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NATURAL FREQUENCIES OF TUNNEL DIODES WITH PARASITICS

by

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ABSTRACT

The regions of allowed natural frequencies in Re p $\geq$ 0 are determined for two different circuits which correspond to linearized equivalent circuits for tunnel diodes. Passive networks to obtain any possible natural frequency are given. Multiple-diode circuits are treated, and for identical diodes it is shown that the best results occur with two diodes.

CONTENTS

ILLUSTRATIONS

I. INTRODUCTION

In a recent paper Kinariwala [Ref. 1] found the bounds on the natural frequencies obtainable from a single tunnel diode embedded in a lossless but passive network. In obtaining these results the linearized equivalent circuit was assumed to be a negative resistance in parallel with a capacitance. Here we begin by considering essentially the same problem, except that more general linearized equivalent circuits are used. Two such circuits are considered. The first adds a series (loss) resistance to the parallel resistance-capacitance circuit, while the second adds a series (lead) inductance. The results are then extended to two or more diodes, with typical curves and passive networks being presented for the case of identical diodes.

The general idea of the method is as follows. After a convenient normalization the results of Desoer and Kuh [Ref. 2] are used to determine the p_o for which $q_+(p_o) \le 0$ with Re $p_o > 0$. With the addition of degenerate points this is the region for which the diodes can be embedded in a passive network N_P to obtain a natural frequency at p_o. For n identical diodes it is shown that n = 2 yields all possible natural frequencies. For one or two diodes N_P can be synthesized by previously developed techniques [Ref. 3]*, and N_P can be chosen lossless [Ref. 4]. In the case of two diodes this lossless N_P may contain a gyrator.

*On p. 165, Ref. 3, case 1, $Y_{RS} = 0$ needs modification since b_1 and b_2 are arbitrary in contrast to the statement made. If $b_1 \neq 0$, the method of case 3_a can be used. On p. 172 another instance occurs in case 4_{c1} when $q_+ = 0$ with $\epsilon = 1$. This requires $b_1^2 = b_2^2$. Synthesis results by connecting port 2 to port 1 if $b_2 = -b_1$; if $b_1 = b_2$, connect $Y_P = gE$, $g = \sqrt{1 + b_1^2}$. E is the 2x2 skew-symmetric matrix with unit entries.

II. NATURAL FREQUENCIES

We review here the concepts of "natural frequency" and "active at p_o," upon which the remainder of the paper is based. However, in order to cover degeneracies, the definitions are slight extensions of those previously available. For convenience we will work on the impedance basis, a situation which is dual to that treated in the literature [Refs. 2,3]. For the reasons mentioned in the conclusions, we limit the treatment to $p = \sigma + j\omega$ with $\sigma > 0$.

An n-port N_o is said to possess a short-circuit natural frequency p_o, Re $p_o > 0$, if some nonzero current of the form $\underline{i}(t) = \text{Re } Ie^{p_o t}$ can flow into N_o when $\underline{v}(t) = 0$ appears across its terminals, see Fig. 1(a). The modifier "short circuit" comes from $\underline{v}(t) = 0$. Here I is an n-vector of complex constants and $-\infty < t < \infty$. It should be noted that N_o need not be linear, finite, or time-invariant, and that every p_o is a short-circuit natural frequency for a short circuit. Open-circuit natural frequencies are defined in an exactly dual manner. For brevity we will simply refer to either a short-circuit or an open-circuit natural frequency as a *natural frequency*. If N_o is linear and has an impedance matrix $Z_o(p)$, a necessary and sufficient condition for p_o in Re $p_o > 0$ to be a short-circuit natural frequency is det $Z_o(p_o) = 0$ ("det" is written for determinant).

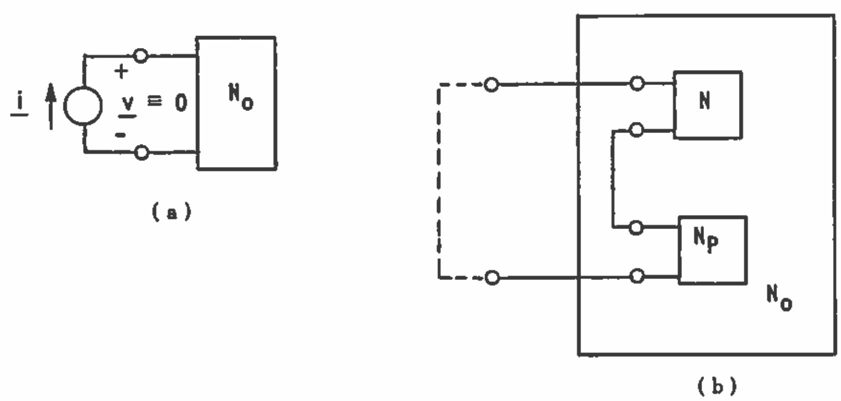

FIG. 1. NETWORKS FOR DEFINING "ACTIVE AT p_o" WHERE p_o IS A SHORT-CIRCUIT NATURAL FREQUENCY IF $\underline{i}(t) = \text{Re } Ie^{p_o t}$ IN (a).

