Conclusions

A distributed transistor model including linear dc crowding has been analyzed. An exact expression for the short-circuit input impedance of the transistor has been obtained. Asymptotic expansion of this impedance has been made for the extreme cases of "small" crowding and "large" crowding. In both cases, the impedance has been found to be equivalent to that of a uniform transmission line of reduced length.

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Noise in FM Receivers with Negative Feedback

***Abstract*—Using a simple approximate formula [1] which can be derived from an exact solution obtained by the authors [2], it can be shown that the output SNR of an FMFB (frequency modulation feedback) system is almost equivalent to that of the conventional system having the same IF filter bandwidth and modulation index as the FMFB system.**

It is shown in this letter that the output SNR of an FMFB system above and below the threshold is almost equivalent to that of the conventional system having the same modulation index and IF filter bandwidth as the FMFB system.

If the frequency deviation is larger than the IF filter bandwidth, the output SNR can be simply approximated using a formula derived previously by the authors [1]:

$$\left(\frac{N_0}{S_0}\right) \simeq \frac{1}{3(2F_e)A^2\left(\dfrac{C}{N}\right)} + \frac{(2F_e)}{\sqrt{3\pi}A^2(1-e^{-C/N})^2}\sqrt{\frac{N}{C}}\left[1+24\left(\frac{A}{2F_e}\right)^2\left(\frac{C}{N}\right)\right]e^{-C/N} \quad (1)$$

$$\simeq \frac{1}{3(2F_e)A^2\left(\dfrac{C}{N}\right)} + \frac{24}{\sqrt{3\pi}(2F_e)}\sqrt{\frac{C}{N}}e^{-C/N}$$

where A represents the rms frequency deviation normalized by the highest baseband frequency and $2F_e$ represents the IF filter bandwidth normalized by the highest baseband frequency.

The first term represents the triangle noise component and the second term represents the noise component which decides the output SNR at the threshold and below the threshold.

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One should note that the first term decreases with increasing modulation index and the second term is independent of modulation index.

Therefore, for large modulation index, the threshold point may be deteriorated by the noise due to modulation.

For example, the authors have calculated the output SNR of the case discussed by Frutiger [3] and Bayles [4], i.e.,

$$A = \frac{\beta}{\sqrt{2}} = \frac{9.0}{\sqrt{2}}$$

$$2F_e = 3.6$$

where β represents the peak modulation index. The sinusoidal modulation is applied to the case presented in [3].

Table I compares the calculated results using (1) with Frutiger's data [4]. From this comparison, it is seen that the approximate formula of (1) is sufficiently accurate to calculate the output SNR of FMFB systems. The formula of (1) can also be applied to the case where the baseband signal is Gaussian.

TABLE I
Comparison with Frutiger's Results

CNR (dB)	$F_e(C/N)$ (dB)	Authors' Results (dB)	Frutiger's Results (dB)
12.00	14.56	38.73	38.7
11.00	13.56	36.82	37.0
10.00	12.56	32.66	32.3
9.00	11.56	26.08	25.0
8.00	10.56	19.85	18.0

It appears that the authors' conclusion is fundamentally different from those of other authors [5].

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Determinations of State-Variable Equations for Admittance Descriptions Suitable for the Computer

***Abstract*—A simplified procedure is described for finding the state equations of an *n*-port network from its admittance description. The restrictions on this procedure are that the network be linear, but perhaps time-varying, and made up of a finite number of lumped elements in a configuration such that an admittance description exists for the nondynamical portions of the network.**

A study of existing computer routines for obtaining network state-space equations has shown that such routines are rather complicated for many purposes. Here we outline a possible procedure suitable for many applications.

In essence, the problem is to find state equations of the form

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$$\dot{x} = Fx + Gu \qquad (1a)$$
$$y = Hx + Ju \qquad (1b)$$

for a linear n-port network given in schematic form. Considering an admittance description, we choose $u=v$ and $y=i$. Through the use of an inductor equivalent network (a capacitor-loaded gyrator) the original n-port network can be transformed to an $(n+c)$-port network (Fig. 1) from which all the dynamic elements have been extracted as capacitors to leave a network with only algebraic constraints. It is understood that all the active devices are replaced by their equivalent circuits and the dynamic elements removed from them in the manner just described.

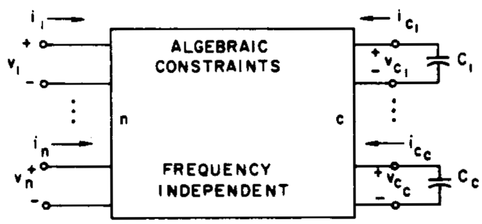

Fig. 1. Dynamic element extraction.

This $(n+c)$-port network can now be described in an $AV=BI$ manner[1] such that $Y_rV=I$ where Y_r is an $(n+c)\times(n+c)$ admittance matrix describing only the resistive network, and V and I are (time-domain) column matrices with $(n+c)$ elements.

