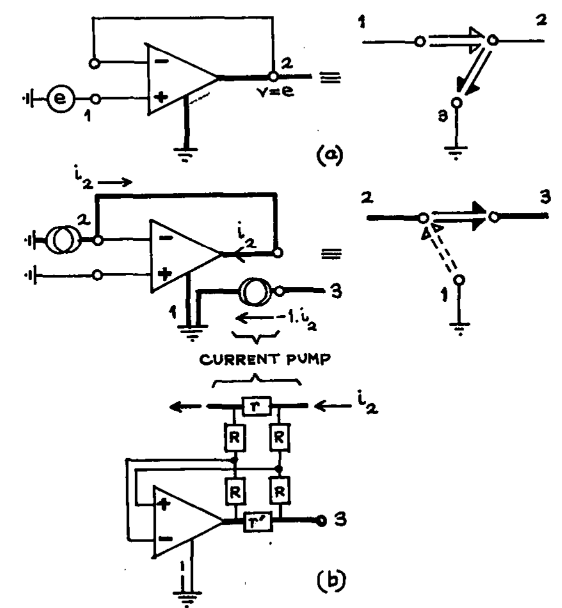

Fig. 2. (a) Voltage follower operational amplifier realization of unitor in its $1\bar{3}2$ orientation. (b) Addition of current pump to (a) to extract current output and thereby obtain accessible realization of unitor in its $2\bar{1}3$ orientation.

pump an equal current out of ground to an access terminal. Such an amplifier pair would comprise a single balanced-input balanced-output operational amplifier and would, of course, have a wider range of application than the conventional single-grounded output units.

A suitable current pump is shown in Fig. 2(b). The present writer has employed this pump extensively in operational amplifier circuit technique and has given a number of applications in a previous paper [8]. Here we wish particularly to show how this pump may be associated with the voltage follower to obtain the T_{213} transfer property of the unitor element and thereby obtain a useful unit for active *RC* realization purposes. In Fig. 2(b) we have inserted a small resistor r in the path of the output current of the voltage follower (but inside the feedback connection, so that the zero output impedance property is preserved). The potential difference developed across r is applied as a voltage reference input to the current pump. The current pump output is monitored in a similar manner by $r'=r$ and feedbacks are applied by two pairs of equal large resistors (it is convenient to make all four the same high value) to the pump input pair to force the two potential differences and therefore (for $r'=r$) the two currents to have the same value. Thus we obtain unilateral unity-gain current transfer (*current follower*) action from the voltage follower output terminal (2) to the output terminal (3) of the pump, relative to the common ground terminal (1). The pump reproduces the inaccessible current follower action that occurs in the primary amplifier, but makes it accessible for external application. We have a four-terminal unit, with one terminal grounded (0), but with floating unitor action between the other three terminals (1, 2, 3). If the direct voltage feedback connection were removed, we would have a five-terminal unit (four-terminal floating amplifier with associated ground connection), which, incidentally, is not a nullor.

III. Extension and Applications

The basic unitor (pair of unity-gain unilateral paths) concept may be generalized in various ways [9], and the circuit that we have obtained readily allows some of these to be realized.

1) The direct voltage feedback path may be replaced by a resistive voltage divider to obtain a voltage gain greater than unity (voltage-amplifying unitor). We remark also that this feedback may alternatively be made on the other side of the double-amplifier unit.

2) The input voltage may be applied under this voltage divider, with the normal voltage follower input grounded, to obtain voltage inversion (with amplification), but with finite input impedance (voltage inverting unitor).

3) The resistor r monitoring the pump output current may be made greater or less than r to obtain a current transfer ratio other than unity.

4) The voltage-input leads to the pump may be transposed in order to invert the current ratio.

5) A voltage attenuator may be inserted before the voltage follower or after the current follower (or both).

Finally, the more basic applications include the following.

1) Connection of the pump current output terminal to the voltage follower input, thereby completing a feedback loop and providing an immittance converter realization, which, with appropriate choice of the above modification, allows a comprehensive range of convertors to be realized [e.g., modification 4)] above gives a negative impedance converter.

2) Coupling of the two paths in cascade by connection of an admittance Y from the voltage follower output to ground to obtain transfer admittance $y_{21}=Y$ [with reversal of sign by 4)]. Two such cascades in reverse parallel make a general immittance inverter, in particular a gyrator.