An n-port N is called *active at* p_o, Re $p_o > 0$, if there is a passive network N_P such that the series combination of N and N_P has a natural frequency at p_o, see Fig. 1(b) [Ref. 2, p. 418]. Clearly then, if N and

N_P possess impedance matrices Z and Z_P which are analytic at p_o, a necessary and sufficient condition for N to be active at p_o is $\det[Z(p_o) + Z_P(p_o)] = 0$.

Assuming that N is a finite (lumped) network possessing an impedance matrix which is analytic at p_o, Desoer and Kuh [Ref. 2, p. 427] have proven that if N is active at p_o, then $q_+(p_o) \leq 0$, where

$$q_+(p) = \min_{\tilde{I}^* I = 1} \begin{cases} \tilde{I}^* Z_H(p) I + \frac{\sigma}{|p|} \left|\tilde{I} Z(p) I\right| & \text{if } \omega > 0 \\ \tilde{I}^* Z_H(p) I & \text{if } \omega = 0 \end{cases} \tag{1}$$

In this equation, I is again a vector of complex constants; a superscript tilde represents matrix transposition; a superscript asterisk represents complex conjugation; Z_H is the Hermitian part of Z, i.e., $2Z_H = \tilde{Z}^* + Z$; and $| \; |$ represents the absolute value. The region $\omega < 0$ need not be considered since $Z^*(p) = Z(p^*)$ in Re $p > 0$ is assumed. If Z is not analytic at p_o, N already possesses an open-circuit natural frequency at p_o (under the assumption of a finite N). If a finite N is a one or two-port, it is known that $q_+(p_o) \leq 0$ is a sufficient condition for N to be active at p_o [Ref. 3]. This last result is also known to hold for many n-ports and appears to be valid for all n-ports [Ref. 5*].

Treating N as a network of uncoupled tunnel diodes, we will apply the above facts and search for the region in which $q_+ \leq 0$, as well as for the poles of Z.

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*The known n-port synthesis is contained on pp. 32-33, Ref. 5. The theory presented in pp. 1-22 is more readily available in Ref. 3, which contains corrected examples.

III. EQUIVALENT CIRCUITS

The equivalent circuits to be used for a diode biased in the active region are shown in Fig. 2. In (a) the resistance R_s is the series resistance of the diode; in practice $R_s << R$ [Ref. 6] In many cases the series lead inductance, mostly due to mounting, is important and (b) then becomes of interest. For clarity of presentation it is convenient to

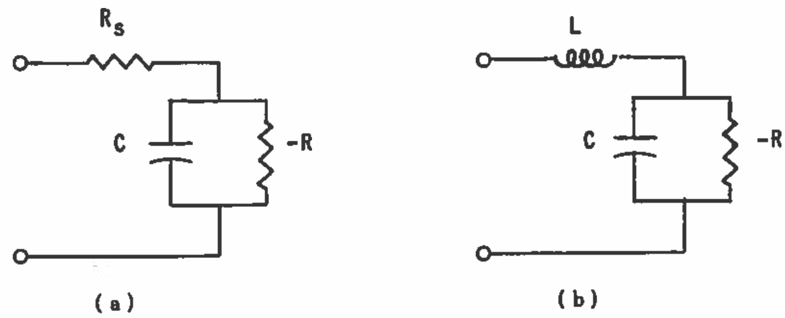


FIG. 2. ACTUAL EQUIVALENT CIRCUITS.

normalize the circuits of Fig. 2. By impedance scaling and frequency normalization we can bring both R and C to unity, the process being the same for both circuits. We then have for the normalized circuits of Fig. 3

$$s = \frac{1}{RC} p, \quad \ell = \frac{L}{R^2C}, \qquad r = \frac{R_s}{R} \tag{2}$$

where s is the actual frequency, and p the normalized frequency. The impedances for the circuits of Fig. 3 are

$$z_a = r + \frac{1}{p-1} = r + \frac{\sigma-1}{(\sigma-1)^2+\omega^2} + j\,\frac{(-\omega)}{(\sigma-1)^2+\omega^2} \tag{3a}$$

$$z_b = \ell p + \frac{1}{p-1} = \ell\sigma + \frac{\sigma-1}{(\sigma-1)^2+\omega^2} + j\left[\ell\omega - \frac{\omega}{(\sigma-1)^2+\omega^2}\right] \tag{3b}$$

