State-Space Techniques
With Applications to Integrated Circuits*

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Abstract: The generality of state-variable theory allows the development of synthesis techniques of particular interest to linear integrated circuits. This paper reviews presently available state-variable results while discussing several extensions. Included are time-variable, lumped-distributed, and passive techniques with reference to the use of integrated operational amplifiers and gyrators.

"My grandmother read me a tale about a mermaid who had acquired a pair of feet." [1, p. 116]

I. Introduction

As becomes more and more apparent with time the theory of linear integrated circuits, and with it the more general theory of micro-systems, is bound to have a rather profound effect upon the capabilities of mankind. For example, more convenient biological monitors and transducers, such as hearing aids and timing pieces, are already under consideration. However, because of the precision required and the detailed nature of the processing, one can not progress effectively with cut-and-try methods. What is needed is a set of solid relevant

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theories. In our view the theory of state-variables, which is well developed from system and control applications, offers the most promise for significant advancement. Here we outline some of the results available for immediate use as well as some of those having implication for the future.

To be sure the most accurate methods of designing integrated circuits would be to take into detailed account the total material properties of the media, for example the effects of diffusion gradiants in monolithic structures. Unfortunately such theories are either unavailable or too difficult to apply except in the simplest of design situations. Consequently it appears best to set up macroscopic designs on a rigorous basis, as we outline here, and then apply computer aided analysis techniques to incorporate material properties in preliminary evaluations. Since state-variable theory can be applied at all of these stages, the state and its properties seem most appropriate for study within the integrated circuit framework.

The material can be outlined as follows. In section II we review some of the mathematics associated with state-variables as well as some of the circuit components available in integrated form. In section III mathematical realization techniques are discussed while in section IV these results are used for synthesis. A technique being used for commercially available integrated filters is discussed in section V where low sensitivity degree two realizations are investigated. In most places the treatment is summary and tutorial with details being adequately developed elsewhere, although in several instances new results are incorporated.

"The inquisitive breeze would join in the reading and roughly finger the pages so as to discover what was going to happen next. That is about all I remember of the voyage." [1, p. 116]

II. Preliminaries

A. Mathematical

As background we recall the basic details of state-variable theory needed for the development. A thorough treatment can be found in [2] while [3] gives the state-variable position within linear network theory from which much of the following is justified.

We begin by realizing that almost any finite linear circuit can be described by the canonical state-variable equations

$$\dot{\underline{x}}(t) = \underline{A}(t)\underline{x}(t) + \underline{B}(t)\underline{u}(t) \tag{2.1a}$$

$$\underline{y}(t) = \underline{C}(t)\underline{x}(t) + \underline{D}(t)\underline{u}(t) \tag{2.1b}$$

Where $\underline{u}$ is the input n-vector, $\underline{y}$ is the output m-vector and the state k-vector $\underline{x}$ represents the internal behavior of the system; the set of matrices $R = \{\underline{A}, \underline{B}, \underline{C}, \underline{D}\}$ is called a realization and is assumed independent of the input, output, and state, for linearity. If the realization matrices (which are assumed real) are constant in time t then the system is time-invariant; conversely, given a time-invariant system having a canaonical state-variable representation, there exists a constant realization R to describe it.

Of particular interest to us will be the fact that the canonical state-variable equations have the block diagram representation of Fig. 2.1 where all of the dynamics is seen to be taken up by the k uncoupled integrators denoted through the k×k identity matrix $\underline{1}_k$.

For specification purposes a linear system is most often described in terms of its m×n impulse response matrix $\underline{h}(t,\tau)$; more precisely

$$\underline{y}(t) = \int_{-\infty}^{\infty} \underline{h}(t,\tau)\underline{u}(\tau)d\tau \tag{2.2}$$

where the integral rigorously denotes a scalar product between the distributional kernel $\underline{h}$ and input $\underline{u}$ [4, p. 143]. If we define the fundamental matrix $\underline{\Phi}(t,\tau)$ for Eq.(2.1) through

$$\frac{d\underline{\Phi}(t,\tau)}{dt} = \underline{A}(t)\underline{\Phi}(t,\tau); \quad \underline{\Phi}(t,t) = \underline{1}_k \tag{2.3}$$

then all of the above equations yield

$$\underline{h}(t,\tau) = \underline{D}(t)\delta(t-\tau) + \underline{C}(t)\underline{\Phi}(t,\tau)\underline{B}(\tau)1(t-\tau) \tag{2.4}$$

In this expression $1(\cdot)$ is the unit step function with $\delta(\cdot)$ its derivative, the unit impulse. When the realization is constant, time-invariance obtains in which case $\underline{h}(t,\tau) = \underline{h}(t-\tau,o)$ and $\underline{\Phi}(t,\tau) = \exp[\underline{A}(t-\tau)]$; on taking Laplace transforms we find the transfer-function m×n matrix $\underset{\sim}{H}(p)$,

$$\underset{\sim}{H}(p) = \mathcal{L}[\underset{\sim}{h}(t,o)] = \underset{\sim}{D} + \underset{\sim}{C}[p\underset{\sim}{1}_k - \underset{\sim}{A}]^{-1}\underset{\sim}{B} \tag{2.5}$$

where $p = \sigma + j\omega$ is the complex frequency variable. Note that $\underset{\sim}{H}(p)$ is rational with real coefficients, called real-rational, and finite at $p = \infty$.

Given any one realization R_o of the canonical state-variable equations one can find any other realization R having the same impulse response matrix $\underset{\sim}{h}$ by the use of transformations on the state and canonical equations [5, p. 536][6, p. 373]; R and R_o are naturally called equivalent realizations. In particular if $\underset{\sim}{A}$ and $\underset{\sim}{A}_o$ both are of minimal size δ, $k = k_o = \delta$, in which case R and R_o are called minimal realizations, there exists a nonsingular transformation $\underset{\sim}{T}(t)$ on the state, $\underset{\sim}{x}_o = \underset{\sim}{T}\underset{\sim}{x}$, such that

$$\underset{\sim}{A} = \underset{\sim}{T}^{-1}\underset{\sim}{A}_o\underset{\sim}{T} - \underset{\sim}{T}^{-1}\dot{\underset{\sim}{T}}, \quad \underset{\sim}{B} = \underset{\sim}{T}^{-1}\underset{\sim}{B}_o, \quad \underset{\sim}{C} = \underset{\sim}{C}_o\underset{\sim}{T}, \quad \underset{\sim}{D} = \underset{\sim}{D}_o \tag{2.6}$$

If $\underset{\sim}{T}$ is a constant matrix, as for the time-invariant situation, then $\underset{\sim}{A}$ and $\underset{\sim}{A}_o$ are related by similarity. It is worth commenting that this minimal state dimension δ is called the degree; since the fundamental matrix has the transition property $\underset{\sim}{\Phi}(t,\tau) = \underset{\sim}{\Phi}(t,t_o)\underset{\sim}{\Phi}(t_o,\tau)$ for any t_o, the impulse response is separable, that is it can be written with

$$\underset{\sim}{C}(t)\underset{\sim}{\Phi}(t,\tau)\underset{\sim}{B}(\tau) = \underset{\sim}{\Psi}(t)\underset{\sim}{\Theta}(\tau) \tag{2.7}$$

in which case δ is the smallest number of linearly independent columns in $\underset{\sim}{\Psi}$. Alternatively, the degree δ can be calculated directly from the transfer function $\underset{\sim}{H}(p)$ in the time-invariant case [7].

B. Integrated Circuits

In developing theoretical results for physical entities it is important to observe the nature of the physical constructs available. This is particularly true for integrated circuits for which we here briefly review several devices and their characteristics.

For integrated circuits, by which we will mean monolithic or thin film or their hybrid combination, we will consider the prime elements available to be monolithic resistors & bipolar transistors and thin film type capacitors, MOS transistors & precision resistors. For conciseness of treatment we will generally limit ourselves to the first three mentioned components while realizing that of these the capacitors cause the most difficulty in fabrication. For this reason, and availability of limited (chip) construction space, we desire to minimize the total number of capacitors as well as keep a common ground, if possible, for all capacitors present. There are of course other prime elements, as the monolithic FET, and one of these, the distributed RC line will briefly be used later.

It is certainly possible to consider all circuits viewed as a connection of prime elements, but we know from experience that for design purposes it is more convenient to introduce additional building blocks, called generating elements, constructed from the primes. The several synthesis methods to be discussed will depend upon the availability of three specific generating elements, the differential voltage-to-current converter (DVCCS), the gyrator, and the differential voltage controlled voltage source (DVCVS), often of infinite gain (operational amplifier). For example the DVCCS allows direct admittance matrix synthesis while it is also key for the construction of the gyrator; any finite circuit can be constructed from DVCCS's and capacitors [8]. The gyrator yields passive syntheses, generally having low sensitivities, while operational amplifiers almost directly allow implementation of Fig. 2.1. Again there are other generating elements, such as the NIC, but the above, all of which have been integrated, suffice for our purposes.

For integration all of the generating elements should be direct coupled with differential source inputs for symmetry and convenience of use. Commercial usage often necessitates rather complex circuitry, but here we exhibit some simple circuits to illustrate the basic concepts. Also, since we are generally interested in the time-variable or adjustable, situation we will incorporate a means of time-variation.

