_{12}(p) \\ s_{21}(p) & s_{22}(p) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{y(p)-1}{y(p)+1} & 0 \end{bmatrix} \tag{1}$$

where $y(p) = -y(-p)$ is a reactance constructed from lumped elements. In those situations where fabrication procedures permit the use of lossless distributed elements (such as strip lines) the driving-point reactance $y(p)$ can be produced by an LC transmission line of length d terminated by a short (or an open) circuit. For the case of a short circuit termination, the transmission line input admittance is [2, p. 350]

$$y(p) = \frac{\cosh p\theta}{z_0 \sinh p\theta}, \qquad \theta = \sqrt{LC}d \tag{2}$$

where z_0 is the characteristic impedance, and L and C are inductance and capacitance per unit length. Substituting (2) into (1), the all-pass transfer functions are

$$s_{12}(p) = 1$$

and

$$s_{21}(p) = \frac{\cosh p\theta - z_0 \sinh p\theta}{\cosh p\theta + z_0 \sinh p\theta}. \tag{3}$$

The characteristic impedance can be normalized to unity by a proper choice of circuit values, so that the phase and delay functions [3, p. 24] of $s_{21}(j\omega)$ are, respectively,

$$\phi_{21}(\omega) = -2\theta\omega$$

and

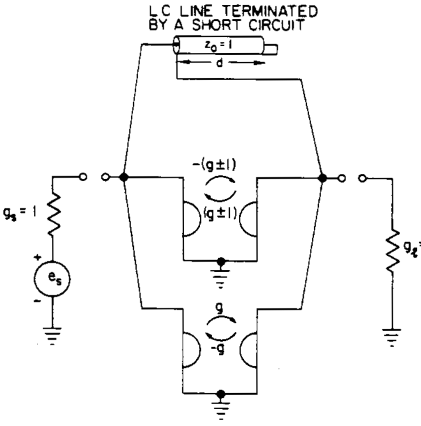

Fig. 2. Adjustable nonreciprocal delay network.

$$\tau_{21}(\omega) = \frac{-d\phi(\omega)}{d\omega} = 2\theta. \tag{4}$$

Of course, the phase and delay functions of $s_{12}(j\omega)$ are both zero. To this point an all-pass network described by (1) and (2) has been obtained which produces an ideal delay for signals passing from port one to port two, but acts as a direct connection (no delay) for signals passing from port two to port one. Thus, a transmission delay for signals passing through the network described by (1) is available which is either 0 or 2θ: one value is obtained from the other by physically transposing the network ports, i.e., reversing the terminal connections of the circuit in Fig. 1. Therefore, this simple network can be used to switch a fixed transmission delay in and out of a larger network merely by reversing the ports.

A relay or other electrically operated switch could be added to the circuit in Fig. 1 to perform this function. However this addition is not necessary since the entire network in Fig. 1 can be reversed electrically by reversing only the gyrator, i.e., changing the polarity of the gyrator conductances. The network is all-pass, so that several of these networks can be cascaded without difficulty to produce a larger total delay (the delays produced by the sections add directly), and the magnitude of the total delay can be adjusted in increments by reversing the gyration conductance polarity of one or more of the all-pass sections in the overall cascade.

To assemble a working circuit, one must note that it is inconvenient to reverse the polarity of gyration conductances in currently available gyrators. However, these difficulties can be circumvented as shown in Fig. 2 by using two gyrators in parallel to produce an equivalent gyrator with electrically reversible gyration conductance polarities. This is possible if a gyrator of conductance $g > 1$ is paralleled with a reversed gyrator of variable conductance which assumes either of the two values $g+1$ or $g-1$. Thus, the equivalent gyration conductance can be switched between $+1$ and -1 by adjusting the "reversed" gyrator between $g+1$ and $g-1$. Since changes in gyration conductance can be produced by changes in the bias voltages of field effect transistors, the switching speed of this network can be very fast.