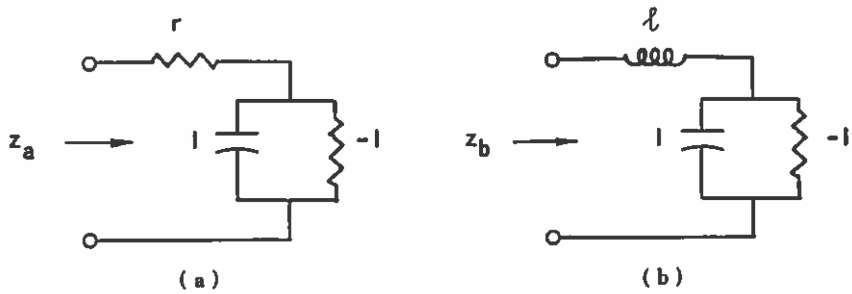


FIG. 3. NORMALIZED EQUIVALENT CIRCUITS.

IV. SINGLE DIODE N

In this section we will determine the active regions and the corresponding passive networks, assuming a single diode.

We first note that the diode is active at p = 1 since it has an open-circuit natural frequency at that point. Since z_a and z_b are analytic at all other p, we next determine where $q_+ \leq 0$.

For $\omega > 0$, q_+ for a one-port can be written as

$$q_+ = \text{Re } z + |\text{Re } z| \sqrt{\frac{1 + (\text{Im } z/\text{Re } z)^2}{1 + (\omega/\sigma)^2}} \tag{4}$$

if Re z $\neq$ 0; here z is written place of Z for a one-port. If Re z = 0, then $q_+ \leq 0$ requires Im z = 0 also, and in fact $q_+ = 0$ in this case. If Re z < 0, then (4) clearly shows that we wish $(\text{Im } z/\text{re } z)^2 \leq (\omega/\sigma)^2$ for $q_+ \leq 0$. Even if Re z = 0, we have then shown that for $\omega > 0$, $q_+ \leq 0$ requires

$$\text{Re } z \leq 0 \tag{5a}$$

$$(\sigma \text{ Im } z)^2 - (\omega \text{ Re } z)^2 \leq 0 \tag{5b}$$

In other words the region of activity when $\omega > 0$, excluding poles, for a one-port is the region in Re p > 0 for which the two inequalities of (5) are satisfied. If $\omega = 0$, (1) shows that only Re z $\leq$ 0 in (5) need be considered.

Turning our attention to the equivalent circuit of Fig. 3(a) we see that Re $z_a \leq 0$ occurs where $r(\sigma-1)^2 + r\omega^2 + \sigma-1 \leq 0$ since p $\neq$ 1 forces a positive denominator. Assuming r > 0 and letting g = 1/r, we see by completing the square that a necessary and sufficient condition for Re $z_a \leq 0$ is

$$\left[\sigma - \left(1 - \frac{g}{2}\right)\right]^2 + \omega^2 \leq \frac{g^2}{4} \tag{6}$$

This describes the interior of a circle of radius g/2 centered at $\sigma = 1 - (g/2)$, $\omega = 0$. Since in practice g > 2 for a tunnel diode, a typical region described by (6) is shown in Fig. 4. Note that the circle lies entirely in Re p > 0 if g < 1. For $\omega = 0$ these arguments have shown that $0 < \sigma \leq 1$ is in the active region if g > 1.

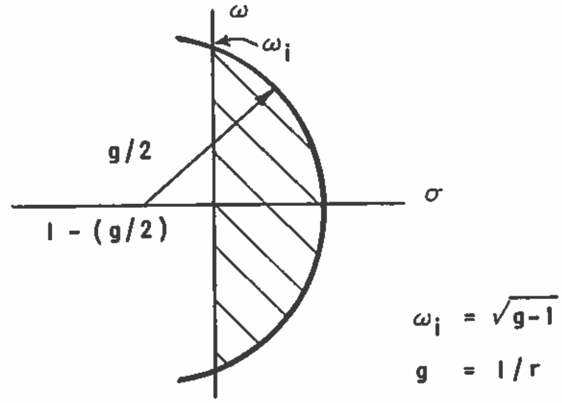


FIG. 4. REGION FOR Re $z_a \leq 0$ WHICH COINCIDES WITH THE ACTIVE REGION FOR TWO IDENTICAL DIODES OF FIG. 3(a).