Figure 2.2 shows a simple scheme for obtaining a DVCCS; a differential voltage amplifier is cascaded with a voltage to current converter which is fed by a constant current source for bias purposes. The transconductance of the device depends directly upon the bias voltage

V_b and, hence, can be readily varied electronically. The use a pnp transistor presently necessitates a lateral transistor [9], but, except for this, the device is easily integrated with acceptable performance presently available into the mid-KHz region.

Ideally the DVCCS has infinite input and output impedances. Consequently, the back-to-back connection of Fig. 2.3a) yields the admittance matrix

$$\underset{\sim}{Y} = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} \tag{2.8}$$

of the grounded gyrator symbolized in Fig. 2.3b). By this technique the gyrator has been integrated [10] while Fig. 2.3c) shows a means for obtaining a floating (input) port. Since it is difficult to time-vary the <u>gyration conductance</u> G through zero, Fig. 2.3d) shows a scheme for changing sign [J. Woodard]. The presently integrated gyrator has reasonable quality again into the high KHz range, but commercially explored circuits using thin film techniques indicate extremely high quality availability [11][12]. We would comment that most often in practice $y_{12} \neq -y_{21}$ in which case the gyrator loses its passive property. Also in practice the high frequency performance is primarily limited by the phase shift present in G, $G = G_o \exp(-pT)$ [13, p. 27].

The final generating element to be mentioned is the DVCVS shown in Fig. 2.4a) where a unity gain emitter follower is used to provide isolation of the amplifying portion; a zener diode, T_4 with $V_z \approx 6.5$v, is used to provide a zero level dc output voltage (at the expense of noise which can be avoided by more elaborate level shifters). Again the amount of time-variation, provided by a variable V_b, is limited. The variable range can be extended by the feedback arrangement of Fig. 2.4d) where the infinite gain device can be approximated by the cascade of several finite gain amplifiers. To point out an interrelation between the various generating elements the equivalence of Fig. 2.4c) is worth noting. Although the circuit of Fig. 2.4a) is relatively easily integrated it is of more significance to realize the ready commercial availability of operational amplifiers. Ideally the device has infinite input impedances and zero output impedance, but parasitic effects again limit the frequencies of operation to below the hundreds of KHz.

The importance of the gyrator for integrated circuits lies in the fact that it can convert capacitors into equivalent inductors, as illustrated in Fig. 2.5a); for example an integrated gyrator with $G = 3.33\times10^{-4}$ ℧ loaded in a 100 pf capacitor yields a 1 mhy inductor. The interest in the DVCVS primarily lies in their use for construction of integrators and summers as shown in Fig. 2.5b),c); note that multiple inputsummers are desired to handle the matrix operations of Eq. (2.1). The gain expressions for the integrator and summer are respectively

$$v_o = \frac{2}{RpC} v_i, \qquad v_o = \sum_{j=1}^{m_+} \frac{RG^- + 1}{G^+} G_j^+ v_j^+ - \sum_{j=1}^{m_-} RG_j^- v_j^- \qquad (2.9)$$

where G^+(or G^-) is the sum, $j = 0, \ldots, m_+$(or m_-), of the input conductances $G_j = 1/R_j$. Note that to time vary one of the sum coefficients, as may be needed for $\underline{A}(t)$, say, it is easiest to insert a variable gain amplifier [Fig. 2.4a)] before the summer resistor while keeping the summer time-invariant.

It is worth mentioning that in monolithic form resistor ratios but not absolute values are relatively easy to control with most convenient values in the range 100-50,000 Ω. Thin film resistors yield higher values with higher precision. Integrated capacitor values depend of course upon the area used but reasonable values lie in the 10-1,000 pf range. Distributed losses play a large role in limiting device operation, but in some cases the distributed effects can be used to advantage for cutting down size and extending the useable frequency range [14].

At this point it may be valuable to exhibit photomicrographs of the above mentioned generating elements. Thus actual integrated layouts are illustrated in Fig. 2.6 where part a) gives the DVCCS of Fig. 2.3a) [J. Miller, Stanford Integrated Circuits Laboratory], part b) the DVCVS of Fig. 2.4a) [University of California, Berkeley, Integrated Circuits Laboratory], and part c) a gyrator [10]

With these thoughts and results in mind we can turn to concepts directly related to synthesis.

III. State Realizations

Here we discuss means of setting up the canonical state-variable equations. Of particular interest is that of finding a minimal realization from a given transfer function or from a given impulse response which we know must be of the form

$$\underline{h}(t,\tau) = \underline{\Lambda}(t)\delta(t-\tau) + \underline{\Psi}(t)\underline{\Theta}(\tau)1(t-\tau) \tag{3.1}$$

where all matrices are real valued. Assuming matrices $\underline{\Lambda}$, $\underline{\Psi}$, and $\underline{\Theta}$ of infinitely differentiable functions, for convenience, we will call $\underline{h}$ a separable kernel of order zero this latter denoting the distributional order of the kernel [15, p. 133] (that is, no $\delta^{(j)}$, $j > 0$ are present).

Since $\underline{\Phi}(t,\tau) = \underline{1}_k$ when $\underline{A}$ is the zero matrix, $\underline{A} = \underline{0}_k$, we can immediately obtain the realization, on comparing Eqs. (3.1) and (2.4),

$$\underline{A} = \underline{0}_k, \quad \underline{B} = \underline{\Theta}, \quad \underline{C} = \underline{\Psi}, \quad \underline{D} = \underline{\Lambda} \tag{3.2}$$

which is minimal if the columns of $\underline{\Psi}$ have been made linearly independent. Any other minimal realization can be obtained by the use of Eq. (2.6); in fact, if $\underline{T}$ is chosen as a Lyapunov transformation [16, p. 117] (essentially, bounded and of bounded derivative and inverse) then the stability properties are preserved. Although not directly applicable to the realization of Eq. (3.2) it is of special interest to know that if an initial realization R_o has $\dot{\underline{x}}_o = \underline{A}_o\underline{x}_o$ exponentially asymptotically stable, then given any positive definite (symmetric) $\underline{Q}(t)$ there exists a (symmetric) positive definite $\underline{P}(t)$ such that [17, p. 95] (the tilde denotes matrix transposition)

$$\dot{\underline{P}} + \tilde{\underline{A}}_o\underline{P} + \underline{P}\underline{A}_o = -\underline{Q} \tag{3.3a}$$

If one then chooses for the transformation on the state the unique positive definite square root of the inverse of $\underline{P}$, $\underline{T} = \underline{P}^{-\frac{1}{2}}$, then on using Eq. (2.6) in (3.3a) one finds

$$\tilde{\underline{A}} + \underline{A} = -\tilde{\underline{T}}\underline{Q}\underline{T} \tag{3.3b}$$

That is, in this case the symmetric part of $\underline{A}$ becomes negative definite (for all time).

Turning to the time invariant case, we see that the realization of Eqs. (3.2) will in general be time-varying. Thus we actually prefer to proceed through the transfer function from which Eq. (2.5) yields on expansion about infinity

$$H(p) = D + \sum_{i=0}^{\infty} C \frac{A^{i}}{p^{i+1}} B = \sum_{i=-1}^{\infty} \frac{A_i}{p^{i+1}} \qquad (3.4a)$$

The right hand side of this expression defines the A_i which can then be assumed as given and from which we wish to find a minimal constant realization. The process is relatively difficult to justify [18, p. 13] [3, p. 63] but a construction can proceed as follows. From the least common denominator $g(p) = p^{r} + a_1 p^{r-1} + \ldots + a_r$ of elements of the m×n $H(p)$ we form the rm×rm generalized companion matrix Ω, and from the A_i of Eq. (3.4a) we form the rm×rn generalized Hankel matrix S_r; precisely

$$\Omega = \left[\begin{array}{c|c|c|c} 0 & 1_m & 0 & 0 \\ 0 & 0 & 1_m & 0 \\ & & \ddots & \\ 0 & 0 & 0 & 1_m \\ \hline -a_r 1_m & -a_{r-1} 1_m & \cdots & -a_1 1_m \end{array}\right], \quad S_r = \begin{bmatrix} A_o & A_1 & \cdots & A_{r-1} \\ A_1 & A_2 & \cdots & A_r \\ \vdots & \vdots & & \vdots \\ A_{r-1} & A_r & \cdots & A_{2r-2} \end{bmatrix} \qquad (3.4b)$$

Letting δ be the rank of S_r, which is also the degree of H, and introducing $1_{\alpha,\beta}$ to denote the α×β matrix whose first α columns are 1_α and whose last β−α columns are zero, we find matrices M and N to diagonalize S_r; thus

$$M S_r N = \tilde{1}_{\delta,rn}\, 1_{\delta,rm} \qquad (3.4c)$$

A minimal realization is then given by

$$A = 1_{\delta,rm} M \Omega S_r N \tilde{1}_{\delta,rn}, \quad B = 1_{\delta,rm} M S_r \tilde{1}_{n,rn} \qquad (3.5a)$$

$$C = 1_{m,rm} S_r N \tilde{1}_{\delta,rn}, \quad D = A(\infty) \qquad (3.5b)$$

For example, one can check that $A_i = C A^{i} B$ to see that a realization is obtained.