As an example of the overall delay circuit operation, if four transmission lines are chosen with the parameters of the four lines normalized so that $\theta = 0.5$, 1.0, 2.0, and 4.0 and $z_0 = 1$, and if each of the four lines is connected across a parallel gyrator pair to produce four nonreciprocal all-pass sections (a single section is shown in Fig. 2), then (4) shows that the four sections can generate (normalized) delays of 1.0, 2.0, 4.0, and 8.0, respectively. A cascade of these four sections permits a total ideal delay ranging from 0 to 15 (in integer steps) to be achieved. The magnitude of this delay is adjusted in increments between these limits by adjusting the appropriate gyration conductances so that the corresponding delay section is switched in or out of the overall circuit. For instance, if the second and fourth sections are biased "forward" with the first and third "reversed," then the total delay through the cascade is $0+2+0+8=10$. The delay can be reduced to 9 by changing the first section to "forward" and "reversing" the second section $(1+0+0+8=9)$.

We remark that nonreciprocal all-pass networks with fixed parameters have been described by several researchers [4]–[7]. However, it appears that studies of nonreciprocal all-pass networks with variable parameters are limited [1], [8]. A properly developed theory should play a key part in synthesizing time-varying networks.

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REFERENCES

[1] J. C. Woodard and R. W. Newcomb, "Adjustable all-pass network suitable for phase shifting and time delay applications in integrated circuits," *IEEE Trans. Industrial Electronics and Control Instrumentation*, vol. IECI-15, pp. 17–21, November 1968.
[2] L. A. Pipes, *Matrix Methods for Engineering*. Englewood Cliffs, N. J.: Prentice-Hall, 1963.
[3] E. S. Kuh and D. O. Pederson, *Principles of Circuit Synthesis*. New York: McGraw-Hill, 1959.
[4] D. Hazony, *Elements of Network Synthesis*. New York: Reinhold, 1963, p. 124.
[5] K. L. Su, *Active Network Synthesis*. New York: McGraw-Hill, 1965, p. 278.
[6] R. W. Newcomb, *Linear Multiport Synthesis*. New York: McGraw-Hill, 1966, p. 148.
[7] A. W. Keen, "Active and nonreciprocal networks," in *Network and Switching Theory*, G. Biorci, Ed. New York: Academic Press, 1968.
[8] R. W. Newcomb, *Active Integrated Circuit Synthesis*. Englewood Cliffs, N. J.: Prentice-Hall, 1968.

Direct Determination of the Pinch-Off Voltage of a Depletion-Mode Field-Effect Transistor

***Abstract*—A direct method of determining V_p for a depletion-mode FET is presented and compared with the values determined from a square-law transfer characteristic, the output characteristic, and the cutoff of drain current. The accuracy of the direct method makes it a useful technique for the screening and matching of devices.**

INTRODUCTION

The accurate determination of the pinch-off voltage directly from the output characteristics of an FET is difficult since neither drain current saturation nor cutoff is sharply defined in terms of V_p. An alternate approach suggested by Richer and Middlebrook [1] is to assume a power-law relationship for the transfer characteristic in the pinch-off region. This yields a linear relationship between the ratio I_D/g_m and V_{GS}, such that the pinch-off voltage is given by the intercept of the line with the V_{GS} axis. Although this method yields quite accurate results, especially from an engineering applications point of view, it requires the measurement of an incremental quantity g_m at several operating points and is therefore somewhat cumbersome.

Richman [2] has suggested a simplified graphical technique which is based on an assumed square-law transfer characteristic in the pinch-off region of operation. Thus a plot of $\sqrt{I_D}$ versus V_{GS} yields a zero-current intercept equal to V_p. The assumption of a square-law transfer characteristic for all FET's has been justified numerically by Sevin [3], by the results of the power-law analysis [1], which yielded an exponent near 2 in all cases, and by Middlebrook's [4] approximate derivation of the junction FET characteristics.

A fourth technique [5], generally used by manufacturers to produce data-sheet information, is to define V_p (or $V_{GS(\mathrm{off})}$) as the value of V_{GS} that reduces the drain current in the pinch-off region to some small fixed value (typically 1.0 μA or 0.1 μA). This measurement requires the adjustment of V_{GS} while I_D is monitored for the desired value.

DIRECT MEASUREMENT OF THE PINCH-OFF VOLTAGE

The basis for the direct measurement of the pinch-off voltage of a depletion-mode FET (junction or MOS) is the behavior of the channel depletion layer under the condition of a reverse bias on the gate, applied between drain and gate, with the source terminal open-circuited. If gate leakage

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