By direct substitution of (3a) into (5b) with the use of $\omega > 0$, the second constraint of (5) yields $|\sigma g| \leq |\sigma^2 + (g-2)\sigma + \omega^2 + (1-g)|$. The term inside the absolute value on the right of this is nonpositive, since Re $z_a \leq 0$, while that on the left is positive in Re $p > 0$. We can thus delete the absolute value signs to obtain, after again completing the square,

$$[\sigma - (1-g)]^2 + \omega^2 \leq g^2(1-r) \tag{7}$$

Equation (7) describes the interior of a circle of radius $g\sqrt{1-r}$ centered at $\sigma = 1-g$, $\omega = 0$. If $r > 1$, there is no active region for $\omega > 0$. This circle intersects the ω axis at $\omega_i = \pm\sqrt{g-1}$, which is also the intersection point for the circle of (6). Since the circle of (7) intersects the σ axis at $\sigma_i = 1-g + g\sqrt{1-r} < 1$, this circle lies inside the circle for Re $z_a \leq 0$. Thus, for $\omega > 0$, the active region for a single diode of Fig. 3(a) is described by (7). Typical regions of activity are then plotted in Fig. 5. Here the bound on ω should be observed as well as the fact that the region consists only of a portion of the real axis when $r > 1$.

The results for the equivalent circuit of Fig. 3(b) are a little harder to obtain. As before, we first determine where Re $z_b \leq 0$. Assuming $\ell > 0$ and setting $\gamma = 1/\ell$, (3b) directly shows that the region for Re $z_b \leq 0$ is described by

$$(\sigma-1)(\sigma^2-\sigma+\gamma) + \sigma\omega^2 \leq 0 \tag{8}$$

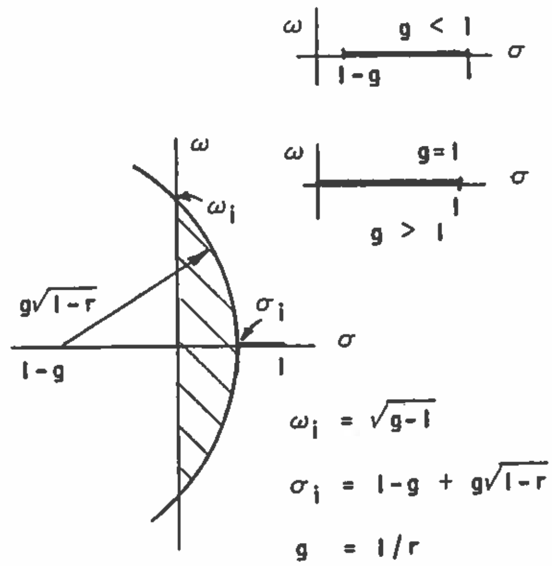


FIG. 5. ACTIVE REGIONS FOR A SINGLE DIODE OF FIG. 3(a).

Typical regions described by (8) are illustrated in Fig. 6. From the figure it is seen that given any ω_0 there is a σ_0 near $\sigma = 0$ such that $\mathrm{Re}\ z_b(p_o) < 0$. For $\ell > 4$ the region consists of two parts whose intersections on the σ axis are found by using the equality sign in (8) with $\omega = 0$. These are

$$2\sigma_+ = 1 + \sqrt{1-4\gamma} \tag{9a}$$

$$2\sigma_- = 1 - \sqrt{1-4\gamma} \tag{9b}$$

For $\omega = 0$, the active region is the union of $0 < \sigma \leq \sigma_-$ with $\sigma_+ \leq \sigma \leq 1$ if $\ell \geq 4$, or $0 < \sigma \leq 1$ if $\ell < 4$.

In order to find the constraints imposed by (5b) the latter is written in the form $|\sigma\ \mathrm{Im}\ z_b| \leq |\omega\ \mathrm{Re}\ z_b|$. Since $\mathrm{Re}\ z_b \leq 0$ is required and $\omega > 0$ is assumed, this is equivalent to the following two equations

$$\sigma\ \mathrm{Im}\ z_b - \omega\ \mathrm{Re}\ z_b \geq 0 \tag{10a}$$

$$\sigma\ \mathrm{Im}\ z_b + \omega\ \mathrm{Re}\ z_b \leq 0 \tag{10b}$$

In conjunction with $\mathrm{Re}\ z_b \leq 0$ these two equations define the active region, $\omega > 0$, for a single diode. After combining terms, the direct