Of course Eqs. (3.3) remain valid in the time-invariant case where a constant $\underset{\sim}{P}$ can relatively easily be determined by solving the linear algebraic equations of (3.3). However, it is also true that [19, p. 83]

$$\underset{\sim}{P} = \int_{-\infty}^{t} \underset{\sim}{\Phi}(t-\tau,0)\underset{\sim}{Q}\tilde{\underset{\sim}{\Phi}}(t-\tau,0)d\tau \qquad (3.6)$$

which clearly indicates the positive definite property of $\underset{\sim}{P}$ with $\underset{\sim}{Q}$.

If the given matrix $\underset{\sim}{H}(p)$ is a positive-real admittance matrix [20, p. 96] then it will be of interest to form an auxiliary admittance matrix (from a particular realization)

$$\underset{\sim}{Y}_c = \begin{bmatrix} \underset{\sim}{D} & \underset{\sim}{C} \\ -\underset{\sim}{B} & -\underset{\sim}{A} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{D} & \underset{\sim}{C}_o\underset{\sim}{T} \\ -\underset{\sim}{T}^{-1}\underset{\sim}{B}_o & \underset{\sim}{T}^{-1}\dot{\underset{\sim}{T}}-\underset{\sim}{T}^{-1}\underset{\sim}{A}_o\underset{\sim}{T} \end{bmatrix} \qquad (3.7)$$

By a proper choice of the indicated $\underset{\sim}{T}$, any initial constant realization R_o can be brought [by the use of Eq. (2.6)] to yield a positive-real $\underset{\sim}{Y}_c$. Again the details are rather complicated [21], but the process can be described as follows. One forms and factors (the para-Hermitian part of $\underset{\sim}{H}$)

$$\underset{\sim}{H}(p) + \tilde{\underset{\sim}{H}}(-p) = \tilde{\underset{\sim}{W}}(-p)\underset{\sim}{W}(p) \qquad (3.8)$$

to obtain $\underset{\sim}{W}$, which is assumed to be the unique factor (to within orthogonal multipliers) which is holomorphic together with its right inverse in Re $p > 0$ [22][23, p. 175]. A minimal realization is found for $\underset{\sim}{W}$ which is of the form $\underset{\sim}{A}_o$, $\underset{\sim}{B}_o$, $\underset{\sim}{L}$, $\underset{\sim}{W}(\infty)$ where $\underset{\sim}{A}_o$ and $\underset{\sim}{B}_o$ are those for $\underset{\sim}{H}$ itself. This $\underset{\sim}{L}$ is used to define $\underset{\sim}{Q} = \underset{\sim}{L}\tilde{\underset{\sim}{L}}$, the resulting $\underset{\sim}{P}$ for Eq. (3.3a) found, and $\underset{\sim}{T} = \underset{\sim}{P}^{-\frac{1}{2}}$ chosen. Equation (3.3b) can then be used, with $\underset{\sim}{B} = \tilde{\underset{\sim}{C}}-\underset{\sim}{T}\underset{\sim}{L}\tilde{\underset{\sim}{W}}(\infty)$, to show that the symmetric part of $\underset{\sim}{Y}_c$ is positive semi-definite and, hence, that $\underset{\sim}{Y}_c$ is PR. In view of the fact that Eqs. (3.3) hold in the time-varying case, almost identical results hold to show that a suitable passive admittance impulse response matrix has a passive $\underset{\sim}{Y}_c(t)$ [24].

To this point we have discussed finding a state-space realization when the impulse response or transfer function is given. The results on hand, then, are appropriate for synthesis. However, for integrated circuits it is imperative to have a complete analysis of a complicated design available, including all known parasitic effects, before actual construction is undertaken. For this, analysis via the computer using state-space methods and appropriate equivalent circuit models is of considerable assistance. By use of the capacitor-loaded-gyrator inductor equivalent of Fig. 2.5a), as well as a capacitance scaling through cascade gyrator transformer equivalents, all dynamical elements can be removed as unit capacitors. Between inputs, outputs, and unit capacitors (which serve to define the state) there exists a (resistive) network which is relatively easily characterized since it is described by algebraic relationships.

For example if an admittance impulse response is under consideration, the structure of Fig. 3.1a) results where the (n+k)-port resistive network is described by its coupling admittance $\underset{\sim}{Y}_c(t) = [\underset{\sim}{y}_{ij}]$; noting that $\underset{\sim}{u} = \underset{\sim}{v}$, $\underset{\sim}{y} = \underset{\sim}{i}$, $\underset{\sim}{v}_c = \underset{\sim}{x}$, $\underset{\sim}{i}_c = -\dot{\underset{\sim}{x}}$ gives [25]

$$\underset{\sim}{Y}_c \begin{bmatrix} \underset{\sim}{u} \\ \underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{y}_{11} & \underset{\sim}{y}_{12} \\ \underset{\sim}{y}_{21} & \underset{\sim}{y}_{22} \end{bmatrix} \begin{bmatrix} \underset{\sim}{u} \\ \underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{y} \\ -\dot{\underset{\sim}{x}} \end{bmatrix} \qquad (3.9)$$

from which the canonical equations are readily read off yielding $R = \{-\underset{\sim}{y}_{22}, -\underset{\sim}{y}_{21}, \underset{\sim}{y}_{12}, \underset{\sim}{y}_{22}\} = \{\underset{\sim}{A}, \underset{\sim}{B}, \underset{\sim}{C}, \underset{\sim}{D}\}$. Note that this realization justifies Eq. (3.7). But on the computer $\underset{\sim}{Y}_c$ is readily found via reduction of the indefinite admittance matrix or through topological means [3, p. 35]. If the impulse response is of the transfer voltage type, Fig. 3.1b) results and one can proceed by setting up the hybrid matrix $\underset{\sim}{H}_c = [\underset{\sim}{h}_{ij}]$ for the resistive coupling structure

$$\underset{\sim}{H}_c \begin{bmatrix} \underset{\sim}{v}_1 \\ \underset{\sim}{i}_2 \\ \underset{\sim}{v}_3 \end{bmatrix} = \underset{\sim}{H}_c \begin{bmatrix} \underset{\sim}{u} \\ \underset{\sim}{0} \\ \underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{0} \\ \underset{\sim}{y} \\ -\dot{\underset{\sim}{x}} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{i}_1 \\ \underset{\sim}{v}_2 \\ \underset{\sim}{i}_3 \end{bmatrix} \qquad (3.10)$$

from which again the state-variable realization is read off as $R = \{-\underset{\sim}{h}_{33}, -\underset{\sim}{h}_{31}, \underset{\sim}{h}_{23}, \underset{\sim}{h}_{21}\}$. To set up $\underset{\sim}{H}_c$ one can adjoin unit gyrators at the second set of ports and again find the coupling admittance, which is numerically equal to $\underset{\sim}{H}_c$, by reducing the easily established indefinite admittance. Almost identical results hold if nonlinear capacitors are present by choosing the charge q for the state [26].

In summary to this point, given either a circuit, an impulse response, or a transfer function, the methods of this section apply to allow one to obtain a state-space realization if one exists. In particular, a minimal realization results from either $\underset{\sim}{h}(t,\tau)$ or $\underset{\sim}{H}(p)$, while several transformations which preserve stability properties have been discussed for passive synthesis purposes. Given a circuit, a simple and computer oriented analysis exists for setting up the canonical equations. With these results now at our disposal we can next go to actual synthesis.

IV. Synthesis Techniques

Using the results developed above we next obtain synthesis methods based upon the state. The first uses classical analog computer simulations while the final one employs gyrators for a completely passive synthesis. All methods employ, in the first instance, a minimum number of capacitors all of which can be assumed grounded and equal.

Throughout this section we will assume that an $m \times n$, separable, order zero, impulse response $\underset{\sim}{h}(t,\tau)$ or real-rational transfer function $\underset{\sim}{H}(p)$, $\underset{\sim}{H}(\infty)$ finite, is given with a desire for a finite circuit realization in a form suitable for integration.

A. Block Diagram Synthesis

From Eqs. (3.2) or (3.5) we can obtain a minimal state-space realization $R = \{\underset{\sim}{A}, \underset{\sim}{B}, \underset{\sim}{C}, \underset{\sim}{D}\}$. The block diagram of Fig. 2.1 is then implemented by the use of the integrator and summer of Fig. 2.5 where variable gain amplifiers [Fig. 2.4] are cascaded with the latter if the realization is time-dependent. The method is most appropriate for voltage-to-voltage transfer since then the use of terminating converters is avoided.

Although this technique is easily described, it is extremely practical especially in the scalar case (m=n=1) where low sensitivity results occur by factoring $\underset{\sim}{H}(p)$ to yield a cascade of degree one or two sections [27]. These degree two structures will be described in more detail in the next section. But we point out that synthesis of a 1×1 $\underset{\sim}{H}(p)$ by synthesizing low degree factors has several advantages besides that of sensitivity over a direct synthesis of $\underset{\sim}{H}(p)$; for example each section can be separately frequency scaled to assist in obtaining reasonable element values while also practical stability properties are improved by the avoidance of multiple loop feedback paths.

