COMMENTS ON 2-PHASE DRAG-CUP ACCELEROMETER PARAMETER DETERMINATION

It is pointed out that a recently proposed method of determining sufficient parameters to describe the terminal performance of a drag-cup induction accelerometer suffers from implicit assumptions of doubtful validity which call into question the value of the method and the claims made for it.

In a recent letter,[1] a method for determining the parameters of a drag-cup accelerometer was suggested. The method is interesting, but we believe it overlooks some significant factors which will seriously affect the results so obtained. These factors are associated with the validity of the usual linear model of the induction motor when used with the drag-cup machine over a range of operating speeds and excitation frequencies. In this respect, it must be noted that the drag-cup rotor permits current flow in various directions rather than in well defined current paths, as for a squirrel-cage rotor. For this reason alone, one would expect the effects of speed and frequency to have an effect on the parameters of the assumed linear model. We wish to make the following specific observations on the method of parameter determination proposed.

(*a*) The transfer function

$$\frac{V_b(S)}{I_a(S)} = \left(\frac{M^2}{r}\right)\frac{\omega_m S}{\{(1+T_r S)^2 + T_r^2\omega_m^2\}} \quad . \quad . \quad . \quad (1)$$

which is eqn. 10 in the referenced letter,[1] is based, of course, on the linear model. The authors propose to determine the parameters M^2/r and T_r from this transfer function by supplying one phase of the machine from a sinusoidal source and recording input current $I_a(j\omega)$, and output voltage $V_b(j\omega)$, of the other phase at a known speed ω_m. In fact, the values of M^2/r and T_r thus obtained are strictly valid only for the conditions under which they are measured; i.e. for a certain speed ω_m, and a certain excitation frequency ω. Our recent experiments have shown that the parameters of the assumed linear model change considerably with excitation frequency and to some extent with speed. To be more specific, in testing a small drag-cup machine using a transfer-function method similar to that suggested by the authors, we obtained a change in M^2/r of about 25% and an even greater change in T_r in the frequency range 60–200 Hz. The changes in the parameters with speed are less pronounced, but must also be considered. Thus, although the method proposed may give results that are sufficiently good approximations for some purposes and for some particular machines, it does not seem to be a good general method to use for accelerometer parameter determinations, since the accelerometer operates at an excitation frequency $\omega = 0$, and under transient speed conditions.

(*b*) As the authors point out, the two parameters M^2/r and T_r are sufficient to describe terminal behaviour of the machine if the excitation current is known. The method of parameter determination suggested does not enable one to determine the third parameter r. One who is interested in the effects of design on accelerometer performance may, for this reason, find the method unsatisfactory. To enable determination of r, it is necessary to have another equation relating the parameters. Law and Novotny[2] used an empirical relationship between M and l for this third equation. In a recent project,[3] we have obtained acceptable results using an independent method to determine r and steady-state tests to determine l and M. We are currently studying the parameter problem in detail, since our method also fails to consider frequency effects.

(*c*) In view of the above observations, we must question the authors' claim that, in their method, 'no approximations are made'. We also question the contention that the method is 'more exact than that of Law and Novotny'. No experimental proof is provided in support of this claim. Furthermore, Law and Novotny's parameter tests were conducted at the proper accelerometer excitation frequency $\omega = 0$. It is by no means obvious that the assumptions they have used are less valid than those used implicitly by the authors in their proposed method.

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References

1 RAMASWAMI, B., and SASIDHARA RAO, P.: 'Determination of the parameters of a 2-phase drag-cup accelerometer', *Electronics Letters*, 1967, 3, p. 552
2 LAW, J., and NOVOTNY, D. W.: 'The two-phase accelerometer', *IEEE Trans. Power Apparatus Syst.*, 1964, 83, pp. 614–619
3 BURRIDGE, R. E., and UMAMAHESWARAN, V. S.: 'Accelerometer characteristics of a drag-cup induction machine', paper 68TP101–PWR, IEEE winter power meeting, New York, 1968

STATE-VARIABLE SYNTHESIS OF INTEGRATED-CIRCUIT DISTRIBUTED *RC* TRANSFER FUNCTIONS*

On combining the theory of canonical state-space equations with the theory of *RC* transmission lines, a means of designing distributed circuits for desired integrated-circuit transfer functions is given. The theory rests on a transformation of O'Shea, and is made practical by use of the approximation curves of Barron.

Recent applications of state-variable theory to the synthesis of linear integrated circuits have yielded attractive results in terms of low sensitivity minimal-capacitor realisations for both scalar and matrix transfer functions.[1,2] When coupled with the actual presence of integrated operational amplifiers,[3,4] simulations using state-space theories then become practically important. However, some integrated-circuit configurations are inherently of a distributed nature, whereas the canonical finite-dimensional state-space equations (Reference 5, p. 221 and Reference 6, p. 11) assume lumped structures. Since it has recently been shown that designs based on *RC* distributed lines coupled with operational amplifiers can lead to practical designs using fewer components than in lumped form,[7] it is not unreasonable to turn to state-space-based simulations in extending synthesis methods to distributed networks. Here, on using the frequency transformation of O'Shea,[8] this approach is shown to be applicable to the simple realisation of a wide class of distributed transfer functions.