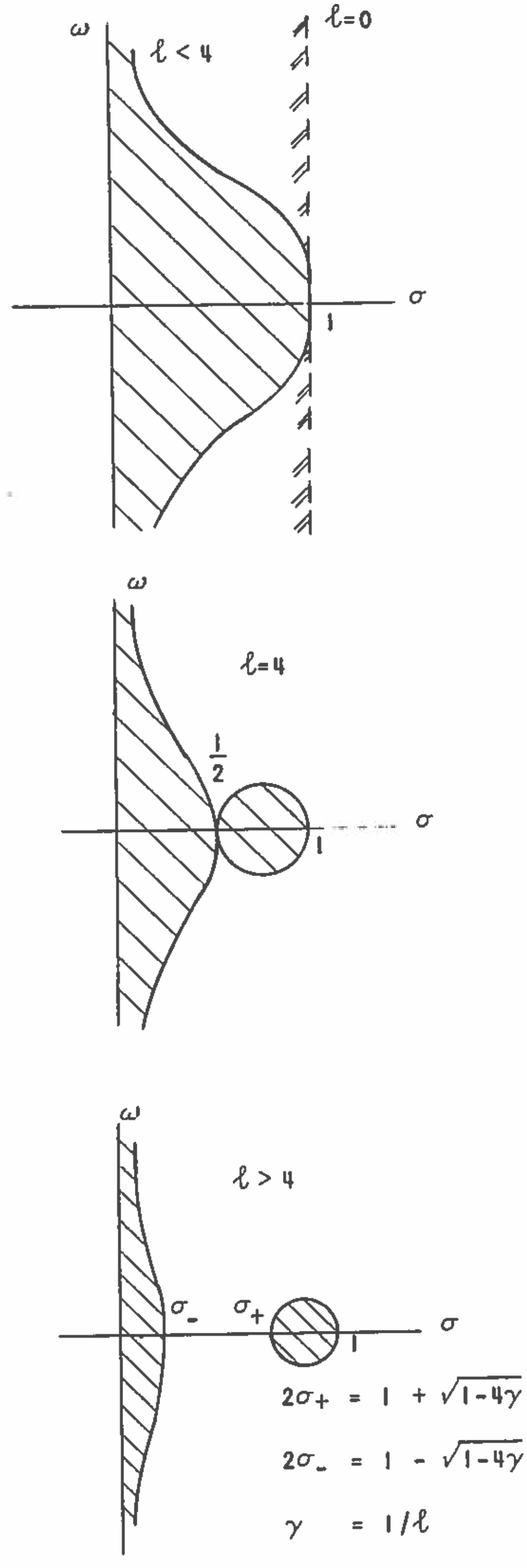


FIG. 6. REGIONS FOR Re $z_b \leq 0$ WHICH COINCIDE WITH THE ACTIVE REGION FOR TWO IDENTICAL DIODES OF FIG. 3(b).

substitution of (3b) into (10a) yields the first of (11), while the introduction of (3b) into (10b) yields the second. Recall that $\gamma = 1/\ell$.

$$1 - 2\,\sigma \geq 0 \tag{11a}$$

$$2\sigma(\sigma-1)^2 + 2\sigma\omega^2 - \gamma \leq 0 \tag{11b}$$

The intersection of the three regions defined by (8), (11a), and (11b) gives the active region when $\omega > 0$. The region defined by (11) always has $\sigma \leq 1/2$ and is in fact contained in that for Re $z_b \leq 0$. To see that indeed this is the case, (8) shows that for any σ in the region where Re $z_b \leq 0$, $\omega^2 \leq (1-\sigma)(\sigma^2-\sigma+\gamma)/\sigma$. For (11) to hold, $\omega^2 \leq [\gamma-2\sigma(\sigma-1)^2]/2\sigma$. Using the maximum allowable ω's in each case, called respectively ω_8 and ω_{11}, we see that $\omega_8^2 - \omega_{11}^2 = \gamma(1-2\sigma)/2\sigma$, which is nonnegative by (11a). Consequently, the active region for $\omega > 0$, using the circuit of Fig. 3(b), is completely described by (11).