B. Admittance Synthesis

If the given impulse response or transfer function matrix is an $n \times n$ admittance it is most convenient to form the coupling admittance $\underset{\sim}{Y}_c$ of Eq. (3.7) from a minimal state-space realization. A circuit results from a synthesis of the $(m+\delta) \times (n+\delta)$ coupling followed by capacitive termination at the δ final ports, as shown in Fig. 3.1.

A synthesis of $\underset{\sim}{Y}_c = \left[y_{ij}\right]$ result in several ways. One method is to realize each scalar y_{ij} by a DVCCS of transconductance y_{ij} connecting port i to port j; this allows all ports and, hence, all capacitors to be grounded. An alternate method of realization is to decompose $\underset{\sim}{Y}_c$ into its symmetric and skew-symmetric parts

$$\underset{\sim}{Y}_c = \underset{\sim}{Y}_{csy} + \underset{\sim}{Y}_{csk} \tag{4.1a}$$

$$2\underset{\sim}{Y}_{csy} = \underset{\sim}{Y}_c + \tilde{\underset{\sim}{Y}}_c, \quad 2\underset{\sim}{Y}_{csk} = \underset{\sim}{Y}_c - \tilde{\underset{\sim}{Y}}_c \tag{4.1b}$$

The skew-symmetric part can be immediately realized as a parallel connection of gyrators while the symmetric part can be diagonalized to ($\dotplus$ denotes the direct sum)[28, p. 298]

$$\underset{\sim}{Y}_{csy} = \tilde{\underset{\sim}{G}}_c[\underset{\sim}{1}_r \dotplus (-\underset{\sim}{1}_\rho)]\underset{\sim}{G}_c \tag{4.2a}$$

Here $\rho = 0$ if $\underset{\sim}{Y}_c$ is positive real while $\underset{\sim}{Y}_{csy}$ is realized by loading a gyrator network, of admittance matrix

$$\underset{\sim}{Y}_G = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{G}_c \\ -\tilde{\underset{\sim}{G}}_c & \underset{\sim}{0} \end{bmatrix} \tag{4.2b}$$

in r positive and ρ negative unit resistors, as shown in Fig. 4.1a). A negative resistor can be obtained from a DVCCS as shown in Fig. 4.1b); note that it and all the capacitors and gyrators can occur with a common ground. We also see that if a given $\underset{\sim}{H}(p)$ is positive real then the transformations on the state mentioned after Eq. (3.8) allow a completely passive synthesis by this alternate method of synthesis of $\underset{\sim}{Y}_C$ [in contrast to the synthesis using the active DVCCS]. Of course our use of state-variable realization theory has limited us to those $\underset{\sim}{H}(p)$ which are finite at infinity, that is, which have no pole at infinity. More general rational positive real admittances can be synthesized by extracting the pole at infinity as a parallel capacitor network and applying this state-space technique to the remainder.

C. Passive Transfer Synthesis

Let us assume that the given m×n impulse response or transfer function is for voltage inputs at one set of ports, $\underset{\sim}{u} = \underset{\sim}{v}_1$, to (short-circuit) current outputs at another set of ports, $\underset{\sim}{y} = \underset{\sim}{i}_2$. We will then be able to obtain a passive realization as follows [29].

From a minimal realization, transformed to have the symmetric part of $\underset{\sim}{A}$ negative definite [Eq. (3.3b)], we set up the canonical state variable equations which are identical to

$$\dot{\underset{\sim}{x}} = \underset{\sim}{A}\underset{\sim}{x} + [\underset{\sim}{B}, \quad \tilde{\underset{\sim}{C}}]\begin{bmatrix}\underset{\sim}{v}_1 \\ \underset{\sim}{v}_2\end{bmatrix} \tag{4.3a}$$

$$\begin{bmatrix}\underset{\sim}{i}_1 \\ \underset{\sim}{i}_2\end{bmatrix} = \begin{bmatrix}\tilde{\underset{\sim}{B}} \\ \underset{\sim}{C}\end{bmatrix}\underset{\sim}{x} + \begin{bmatrix}\underset{\sim}{0} & -\tilde{\underset{\sim}{D}} \\ \underset{\sim}{D} & \underset{\sim}{0}\end{bmatrix}\begin{bmatrix}\underset{\sim}{v}_1 \\ \underset{\sim}{v}_2\end{bmatrix} \tag{4.3b}$$

when $\underset{\sim}{v}_2 = \underset{\sim}{0}$ is chosen and $\underset{\sim}{i}_1$ is ignored. But in this augmented form the augmented input $\tilde{\underset{\sim}{u}}^* = [\tilde{\underset{\sim}{v}}_1, \tilde{\underset{\sim}{v}}_2]$ and output $\tilde{\underset{\sim}{y}}^* = [\tilde{\underset{\sim}{i}}_1, \tilde{\underset{\sim}{i}}_2]$ define an admittance description. We can then form the coupling admittance, analogous to Eq. (3.7) but with the augmented matrices,

$$\underset{\sim}{Y}_C = \left[\begin{array}{cc|c}\underset{\sim}{0} & -\tilde{\underset{\sim}{D}} & \underset{\sim}{B} \\ \underset{\sim}{D} & \underset{\sim}{0} & \underset{\sim}{C} \\ \hline -\underset{\sim}{B} & -\tilde{\underset{\sim}{C}} & -\underset{\sim}{A}\end{array}\right] \tag{4.4}$$

Synthesis of $\underset{\sim}{Y}_C$ then follows in a passive manner using the method described at Eq. (4.1) & (4.2), since this realization satisfying Eq. (3.3b) guarantees that $\underset{\sim}{Y}_{Csy} = \underset{\sim}{0}_{n+m} \dotplus [\tilde{\underset{\sim}{T}}\underset{\sim}{Q}\underset{\sim}{T}]$ is positive semidefinite (in fact of rank δ). Fig. 4.2 gives the final circuit where the constraint $\underset{\sim}{v}_2 = \underset{\sim}{0}$ is readily seen to hold because of the output short circuit; the gyrators are seen to absorb any time variations, if present, while the matrix multiport symbolism should be clear. Actually, by the use of appropriate equivalences the dissipation can be moved to the external ports if so desired (time-variable terminating resistors may be needed as a consequence, however).

If a synthesis of voltage to (open-circuit) voltage response is desired, then n open-circuited gyrators can be used to replace the output short circuits shown in Fig. 4.2, allowing the original synthesis to really be that of voltage to current as just described (recall that a gyrator transforms a load into its dual, hence an open circuit into a short circuit).

In summary our results allow any separable kernel of order zero having an exponentially asymptotically stable realization to be synthesized by a passive circuit using a minimim number of capacitors all of which are grounded and identical.

D. Lumped-Distributed Synthesis

In closing this section we briefly mention some extensions to circuits containing RC transmission lines.

If one analyzes the circuit of Fig. 4.3 he finds [30, p. 152]

$$\frac{v_2}{v_1}(p) = \frac{1}{\cosh\sqrt{pRC}} = \frac{1}{P}; \qquad P = \cosh\sqrt{pRC} \tag{4.5}$$

Consequently, the uniformly distributed RC line acts as an integrator in the P plane (the unity gain amplifier is for isolation). A P-plane transfer function $\underline{H}(P)$ can then be synthesized by using Fig. 2.1 with p replaced by P and a state-space realization for $\underline{H}(P) = \underline{D} + \underline{C}[P\underline{1}_k - \underline{A}]^{-1}\underline{B}$; of course the lumped resistor summer of Fig. 2.5 would be used, giving a lumped-distributed synthesis. Suitable $\underline{H}(P)$ can be obtained from p-plane data using appropriate approximations [31, p. 137].

Using the changes of variables $\lambda = \sqrt{p}$ and $s = \text{ctnh}\sqrt{pRC/2}$ an admittance matrix $\underline{Y}(p)$ describing a passive reciprocal lumped-distributed RC network can be synthesized by appropriate substitutions to obtain a rational two-variable admittance $\underline{H}(s,\lambda) = \underline{Y}(p)/\sqrt{p}$; from the two-variable admittance, which is positive-real if $\underline{Y}(p)$ is, a state-variable type of realization is formed to yield

$$\underset{\sim}{H}(s,\lambda) = \underset{\sim}{D}(\lambda) + \underset{\sim}{C}(\lambda)[s\underset{\sim}{1}_{\delta s} - \underset{\sim}{A}(\lambda)]^{-1}\underset{\sim}{B}(\lambda) \tag{4.5}$$

The "realization" can be found using results similar to Eqs. (3.5)[32] [33], while a synthesis follows on forming an admittance matrix $\underset{\sim}{Y}_c(\lambda)$, as in Eq. (3.7), subject to appropriate transformation; the resultant coupling admittance is synthesized by lumped resistors, capacitors, and gyrator formed transformers, while $\underset{\sim}{Y}(p)$ results by loading in open or short circuited transmission lines [34].

V. Degree Two Realizations

Because sensitivity to circuit element changes is often smallest [35], many practical scalar transfer functions are synthesized as the cascade of degree one or two sections. Here we outline how some of the above ideas apply to such situations.

A. Operational Amplifier Configuration

Let us consider the synthesis of the low-pass voltage transfer function

$$H(p) = \frac{1}{p^2+2\zeta\omega_n p+\omega_n^2} \tag{5.1a}$$

(we have dropped boldfacedness to indicate the scalar nature of H).