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Generation of All Finite Linear Circuits Using the Integrated DVCCS

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Abstract—In conjunction with the capacitor, the differential voltage controlled current source (DVCCS) is shown to yield in a relatively simple manner and in integrated form all components needed for linear circuit construction. The effects of dissipative losses in the DVCCS are tabulated for several configurations. Finally, a new building block having two outputs is introduced.

Introduction

A problem of engineering interest and theoretical importance for a given class of systems is that of choosing an appropriate set of generating elements, that is, the set of "shelf" components from which the systems can be constructed, theoretically or practically, through interconnections. If the class is chosen too large, then, for example, inter-

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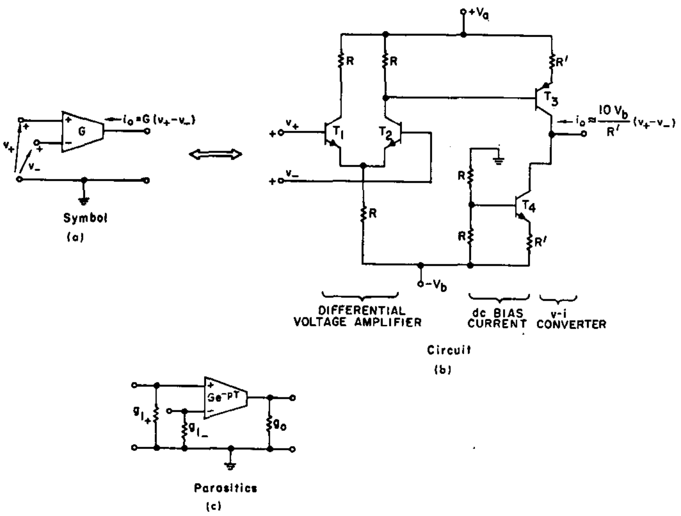

Fig. 1. DVCCS.

connections become overly complicated, whereas if the class is chosen too small, the realizable system characteristics become limited. Consequently, one desires an optimum choice of generating elements, this being especially true where delicate processing is involved, as for integrated circuits. For integrated circuits it is recognized that capacitors, resistors, and transistors form a set of generating elements which is generally too large for the stocking of separate components. Similarly, the set of integrated capacitors and voltage-controlled voltage sources is too limited since, for example, controlled current sources, or even resistors, cannot be obtained. And although the addition of resistors to the set of operational amplifiers and capacitors will yield all circuits through a set of equivalences [1, p. 115], the constructions are often unduly complicated for very simple designs. Here we show that an alternate and perhaps more useful set of generating elements for linear time-invariant integrated circuits consists of the capacitor and differential voltage-controlled current source (DVCCS).

The DVCCS

Fig. 1(a) shows the symbols to be used for the DVCCS, which is assumed ideal in that infinite input and output impedances are obtained in conjunction with the law

$$i_0 = G(v_+ - v_-), \qquad G \geq 0. \tag{1}$$

Fig. 1(b) shows a possible basic circuit [2], which has been integrated [3] and for which the transconductance G can be conveniently varied by adjustment of the bias voltage $G \approx 10V_b/R'$ (at room temperature). Although variation of V_b allows variation of G, we observe that this variation is not extensive since the range of V_b is limited, especially for integrated circuits. Fig. 1(b) shows the possibility of DVCCS realization; however, in industrial practice, for example, more complicated integrated circuits meeting temperature requirements can be used. Principal losses and parasitics for a general DVCCS can be represented by shunt conductances to ground g_{1+}, g_{1-}, and g_0, as shown in Fig. 1(c), as well as a phase shift in the transconductance $G \exp[-pT]$.

Element Generation

In the tabulation of Fig. 2 some of the most useful equivalences are shown with the inclusion of the effects of shunt losses, expressed as fractions of the transconductance:

$$G = g_{1+}/\delta_+ = g_{1-}/\delta_- = g_0/\delta_0. \tag{2}$$

Also $G_{11} = g_{1-} + g_0$ and $G_{22} = g_{1+} + g_0$ are the input and output short-circuit admittances of the gyrator shown in Fig. 2(c). Of course, other circuit elements than those listed can be constructed; for example, the transformer results as a cascade of two gyrators, and the current-controlled voltage source (CCVS) results by cascading gyrators at the input and output of the VCCS. Further, some of the equivalences of Fig. 2 are common knowledge, as for the gyrators, but are given for completeness. Of course one would not ordinarily construct resistors as in Fig. 2(a) and (b), but this possibility shows the generality of the DVCCS especially when compared with the operational amplifier.