The state equations (Reference 5, p. 221) for a linear time-invariant finite-dimensional dynamic system lead to the representation

$$Y(s) = T(s)U(s) = \{D + C(sI_k - A)^{-1} B\}U(s) \quad (1)$$

where $U(s)$ and $Y(s)$ are the Laplace transforms of the input $n$vector $u(t)$ and output $m$vector $y(t)$, respectively; $T(s)$ is the $m \times n$ transfer-function matrix, assumed constant at infinity, I_k is the $k \times k$ identity, where k is the dimension of the state, and A, B, C and D are constant matrices of proper dimensions. For the important case of a scalar transfer function specified as the ratio of two polynomials in s (with the constant at infinity removed),

$$T(s) = d + \frac{c_k s^{k-1} + \ldots + c_2 s + c_1}{s^k + a_k s^{k-1} + \ldots + a_2 s + a_1} \quad . \quad . \quad (2)$$

the decomposition of eqn. 1 directly leads to the configuration of Fig. 1A, where $p(s) = s$ is chosen (Reference 9, p. 178).

The state-variable simulations may be extended by making a transformation in which the Laplace-transform variable s is replaced by some function $p(s)$. The transfer functions for the resulting configurations become

$$T\{p(s)\} = T'(s) = D + C\{p(s)I_k - A\}^{-1}B \quad . \quad . \quad (3)$$

$$T\{p(s)\} = T'(s) = d + \frac{c_k p^{k-1}(s) + \ldots + c_2 p(s) + c_1}{p^k(s) + a_k p^{k-1}(s) + \ldots + a_2 p(s) + a_1} \quad . \quad . \quad . \quad . \quad (4)$$

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Consequently $T'(s)$ can be realised by obtaining a $p$plane integrator and using this, in conjunction with summers and gain blocks,[1] as shown in Fig. 1A.

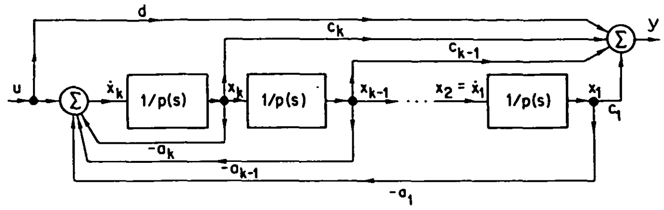

Fig. 1A *State-space transfer-function realisation*

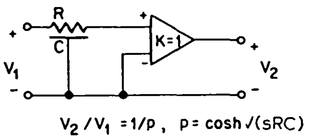

Fig. 1B *p plane integrator*

This general result may be conveniently applied to distributed-network synthesis by considering the specific transformation proposed by O'Shea (Reference 8, p. 547).

$$p(s) = P = \cosh\sqrt{(sRC)} \quad . \; . \; . \; . \; . \; . \; . \quad (5)$$

where R and C are the total series resistance and shunt capacitance of a uniform RC transmission line. A Pplane integrator is as shown in Fig. 1B, where an isolating operational amplifier, or more simply an emitter follower, is placed at the output of the RC line to preserve its integrating function when inserted into the configuration of Fig. 1A. That Fig. 1B results in a Pplane integrator is easily checked from the impedance matrix $Z(s)$ of the uniform RC line (Reference 8, p. 547, and Reference 10)

$$Z(s) = \sqrt{\left(\frac{R}{sC}\right)}\begin{Bmatrix}\coth\sqrt{(sRC)} & \operatorname{cosech}\sqrt{(sRC)} \\ \operatorname{cosech}\sqrt{(sRC)} & \coth\sqrt{(sRC)}\end{Bmatrix} \quad (6)$$

Then

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \left.\frac{V_2/I_1}{V_1/I_1}\right|_{I_2=0}$$

$$= \frac{z_{21}}{z_{11}} = \frac{\operatorname{cosech}\sqrt{(sRC)}}{\coth\sqrt{(sRC)}} = \frac{1}{\cosh\sqrt{(sRC)}} = \frac{1}{P} \quad . \; . \; . \; . \quad (7)$$

which is integration in the Pplane.

Consequently, through Fig. 1 and eqns. 3–5, we are able to apply state-variable simulations in the transformed Pplane to realise any transfer function with real coefficients of the form of eqn. 4. Such functions may be written in factored form, useful for approximation, as

$$T'(s) = k\frac{\prod_{i=1}^{m}\{\cosh\sqrt{(sRC)} + \beta_i\}}{\prod_{i=1}^{n}\{\cosh\sqrt{(sRC)} + \alpha_i\}} = T(P) \quad . \; . \quad (8)$$

where $m \leqslant n$.