To formulate a minimal state-variable realization through Eqs. (3.5) we need to make an expansion about infinity

$$H(p) = \frac{1}{p^2} - \frac{2\zeta\omega_n}{p^3} + \ldots = \frac{A_1}{p^2} + \frac{A_2}{p^3} + \ldots \tag{5.1b}$$

We have $\delta = r = 2$, $m = n = 1$, thus

$$\underset{\sim}{\Omega} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, \quad \underset{\sim}{S}_2 = \begin{bmatrix} 0 & 1 \\ 1 & -2\zeta\omega_n \end{bmatrix} \tag{5.2}$$

With $\underset{\sim}{1}_{\delta,rm} = \underset{\sim}{1}_{\delta,rn} = \underset{\sim}{1}_2$ and $\underset{\sim}{1}_{m,rm} = \underset{\sim}{1}_{n,rn} = [1,0]$ we find directly from Eqs. (3.5)

$$\underset{\sim}{A} = \begin{bmatrix} 0 & -\omega_n^2 \\ 1 & -2\zeta\omega_n \end{bmatrix}, \quad \underset{\sim}{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \underset{\sim}{C} = [0,\ 1], \quad D = 0 \tag{5.3}$$

Using hopefully self-explanatory notation, the appropriate interpretation is given in Fig. 5.1 of the state-variable block diagram of Fig. 2.1.

It turns out that the sensitivity of such structures to component changes is relatively low [27][36]. For example, if K is the gain of the lower integrator of Fig. 5.1 we can calculate the sensitivity

$$S_K^{H(p)} = \frac{K}{H(p)} \frac{\partial H(p)}{\partial K} \tag{5.4}$$

as follows. An analysis of the integrator of Fig. 2.5b) shows that for finite gain K in the DVCVS we have as the integrator transfer function

$$H_i(p) = \frac{2K}{RpC(K + 2) + 4} \tag{5.5}$$

Replacing the p in the (2,2) term of $p\underline{1}_2 - \underline{A}$ by $p(1 + [2/K]) + [2/K]$, assuming RC = 2, gives the actual dependence of H(p) on K; thus

$$H(p) = [0\ 1]\begin{bmatrix} p & \omega_n^2 \\ -1 & p(1 + \frac{2}{K}) + \frac{2}{K} + 2\zeta\omega_n \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{5.5a}$$

$$= \frac{1}{p^2(1 + \frac{2}{K}) + (\frac{2}{K} + 2\zeta\omega_n)p + \omega_n^2} \tag{5.5b}$$

Application of Eq. (5.4) gives the sensitivity

$$S_K^{H(p)} = \frac{2p(p + 1)}{K[(1 + \frac{2}{K})p^2 + (\frac{2}{K} + 2\zeta\omega_n)p + \omega_n^2]} \tag{5.6a}$$

which approaches zero for all p as K becomes infinite as desired. Thus

$$\lim_{K \to \infty} S_K^{H(p)} = 0 \tag{5.6b}$$

In other words, the differential effects of mask misalignment or errant diffusion with regard to the amplifier tend to become unimportant if the gain is sufficiently high. Similar results hold for the quality factor and undamped natural frequency $\left\{ \text{eg., } S_K^{\omega_n} = 2K/[K(K + 2)] \right\}$, while sensitivities with respect to passive elements are also relatively small.

A point of considerable practical interest is that $Q = 1/2\zeta$ is very easily adjusted in Fig. 5.1 since it enters in only one multiplier; Q can be adjusted for example by the insertion of one variable gain amplifier of the type of Fig. 2.4 [in cascade with the bottom multiplier in Fig. 5.1]. Further, by generalizing these results to

$$H(p) = \frac{\alpha_3 p^2 + \alpha_2 p + \alpha_1}{p^2 + 2\zeta\omega_n p + \omega_n^2} \tag{5.7}$$

one circuit can be obtained with pickoff points to give low-pass, band-pass, and high-pass responses with adjustable Q and ω_n and low sensitivities [27]. Such a device has been completely integrated using MOS devices and is available on the commercial market [37]; its photomicrograph is shown in Fig. 5.2.

B. Gyrator Structure

Next let us synthesize the transfer function of Eq. (5.1a), considered as voltage to current, in the passive way exhibited in Fig. 4.2. Without loss of generality we can assume $\omega_n = 1$ by a frequency normalization, thus

$$H(p) = \frac{1}{p^2 + 2\zeta p + 1} = \frac{\mathcal{L}[i_2]}{\mathcal{L}[v_1]} \tag{5.8a}$$

The realization of Eq. (5.3) immediately yields the coupling admittance of Eq. (4.4) as

$$\underset{\sim}{Y}_C = \left[\begin{array}{c|c|cc} 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ \hline -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 2\zeta \end{array}\right] \tag{5.8b}$$

The result of synthesizing $\underset{\sim}{Y}_C$ to yield H(p) in the form of Fig. 4.2 is shown in Fig. 5.3a) where a pickoff point is also indicated for obtaining a voltage transfer function $-H(p) = \mathcal{L}[v_2]/\mathcal{L}[v_1]$. By using the dual-transforming properties of the gyrator the circuit of Fig. 5.3b) having $H(p) = \mathcal{L}[v_2]/\mathcal{L}[i_1]$ is obtained.

The effects of a lossy gyrator represented by

$$\underset{\sim}{Y}_{gy} = \begin{bmatrix} G_1 & 1 \\ -1 & G_2 \end{bmatrix} \tag{5.9}$$

can be taken into account by adding shunt conductances at the gyrator ports; by a simple equivalence the resulting circuit (with loss) can be redrawn as in Fig. 5.3c). For this latter circuit, sensitivities of Q and ω_n with respect to almost all parameters have been shown to be one-half or under [38, p. 95], while the effects of finite bandwidth [38, p. 97] and phase shift [13, p. 27] of the gyrator amplifiers have been shown to have rather large influence on Q sensitivity.

C. Brune Sections

Finally let us mention an application of state-variables to filter design. Since most practical filter characteristics can be realized as a cascade of Brune sections, and since doubly terminated filters constructed internally of lossless Brune sections have about the lowest possible sensitivity to internal parameter variation [39], it is of interest to set up degree two state-variable analogs of the Brune section to obtain cascade integrated filter sections for constructing high quality integrated filters. Details have been worked out and practicality shown by Sheahan [13, pp. 50-92]. For example, the non-reciprocal Brune section of Fig. 5.4a) has the canonical equations [3, p. 13]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -g_2/c_2 \\ g_2/c_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ g_1-g_2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ -i_2 \end{bmatrix} \qquad (5.10a)$$

$$\begin{bmatrix} v_2 \\ i_1 \end{bmatrix} = \begin{bmatrix} -1/c_1 & 0 \\ 0 & (g_1-g_2)/c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ -i_2 \end{bmatrix} \qquad (5.10b)$$

where the input and output variables are chosen conveniently for cascading. On synthesizing these equations using Fig. 2.1, a cascade results as illustrated in Fig. 5.4b) where a two Brune-section (degree four) filter is shown; note that even though currents are labeled all measured quantities are voltages in the analog simulation. The original sections of Fig. 5.4b) are of course dimensioned according to the normal Darlington theory as applied to nonreciprocal sections [40].

"I might have persevered. I might
Have made her tell me more about the white
Fountain we both had seen "beyond the veil"
But ..." [41, p.44]

VI. Discussion

To be sure we have only scratched the surface of available state-variable results, but those results presented, though in the main quite recent, appear to be the most significant available for integrated circuit technology. For example almost all of the topics covered could be treated in considerable more depth, and some have been in the referenced literature, while other topics, such as those covering infinite-dimensional state spaces as applicable to integrated structures, need developing directly from the start. In fact extensions of the ideas in the direction of more general microsystems, as to include moving parts (eg., resonant gate transistors [42], micromotors [43]) represents a fascinating and significant area for scientific advancement. And although we have outlined some of the points specifically for classical filters they are applicable in many other areas, for example in constructing integrated whitening filters [44]. Of course for integrated circuits there are other than state-variable results of interest [45][46]; for example, most of the theory of active RC circuits [47] becomes available, while extensions of the frequency transformation of Eq. (4.5) also seem particularly significant [30][48].

Most of the orientation of the presented material has been toward completely integrated structures, where some phenomena, as temperature sensitivity [46, p. 216], differ from those occuring in only partially integrated structures. Nevertheless almost all the concepts treated here do directly apply to partially integrated circuits. For example the block diagram of Fig. 5.1 covering low sensitivity degree-two sections can be constructed conveniently from integrated operational amplifiers and lumped resistors and capacitors using classical integrator and summer circuits [27, p. 89].

It is interesting to note that all of the structures discussed contain the minimum number of capacitors. In some cases, as for instance in attempting to minimize the total capacitor area, nonminimal realizations may be of interest and the theory of network equivalence via the state comes into play [5,p.536][6]. In other cases it may be of (theoretical) interest to observe that if one chooses for any t_o

$$T = V^{-\frac{1}{2}}; \quad V(t) = \int_{t_o}^{t} \Phi(t,\tau)\tilde{\Phi}(t,\tau)d\tau, \quad t > t_o \tag{6.1a}$$

then Eq. (3.6b) reduces for any initial A_o to

$$A + \tilde{A} = -V^{-1} \quad t > t_o \tag{6.1b}$$

Thus, we see from Fig. 4.2 that any (perhaps unstable [49]) true transfer matrix (which is real-rational and finite at infinity) has a passive circuit realization; a rather paradoxical result.