Since all finite time-invariant circuits can be generated from a finite number of resistors, capacitors, inductors, transformers, and gyrators [4, p. 263], and since the preceding results show that these are all derivable from the DVCCS and the capacitor in convenient form, we have shown that the DVCCS is a convenient generating element. The end result is of particular importance for the area of integrated circuits where the circuit of Fig. 1(b) establishes feasibility. Although it is often of interest to use the gyrator synthesis techniques [5], [6], because of the low sensitivity of passive constructs, to obtain a DVCCS-capacitor structure, it may often be advantageous to directly synthesize in terms of the DVCCS [7], [8]. For example, a constant coupling admittance matrix derived through state-variable techniques [9] can be synthesized directly in terms of DVCCSs and the result loaded in unit capacitors to yield an input admittance matrix. In any event, the above shows, as with the present DVCVS, that it may be profitable to stock integrated circuit DVCCS, especially with adjustable transconductances as available in the realization of Fig. 1(b). Indeed, note that by further making V_b or R' time variable in the DVCCS of Fig. 1(b), *all* linear finite time-variable circuits can be generated [10], [11]; R' can be the gate-controlled source-to-drain resistance of an FET for example.

DVCCS/DVCVS Device

A more comprehensive building block having two outputs acting simultaneously as a DVCCS and a DVCVS (differential operational amplifier) can be also introduced. This can be used to replace some of the equivalences.

The proposed symbol and a basic realization of a DVCCS/DVCVS element are shown in Fig. 3(a) and (b), respectively. If a DVCVS has sufficiently high voltage gain, it is recognized as a differential operational amplifier. In the circuit two complementary transistors T5 and T6 are used in an impedance transformer circuit to assure zero dc voltage level at both outputs. A practically realized DVCCS/DVCVS circuit, using Siemens transistors BC 107(n-p-n) and AF 124(p-n-p), is shown in Fig. 3(c); it has voltage gain on the order of 5000, a high output impedance of 200 kΩ and a low output impedance less than 10 Ω. In engineering practice more complicated integrated circuits meeting temperature and offset requirements can be used.

Placing, for example, two integrated DVCCS/DVCVS elements in one encapsulation, we can obtain a very versatile building block which can fulfill the functions of 1) two DVCCS, with two buffers, 2) two DVCVS; 3) DVCCS with buffer and DVCVS, 4) gyrator, 5) gyrator and buffer, and 6) gyrator with two buffers.

Such a building block which can be divided into four internal parts is more versatile than normally available operational amplifiers. Of special importance is the simplicity of realization of a gyrator for known applications as well as the introduction of a gyrator with two low-output impedance buffers. This last circuit, for example, can be used for simple simultaneous realization of complex poles and real, imaginary, or complex zeros of a transfer function.

Discussion

We have shown the generality of the DVCCS as an integrated circuit generating element when capacitors are the accompanying generating element. Some of this versatility has been previously recognized [12], while a commercially integrated DVCCS is now available.[1] What we have given here is the complete generality of the DVCCS when used

[1] RCA Electronic Components, Linear Integrated Circuits, Model CA 3060: Operational Transconductance Amplifier Array, Data File 404, Mar. 1970.