The approximation of a desired frequency response for a specification in the form of eqn. 8 has been mentioned by O'Shea (Reference 8, p. 549) with some useful approximations carried out by Barron.[11] The locus of $\cosh\sqrt{(j\omega RC)}$ against ω in the Pplane can readily be plotted (Reference 8, p. 549 and Reference 11, p. 135). Poles and zeros are located at α_i and β_i as shown in eqn. 8, and vectors may be drawn from these to the $\cosh\sqrt{(j\omega RC)}$ locus in the customary manner (Reference 12, p. 282) to determine the frequency response for a given pole-zero pattern. From a tabulation, say by computer means, of frequency responses for individual poles and zeros, or conjugate pairs, one can determine a desired $T(P)$ on which to perform a synthesis by adding magnitude responses, in decibels, for proper α_i and β_i. This procedure has been used, for example, (Reference 11, p. 137) to obtain

$$T(P) = \frac{k}{(P + 0{\cdot}75)(P^2 + 0{\cdot}9P + 1{\cdot}89)} = T'(s) \quad (9)$$

for the approximately maximally flat $T'(s)$ response shown in Fig. 2. A realisation for the response of Fig. 2 using three distributed RC lines then follows from Fig. 1.

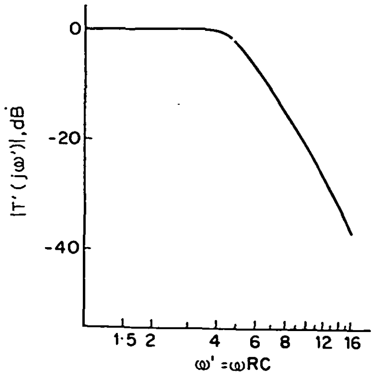

Fig. 2 *Frequency response for eqn. 9*

In conclusion, we have outlined a synthesis technique which can be used to realise a large class of distributed RC filter responses given a graphical description of the requirements, its practicability being limited only by the extent of available pole-zero frequency-response tabulations. The synthesis procedure is quite simple once the desired poles and zeros are specified in the Pplane, this procedure resting on familiar state-space techniques.

This letter has dealt with a specific transformation to the $p(s)$plane, namely $p(s) = P = \cosh\sqrt{(sRC)}$. Although this appears the most convenient for integrated circuits, because of the simplicity of the integrator (Fig. 1B), other possibilities exist, such as $p(s) = \tanh\sqrt{(sRC)}$ for RC lines[13] and $p(s) = \tanh s\sqrt{(LC)}$ for LC lines (Reference 14, p. 66). The consequences of the former transformations have been explored in some detail.[15] Further, the theory extends to cover cases, such as the theory of noncommensurate lines, described by multivariable transfer-function matrices.[16] Because of the simplicity and generality of this synthesis method, deeper investigations are under consideration.

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References

1 KERWIN, W. J., HUELSMAN, L. P., and NEWCOMB, R. W.: 'State-variable synthesis for insensitive integrated circuit transfer functions', *IEEE J. Solid-State Circuits*, 1967, SC-2, p. 87–92
2 NEWCOMB, R. W., and ANDERSON, B. D. O.: 'State variable results for minimal capacitor integrated circuits', Stanford Electronics Laboratories, technical report 6558–14, October 1966
3 FOWLER, W. L.: 'A look at linear integrated circuits', *Electronic Industr.*, 1965, **24**, pp. 64–69, 194
4 WIDLAR, R. J.: 'A monolithic operational amplifier', Fairchild Application Bulletin, APP–105/3, April 1966
5 ZADEH, L. A., and DESOER, C. A.: 'Linear system theory' (McGraw-Hill, 1963)
6 OGATA, K.: 'State space analysis of control systems' (Prentice-Hall, 1967)
7 KERWIN, W. J.: 'Analysis and synthesis of active RC networks containing distributed and lumped elements', Ph.D. thesis, Stanford University, 1967
8 O'SHEA, R. P.: 'Synthesis using distributed RC networks', *IEEE Trans.*, 1965, CT–12, pp. 546–554
9 KALMAN, R. E.: 'Mathematical description of linear dynamical systems', *J. SIAM Control*, 1963, **1**, pp. 152–192
10 RAO, T. N., SHAFFER, C. V., and NEWCOMB, R. W.: 'Realizability conditions for distributed RC networks', Stanford Electronics Laboratories, technical report 6558–7, May 1966
11 BARRON, M. B.: 'Synthesis of distributed RC networks', *in* 'Papers on integrated circuit synthesis', compiled by NEWCOMB, R. W., and RAO, T. N. Stanford Electronics Laboratories, technical report 6560–4, June 1966, pp. 131–152
12 VAN VALKENBURG, M. E.: 'Network analysis' (Prentice-Hall, 1964)
13 SCANLAN, J. O., and RHODES, J. D.: 'Frequency response of distributed RC networks', *Electronics Letters*, 1965, **1**, pp. 267–268
14 NEWCOMB, R. W.: 'Linear multiport synthesis' (McGraw-Hill, 1966)
15 KOZEMCHAK, E. B.: 'State-variable synthesis of distributed RC network transfer functions', *in* 'Papers on integrated circuit synthesis, II', compiled by NEWCOMB, R., and DE OLIVEIRA, R., Stanford Electronics Laboratories, technical report 6560–13, June 1967
16 NEWCOMB, R. W.: 'On the realization of multivariable transfer functions', Cornell University, research report EERL 58, December 1966