In summary, state-variable theory has much to offer the theory and practice of integrated circuits. Its practical significance and structural beauty make the theory of state-variables well worthy of study.

...

"L'été s'en est allé.
Mais sa voix était fraîche comme l'eau des ruisseaux
et ses cheveux sentaient toutes les fleurs de l'été dernier.
Reviendras-tu un jour là-bas?" [50, p. 28]

VII. Acknowledgements-Dedication

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For

L. L. and H. H.

who have brought

sunshine and hope and vitality

when most needed in the world.

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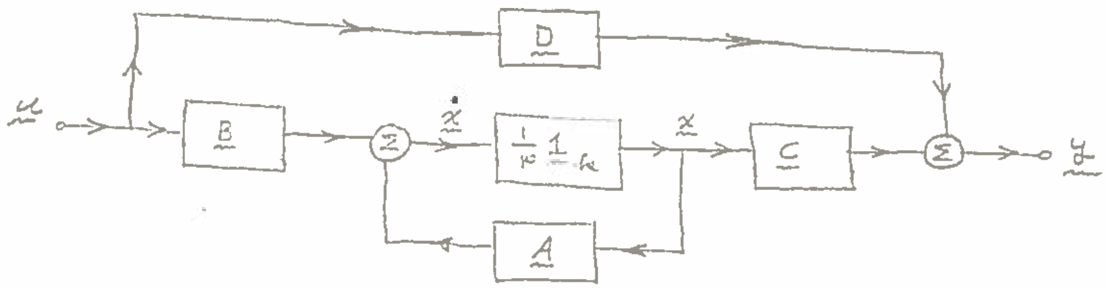
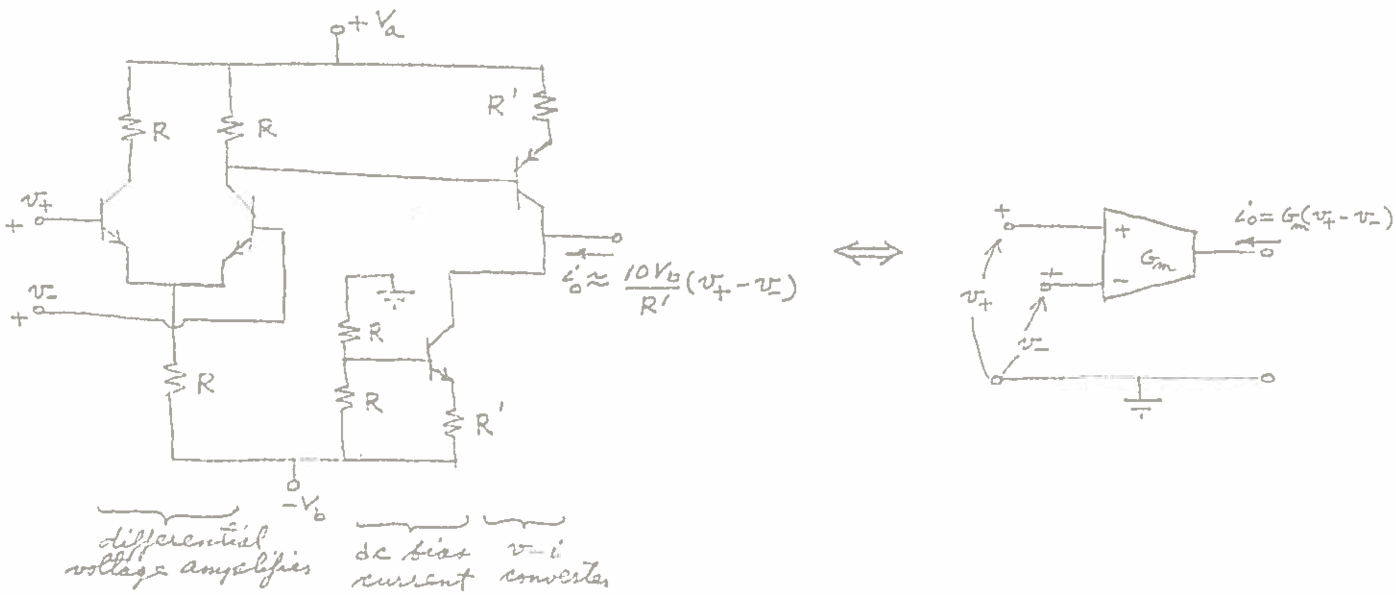