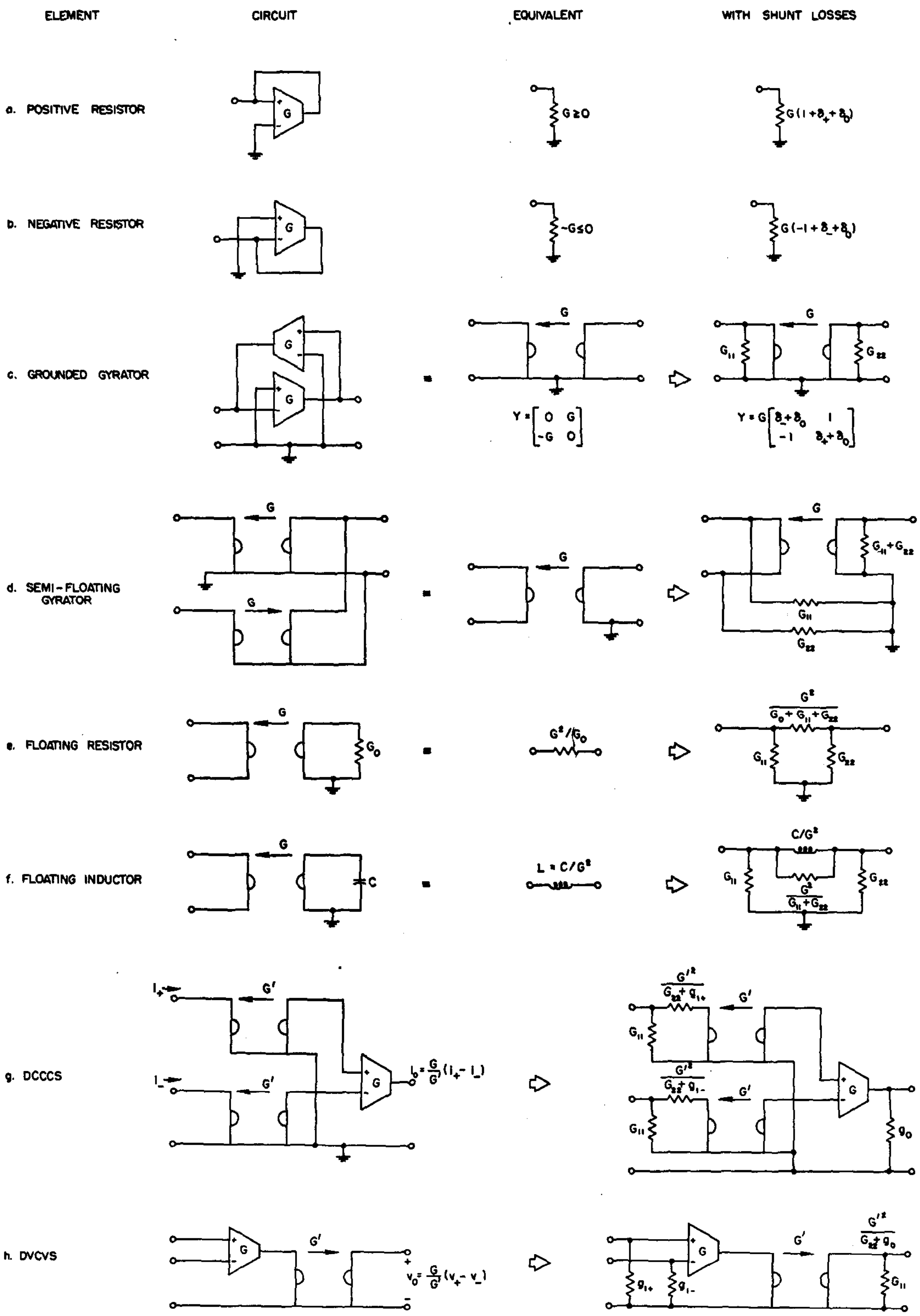

Fig. 2. Some equivalences.

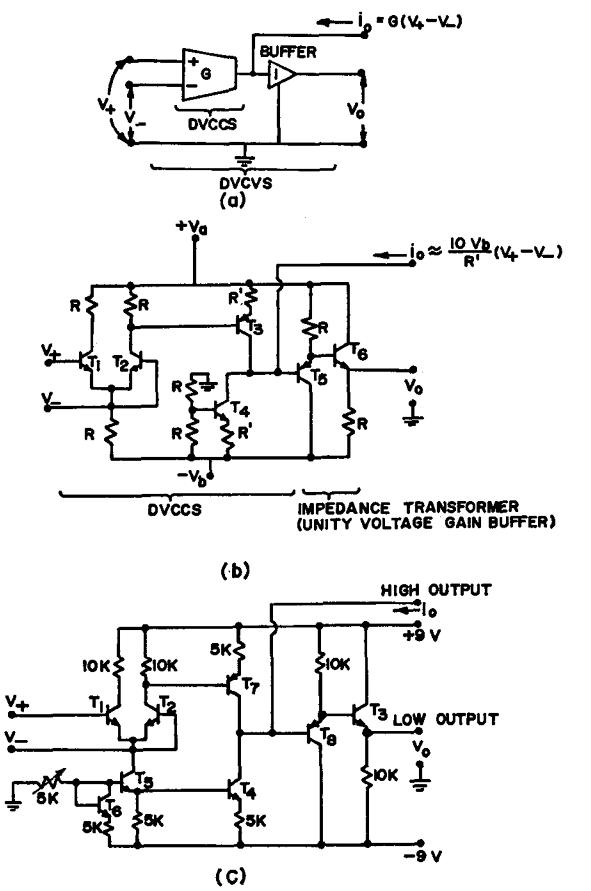

Fig. 3. The DVCCS/DVCVS element. (a) Proposed symbol. (b) Basic realization. (c) Practical realization—T_1–T_6: Siemans BC107; T_7, T_8: Siemans AF124.