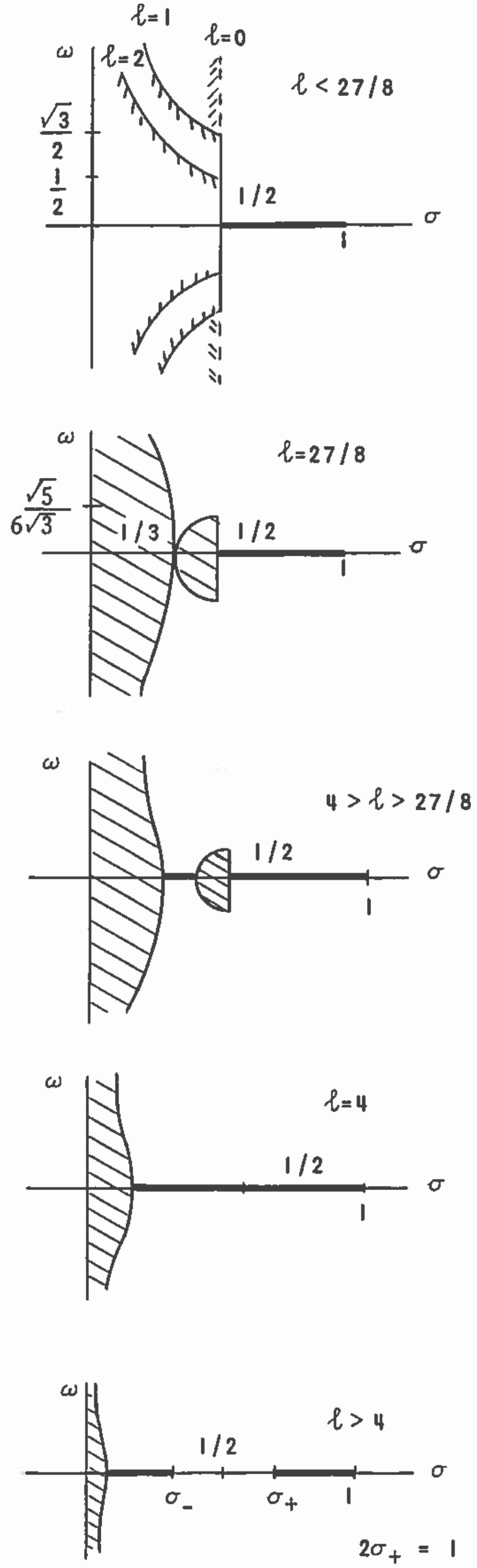
The regions defined by (11) exhibit some interesting properties which are most apparent for small ω's. For large ℓ the region consists of two disconnected subregions. To find the ℓ for which the region splits, we set $\omega = 0$. We then define $f(\sigma)$ by

$$f(\sigma) = 2\sigma(\sigma-1)^2 \leq \gamma \tag{12}$$

Now $[df(\sigma)/d\sigma] = 6\left(\sigma^2 - \frac{4}{3}\sigma + \frac{1}{3}\right) = 0$ occurs at $\sigma = \frac{1}{3}$, 1. Thus, the extrema of $f(\sigma)$ in the region of interest, $0 < \sigma \leq 1/2$, occur at $\sigma = 1/3$, 1/2. At $\sigma = 1/3$, (12) shows that $\ell \leq 27/8$; while for $\sigma = 1/2$, $\ell \leq 4$. Consequently, for $\ell \leq 27/8$ there is one region; for $(27/8) < \ell < 4$, the region splits; while for $\ell \geq 4$, the right-hand region vanishes. This behavior is illustrated by the examples of the typical active regions for z_b given in Fig. 7. It should be observed that a calculation of the point of intersection of the curve for $\omega > 0$ with the σ axis, when $\ell > 27/8$, requires solving the cubic equation of (12) with the equal sign.

The case $\ell = 0$, which is identical to $r = 0$, is a limiting case of the above theory. The region is $0 < \sigma \leq 1/2$ if $\omega > 0$, or $0 < \sigma \leq 1$ if $\omega = 0$, which agrees with the results of previous workers [Ref. 1 and Fig. 6, p. 433, Ref. 2]. This region is also cataloged in Figs. 6 and 7.

Passive networks N_p to yield the natural frequencies in the active region are easily obtained. At $p_o = 1$ there is already an open-circuit



$$2\sigma_+ = 1 + \sqrt{1-4\gamma}$$
$$2\sigma_- = 1 - \sqrt{1-4\gamma}$$
$$\gamma = 1/\ell$$

FIG. 7. ACTIVE REGION FOR A SINGLE DIODE OF FIG. 3(b).

natural frequency and no additional passive network is required. In the remainder of the active region we can write

$$z(p_o) = \alpha + j\beta \tag{13}$$

As can be checked by direct calculation, a possible impedance for the passive network is given by [Ref. 3, p. 164]

$$z_p(p) = \ell_p\, p + \frac{1}{c_p p} \tag{14a}$$

$$\ell_p = \frac{1}{2}\left[(-\alpha/\sigma_o) - (\beta/\omega_o)\right] \tag{14b}$$

$$c_p = 2[\{(-\alpha/\sigma_o) + (\beta/\omega_o)\}\ (\sigma_o^2 + \omega_o^2)]^{-1} \tag{14c}$$

In this if $\omega_o = 0$, then $\beta = 0$, and we replace β/ω_o by zero. ℓ_p and c_p will always be nonnegative [Ref. 3], but c_p may be infinite if $z(p_o) = 0$, in which case $z_p(p) \equiv 0$. The lossless passive network is shown in Fig. 8. For example, for z_a we have

$$2\ell_{pa} = [1-r(\sigma_o-1)^2 - r\omega_o^2]/\{\sigma_o[(\sigma_o-1)^2 + \omega_o^2]\}$$

$$c_{pa} = 2\sigma_o[(\sigma_o-1)^2 + \omega_o^2]/\{(\sigma_o^2 + \omega_o^2)[1-2\sigma_o-r(\sigma_o-1)^2 - r\omega_o^2]\}$$

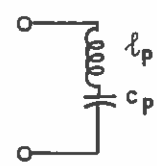
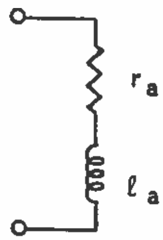


FIG. 8. LOSSLESS N_p FOR A SINGLE DIODE.