Figure 2.1
Block Diagram for
Canonical State-Variable Equations



a) Basic Circuit

b) Circuit Symbol

Figure 2.2
DVCCS

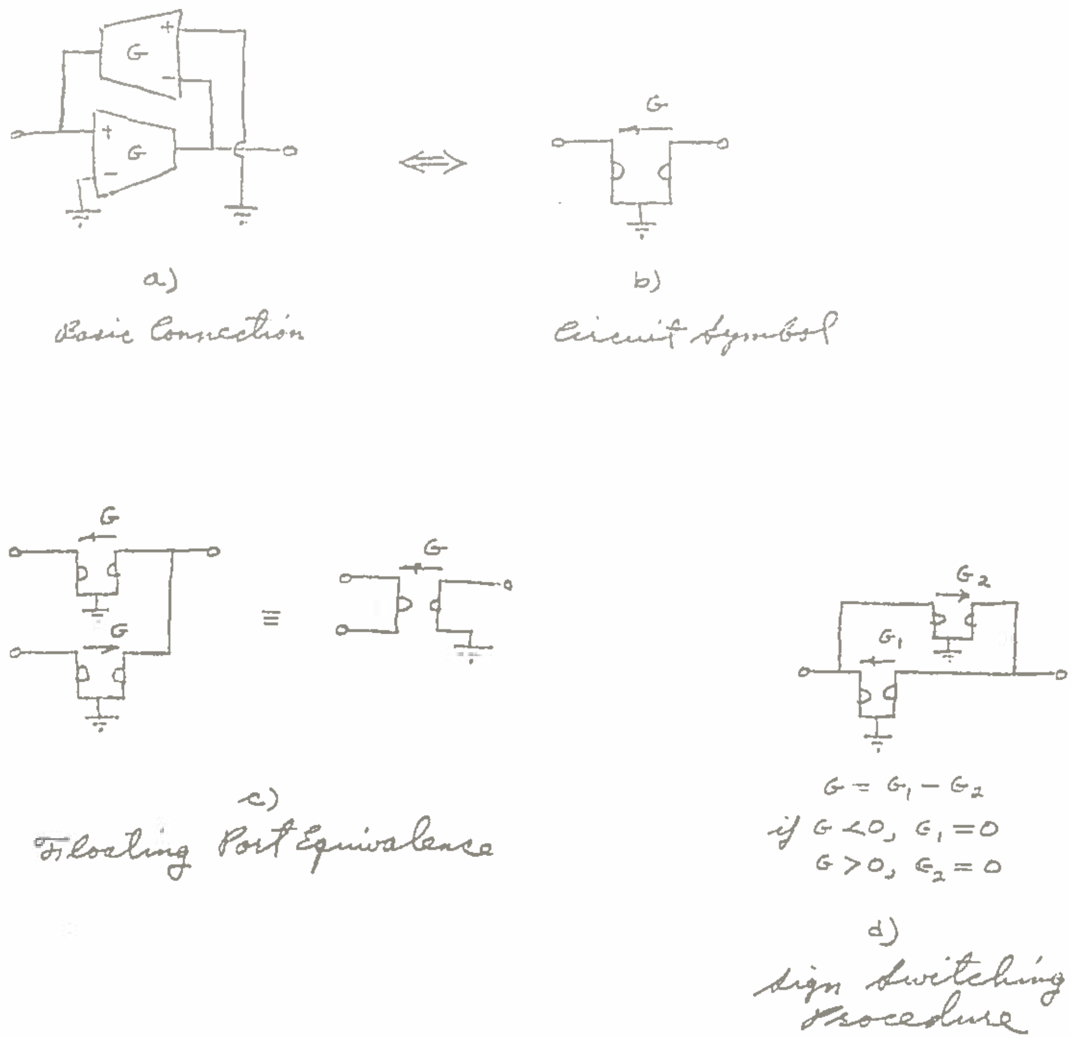


Figure 2.3
The Gyrator

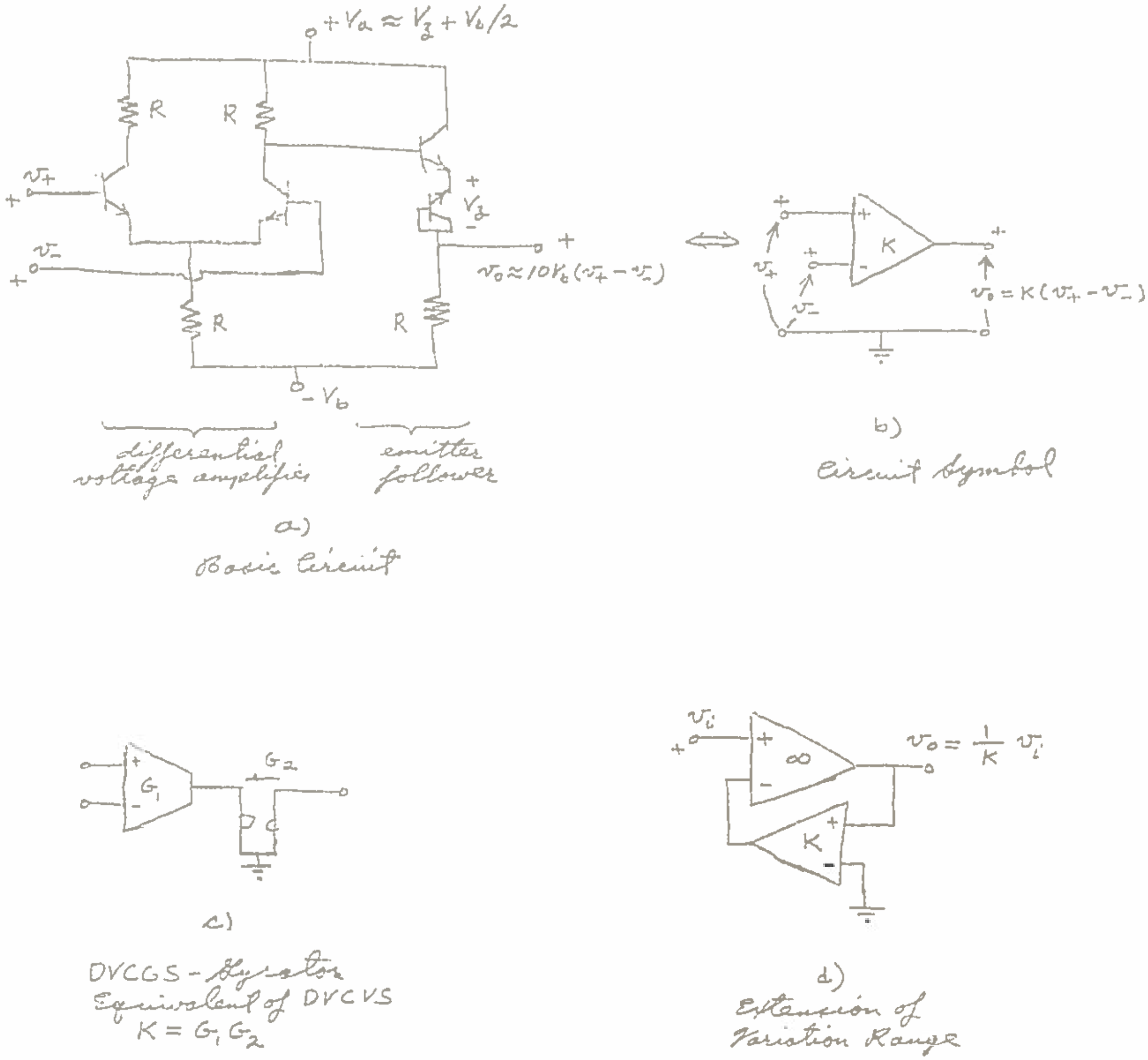
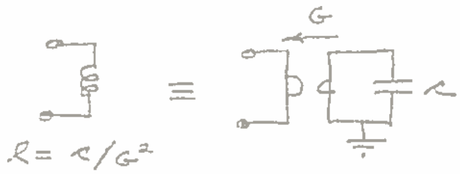
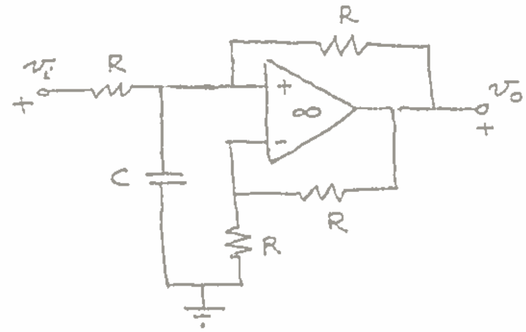