with capacitors, as well as a simple circuit convenient for allowing variation in integrated form. With widespread availability of improved integrated DVCCSs of adjustable transconductances, these devices should prove of value for integrated circuit synthesis. In fact, their importance can be seen from the simple gyrator realization Fig. 2(c) and the fact that the often improved characteristics of gyrator-capacitor filters [5] have led to their worldwide consideration. Although synthesis methods and the integrated DVCCS itself exist, it remains to find improved and optimal techniques and circuits. Perhaps at this time the problem is of somewhat a theoretical nature since, for example, from the practical point of view it is presently usually much simpler to use one integrated resistor than its analog realized of differential voltage-controlled current sources. However, even now transistor circuits are used sometimes as resistors in practical integrated networks.

The two-output DVCCS/DVCVS device could prove quite practical due to the versatility obtained with a very simple circuit. Since it too can generate all finite circuits, work on its application is presently being carried on.

What aim could be more significant than that of promoting mutual knowledge and deeper understanding among nations? This aim, amidst all the dangers and threats of the nuclear age, implies confidence in peace, attachment to life, and respect for man [13, p. 3].

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Synthesis of Networks with Reactive Gyrators

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I. Introduction

The gyrator, as originally defined by Tellegen [1], is a two-port network for which the short-circuit admittance matrix is skew-symmetric and real. This two-port network was shown to be lossless and passive with respect to the ports. A more generalized approach to the gyrator was advanced by Mitra and Howard [2], and later by Ramachandran and Swamy [3]. In general the generalized gyrator can be represented by the short-circuit admittance matrix

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}. \tag{1}$$

If $Y_{11}=Y_{22}=0$, then the gyrator is considered ideal. If $Y_{12}=Y_{21}$, then the gyrator is balanced. If these two terms are constant with respect to s, then the gyrator is called resistive; otherwise, it is reactive. Thus there are eight possible combinations.

It is the purpose of this correspondence to determine the necessary and sufficient conditions which an immittance function must satisfy such that it is realizable with only passive elements and the ideal gyrator. This will be accomplished for only the balanced gyrator.

II. Balanced Gyrator: Necessary Conditions

Consider a one-port network consisting of only passive elements and balanced ideal gyrators, and which is initially relaxed. The graph of the network is formed by considering every nongyrator element to be an edge of the graph and by considering two edges of the graph to correspond to each gyrator in order to be consistent with Brown's [4] general two-port representation. By proper permutation of the edges, the edge impedance matrix can be partitioned as

$$[Z_e] = \begin{bmatrix} Z_p & 0 \\ 0 & Z_g \end{bmatrix} \tag{2}$$

where $[Z_p]$ is the submatrix corresponding to the passive elements and $[Z_g]$ is the submatrix corresponding to the balanced ideal gyrators. It is assumed that for every i and j, either z_{ij} or z_{ji} of the submatrix Z_g is a positive real function. The loop-impedance matrix for the network becomes

$$[Z_m] = [Z_s] + [Z_a] \tag{3}$$

where $[Z_s]$ is the symmetric part resulting from the passive elements and $[Z_a]$ is the skew-symmetric part resulting from the gyrators.

Since only one voltage source was assumed present, the one at port one, $[E]$ has only one nonzero term E_1. Let $Z_1(s)$ be defined as the driving point impedance at port one; then the product $[I]^{*\prime}[E]$ becomes

$$[I]^{*\prime}[E] = |I_1|^2 Z_1(s) = [I]^{*\prime}[Z_s][I] + [I]^{*\prime}[Z_a][I]. \tag{4}$$

For s positive and real, $[I]^* = [I]$ and (4) becomes a quadratic form. Under this condition, since $[Z_a]$ is a real skew-symmetric matrix, the

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