Since it may be desirable to use other than lossless networks for N_p, several other passive networks are presented. For z_a it is always possible to find an R-L series network. This is illustrated by Fig. 9 where the element values, which are nonnegative, are given. For z_b an R-C series circuit can be given to cover part of the active region, while an R-L series circuit will cover the remaining portion. Suitable circuits with their element values are given in Fig. 10. All of these circuits can be checked by direct evaluation to see that $z_p(p_o) = -(\alpha + j\beta)$.

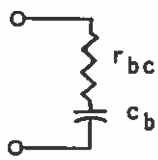
However, systematic procedures are available in the literature for obtaining various passive networks [Ref. 4].



$$r_a = \{1-2\sigma_o - r[(\sigma_o-1)^2+\omega_o^2]\}/\{(\sigma_o-1)^2+\omega_o^2\}$$

$$\ell_a = 1/\{(\sigma_o-1)^2+\omega_o^2\}$$

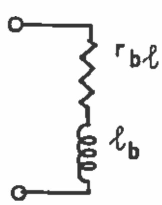
FIG. 9. R-L N_p FOR CIRCUIT OF FIG. 3(a).



$$r_{bc} = \{1-2\ell\sigma_o[(\sigma_o-1)^2+\omega_o^2]\}/[(\sigma_o-1)^2+\omega_o^2]$$

$$c_b = [(\sigma_o-1)^2+\omega_o^2]/\{(\sigma_o^2+\omega_o^2)[\ell\{(\sigma_o-1)^2+\omega_o^2\} - 1]\}$$

(a)



$$r_{b\ell} = (1-2\sigma_o)/[(\sigma_o-1)^2 + \omega_o^2]$$

$$\ell_b = \{1-\ell[(\sigma_o-1)^2+\omega_o^2]\}/[(\sigma_o-1)^2+\omega_o^2]$$

FIG. 10. R-C AND R-L N_p FOR CIRCUIT OF FIG. 3(b).

V. MULTIPLE DIODE N

In general, larger regions can be obtained by using more than one diode. We first consider two identical uncoupled diodes. In this case the impedance matrix is diagonal with identical entries.

$$Z(p) = z(p) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

For $\omega = 0$, Im $z = 0$ and $q_+ =$ Re z. Consequently, the active region on the σ axis is identical to that obtained for a single diode. Connecting the passive network obtained for a single diode to port one while ignoring port two yields a natural frequency. Clearly the second diode adds nothing when $\omega = 0$.

When $\omega > 0$, we again have $q_+ =$ Re z, since in (1) we can obtain $|\tilde{I}ZI| = 0$ by choosing $I_1^2 = -I_2^2$. In this case the regions of activity are larger than those for the single diode. They have already been exhibited in Figs. 4 and 6, since they coincide with the regions for Re $z \leq 0$.

For $\omega > 0$ a passive network can consist of one resistor and a gyrator [Ref. 3, p. 166*] with

$$Z_p(p) = -2 \text{ Re } z(p_o) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \sqrt{[\text{Re } z(p_o)]^2 + [\text{Im } z(p_o)]^2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{16}$$

As can easily be checked, det $[Z(p_o) + Z_p(p_o)] = 0$ and a short-circuit natural frequency occurs as required. If it is desired to obtain a lossless passive network, the resistor can be replaced by substituting the following expression for Re $z(p_o)$ in (16)

$$\frac{\text{Re } z(p_o)}{2\sigma_o} \left[p + \frac{\sigma_o^2 + \omega_o^2}{p} \right] \tag{17}$$

With this result the network to obtain an $\omega > 0$ natural frequency is shown in Fig. 11. Unfortunately, the known passive networks require a gyrator.