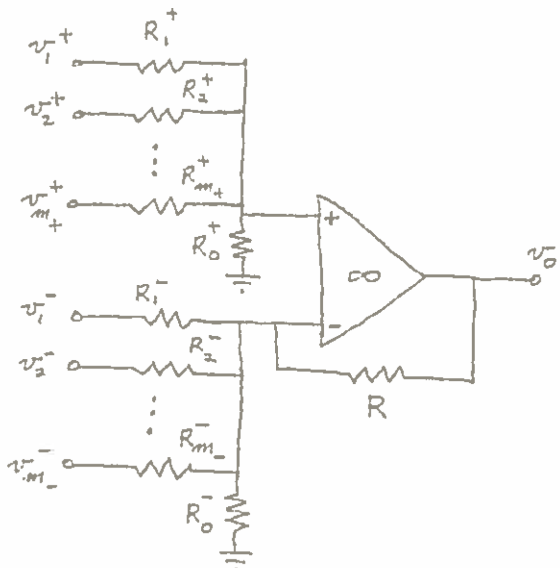
Figure 2.4
DVCVS



a)
Gyrator Usage



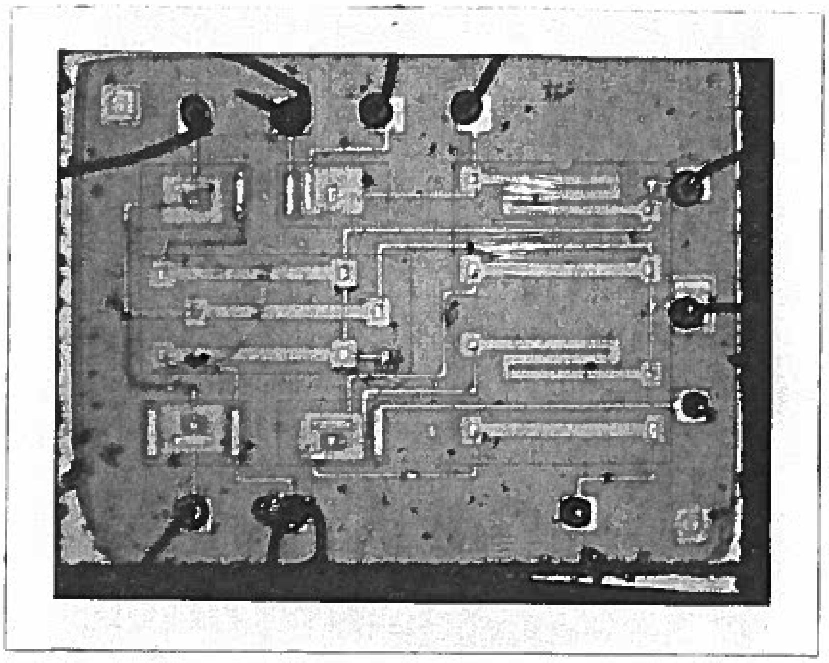
b)
Positive Gain Integrator



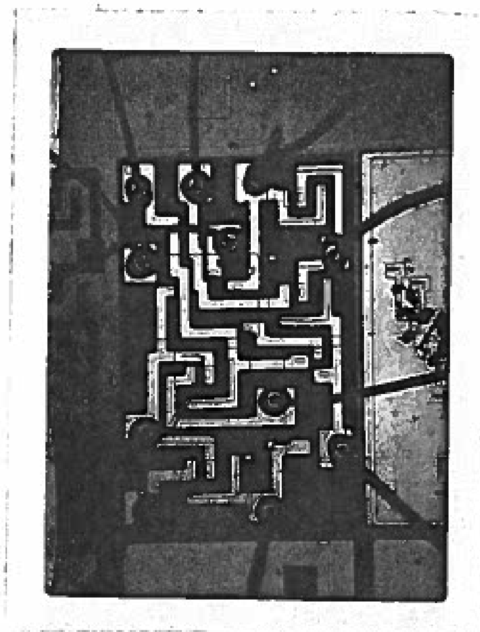
c)
Multiple Input Summer

Figure 2.5

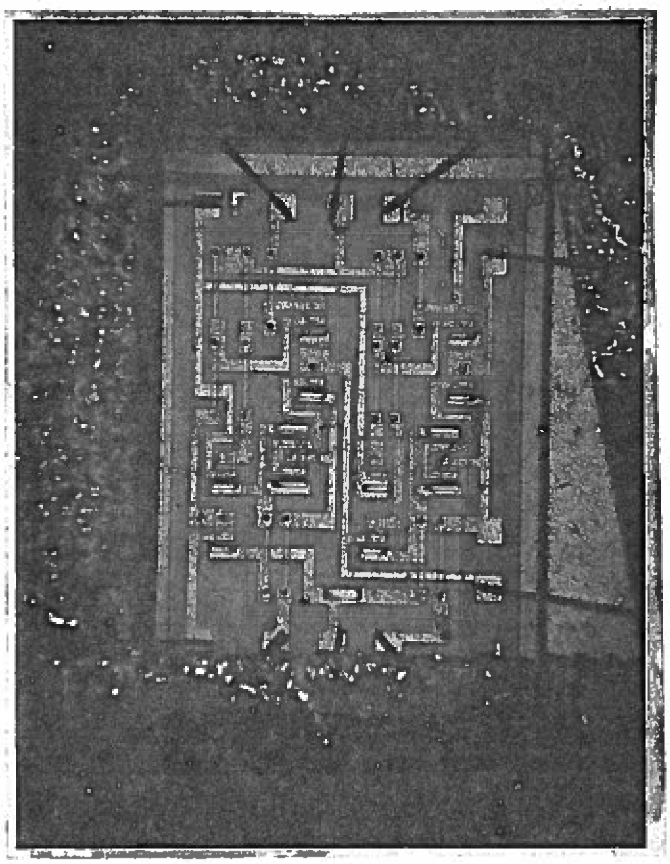
Generating Element Uses



a)
DVCCS

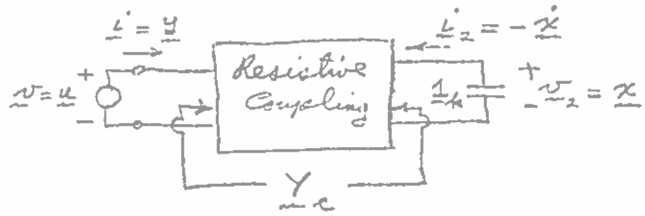


b)
DVCVS



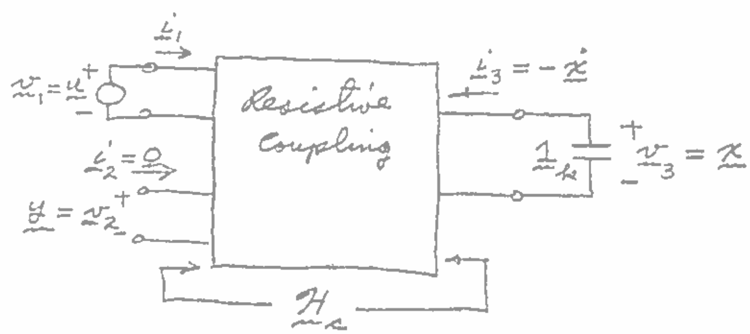
c)
Gyrator

Figure 2.6
Photomicrographs of Integrated
Generating Elements



a)

Admittance Structure



b)

Voltage Transfer Structure

Figure 3.1
Capacitor Extractions
for Analysis

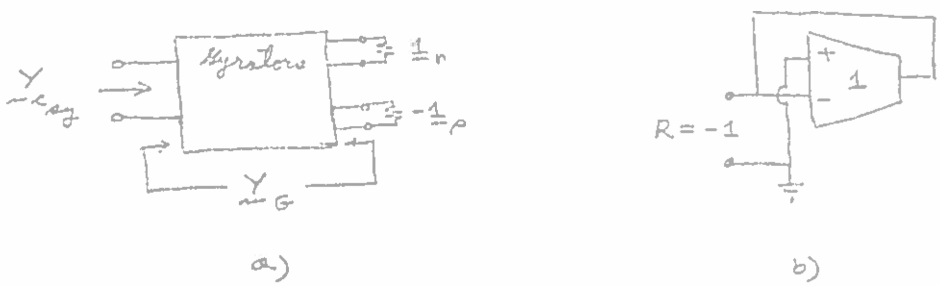


Figure 4.1
Synthesis of $\underset{\sim}{Y}_{e\,sy}$

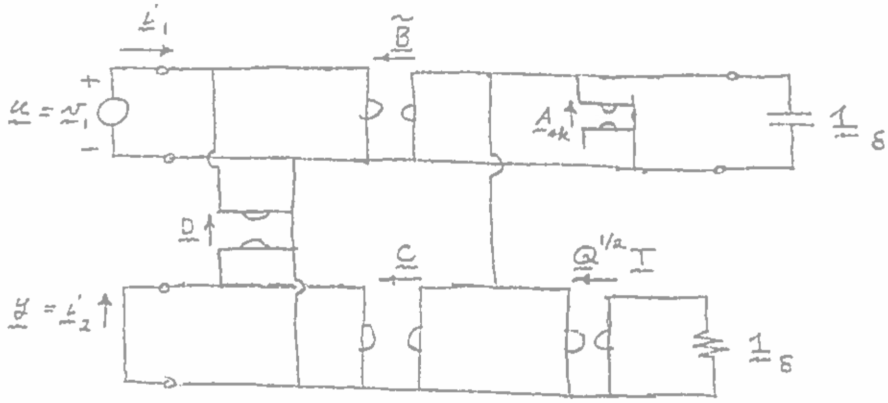


Figure 4.2

Passive Transfer Synthesis

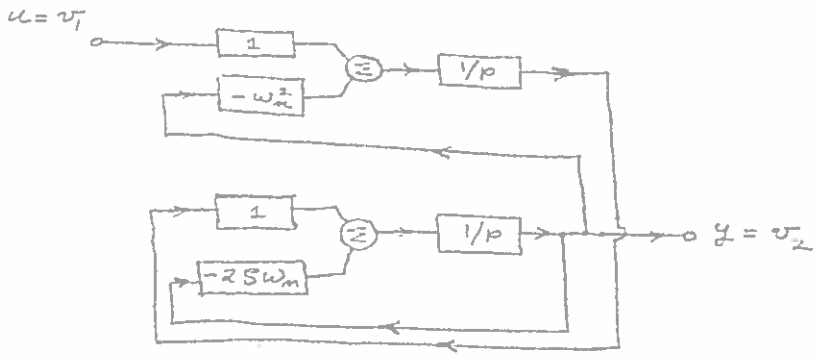


Figure 5.1

Degree Two Low-Pass Schematic

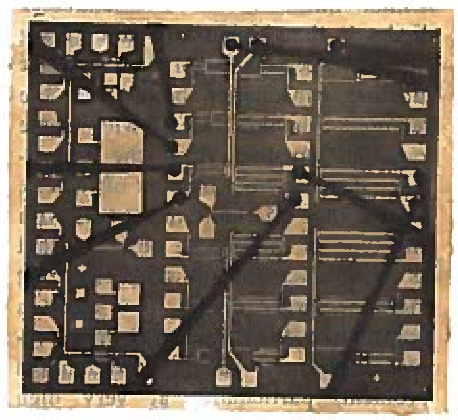
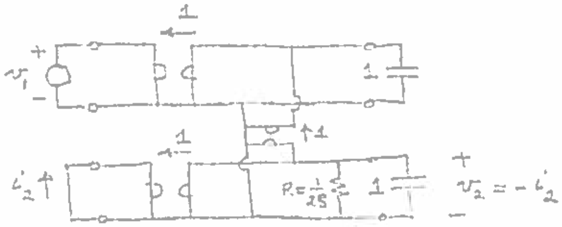
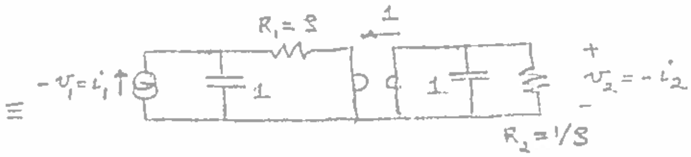


Figure 5.2
Photomicrograph of Degree-Two
IC Active Filter



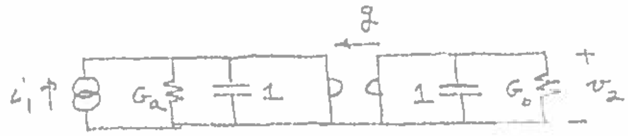
a)

Synthesized Circuit



b)

Transformed Circuit



$$g = \{G_1(G_2+S)+1\} / \{SG_1[G_1(G_2+S)+1] + G_1G_2+1\}$$

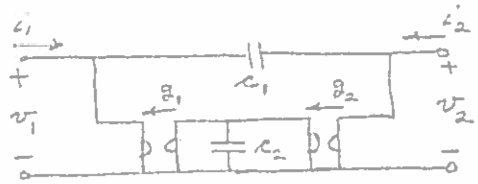
$$G_a = G_1 g \quad , \quad G_b = \{S[G_1(G_2+S)+1] + G_2+S\} g$$

c)

Equivalent with Gyrator Load

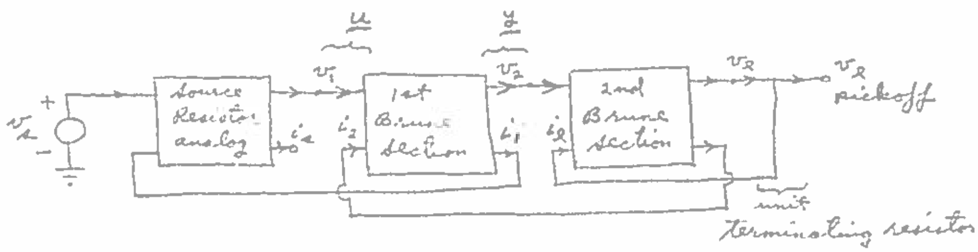
Figure 5.3

Passive Degree Two Structures



a)

Nonreciprocal
Brune Section



b)

Two - Section Analog

Figure 5.4

State - Variable Brune
Section Filtering