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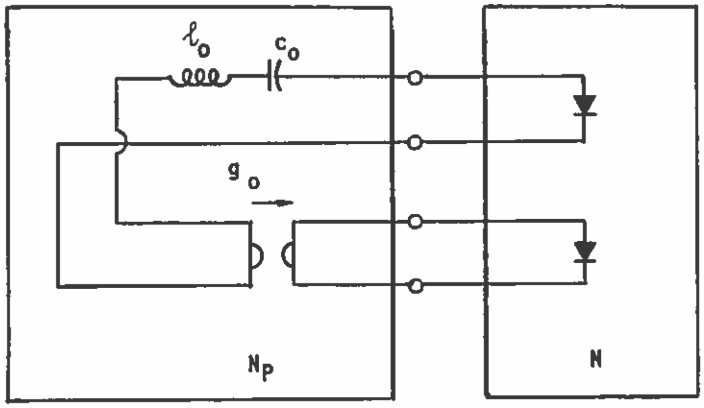
*An alternate results from $Z_p(p) = -\text{Re } z(p_o)1_2 + \text{Im } z(p_o)E$, where 1_2 is the 2x2 unit matrix and E is the 2x2 skew-symmetric matrix with unit entries. This alternate uses two resistors or four reactive elements.



$$\ell_o = -\text{Re } z(p_o)/\sigma_o$$

$$c_o = \sigma_o/\{[-\text{Re } z(p_o)][\sigma_o^2 + \omega_o^2]\}$$

$$g_o = \sqrt{[\text{Re } z(p_o)]^2 + [\text{Im } z(p_o)]^2}$$

FIG. 11. REALIZATION OF A NATURAL FREQUENCY AT p_o WHEN Re $z(p_o) \leq 0$, USING TWO DIODES AND A LOSSLESS N_p.

From the above discussion it should be clear that there is no point in using more than two identical diodes. This is true because we can't hope to have a region larger than that for Re $z \leq 0$; this latter is covered by using two diodes. Thus, if more than two identical diodes are present, we would ignore the extra diodes.

For two nonidentical diodes some 20 separate cases can occur. For any individual case the methods of a previous paper can be used to find the passive network, if $q_+ \leq 0$ [Ref. 3]. Using the methods of another reference, the passive network can be guaranteed lossless [Ref. 4]. Note that only one of the diodes can be normalized as in Fig. 3. For more than two nonidentical diodes it appears that ignoring all but the "best" two diodes gives the best results. However, the constraints, in general, upon an n x n Z matrix to yield $q_+ \leq 0$ are still unknown [Ref. 5]. Thus, this case remains unsolved. We can state, though, that the active region is at least as large as the active region of any one diode.

VI. CONCLUSION AND COMMENTS

Using linearized equivalent circuits which contain parasitic elements, we have determined the natural frequencies that can be obtained from tunnel diodes. For a single diode the results are summarized in Figs. 5 and 7; for two or more identical diodes they are summarized in Figs. 4 and 6. To obtain these results we have limited the treatment to the right half plane, Re $p > 0$. In Re $p > 0$ it is possible to define a natural frequency in a generalized way by using excitations beginning at $t = -\infty$. This procedure actually allows us to extend (1) to distributed parameter networks. Unfortunately, this definition of a natural frequency seems to be inconvenient for Re $p \leq 0$.

In any event we can extend the results to Re $p = 0$ for the finite cases we treated by noting where $q_+(j\omega) \leq 0$. This can be seen directly from the curves presented, and shows which short-circuit natural frequencies can be obtained on $p = j\omega$. Including Re $p = 0$ in the main theory of the report requires that certain distinctions be made which interrupt the important concepts. This situation arises because any open-circuit natural frequency can be obtained on the $j\omega$ axis by using the proper lossless passive network. The passive networks to obtain $j\omega$-axis-short circuit natural frequencies generally require resistance and can be obtained easily from Ref. 3.

Comparing the curves for the circuit with series resistance against those with series inductance shows that the resistance is what limits high-frequency oscillations. A truer representation of actual diode operation would result from incorporating series resistance and inductance simultaneously. One soon gives up such an attempt, since the mathematical analysis, for the general case, becomes intractable. However, the general results of such a treatment can be inferred from the curves presented. With both types of parasitics, the regions could at best be the intersections of the regions obtained in this paper.

We have presented two types of passive networks in Figs. 8 through 11. Lossless networks are theoretically attractive, but the circuits incorporating resistors seem to be practically more feasible. However, R_s of the diode can be used to absorb some of the loss of nonideal lossless networks. The use of two diodes is theoretically appealing since the largest regions are obtained. But this is counterbalanced by the need for a gyrator coupling element. In some cases it may be desirable to transfer the inductance of Fig. 11 through the gyrator by turning it into a capacitor using well-known equivalences.

In treating multiple-diode networks, we have looked at the case where the diodes are uncoupled. If the diodes are somehow connected together the regions of activity can only decrease, since the coupling can be looked upon as a constraint placed on N_p.

The results of this report illustrate the various kinds of regions in which a network can be active at p_o. Of special interest is the splitting of the regions that occurs in Fig. 7. A contribution of the report is the method of circumventing the singularities of Z, a problem essentially ignored in Ref. 2.

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