Let us arrange V and I as

$$\begin{bmatrix} Y_{nn} & Y_{nc} \\ \hline Y_{cn} & Y_{cc} \end{bmatrix}\begin{bmatrix} v \\ \hline V_c \end{bmatrix} = \begin{bmatrix} i \\ \hline I_c \end{bmatrix} \qquad (2)$$

such that v and i refer to the original terminal ports and V_c and I_c refer to the dynamic element ports. Then by simple rearrangement, $Y_rV=I$ can be brought into the following form, where I_n is the $n\times n$ identity matrix

$$\begin{matrix} n\{ \\ c\{ \end{matrix}\begin{bmatrix} Y_{nn} & Y_{nc} & -I_n & 0 \\ \hline Y_{cn} & Y_{cc} & 0 & -I_c \end{bmatrix}\begin{bmatrix} v \\ V_c \\ i \\ I_c \end{bmatrix} = 0. \qquad (3)$$

Carrying out the multiplication of (3) and rearranging gives

$$\begin{matrix} n\{ \\ c\{ \end{matrix}\begin{bmatrix} 0 \\ \hline -I_c \end{bmatrix} = -\begin{bmatrix} Y_{nc} \\ \hline Y_{cc} \end{bmatrix}V_c - \begin{bmatrix} Y_{nn} \\ \hline Y_{cn} \end{bmatrix}v + \begin{bmatrix} i \\ \hline 0 \end{bmatrix}\begin{matrix} \}n \\ \}c \end{matrix}. \qquad (4)$$

To form the state variables, we use the general approach of considering charges on capacitors and fluxes in inductors;[2] since we have removed all the inductors, we need only consider the capacitor charges. That is, let

$$x = CV_c \qquad (5)$$

where C is the $c\times c$ diagonal matrix of capacitances (note extraction numbering in Fig. 1). Assuming C is nonsingular (which will be true for all t in the time-invariant case), (5) can be solved for V_c:

$$V_c = C^{-1}x. \qquad (6)$$

Having already chosen x as the state-variable matrix defined by (5), we then notice that $\dot{x}=-i_c=dCv_c/dt$ for all dynamic elements; or in matrix form,

$$\dot{x} = -I_c. \qquad (7)$$

Using the definitions of (5) and (7) we can go to (4) and make the appropriate substitution to give the final form

$$\begin{matrix} n\{ \\ c\{ \end{matrix}\begin{bmatrix} 0 \\ \hline \dot{x} \end{bmatrix} = -\begin{bmatrix} Y_{nc} \\ \hline Y_{cc} \end{bmatrix}\begin{bmatrix} C \end{bmatrix}^{-1}x - \begin{bmatrix} Y_{nn} \\ \hline Y_{cn} \end{bmatrix}v + \begin{bmatrix} i \\ \hline 0 \end{bmatrix}\begin{matrix} \}n \\ \}c \end{matrix}. \qquad (8)$$

Recalling our previous definition of input and output variables, the equivalent of (1) is, by simple rearrangement of (8),

$$\dot{x} = Fx + Gu \equiv \dot{x} = -[Y_{cc}][C]^{-1}x - [Y_{cn}]v \qquad (9a)$$
$$y = Hx + Ju \equiv i = [Y_{nc}][C]^{-1}x + [Y_{nn}]v. \qquad (9b)$$

If currents are desired as input variables, the first n rows of (8) can be solved for the voltages in terms of the currents and the state variables and then substituted back into the last c rows of (8), such that the impedance matrix is determined.

Conclusion

It should be clear that the method just described is extremely simple and very easily programmed for computer operation. The main characteristic of this method's simplicity is that no transformations are necessary to bring the matrices to proper form, and only simple matrix operations are needed. The method always works if the admittance matrix Y_r exists. The restriction that the nondynamic admittance description exist generally means that no capacitor loops or inductor wyes are allowed, nor poles at infinity. This latter restricts any capacitive path from appearing across any port, but can easily be relaxed by adding a term $K\dot{u}$ to (1b). Dependent sources are handled in a straightforward manner, perhaps through gyrator conversions, allowing for the inclusion of transistors and other active elements. Independent current sources can be handled by the addition of a term, i_s, to the right of (1b), with i_s found through superposition and Norton's theorem. Independent voltage sources are simply brought out as input ports (or converted to i_s with gyrators). A program has been written using the algorithmic structure described here and has satisfactorily performed on the simple circuits analyzed thus far.

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Electromagnetic Matching Considerations for YIG Delay Lines

Abstract—**Results of measurements of the microwave radiation resistance and associated reactance for YIG delay lines, operating in the magnetostatic and magnetoelastic regimes, are given. These are applied in the design of a matching circuit for a one port magnetoelastic delay line, which exhibited experimentally a minimum insertion loss of 6 dB at 1.45 GHz.**

Currently the performance of yttrium iron garnet (YIG) dispersive delay lines at microwave frequencies, utilizing axially biased rod geometries and near axis wave propagation, is impaired by high insertion loss. Untuned structures with grounded fine wire antennas [1] commonly exhibit insertion losses exceeding 30 dB, for magnetoelastic delays less than 1 μs. Typically 10 dB improvement is possible with stub tuners. However, with resonant cavity excitation, single transducer losses of 4 dB have been reported [2]. In this letter we give data for electromagnetic radiation resistance and associated reactance, versus frequency and applied field, for operation in the magnetostatic and magnetoelastic delay regimes using fine wire excitation. These results are applied to the design of a simple electromagnetic matching circuit for a one port magnetoelastic delay line, which exhibited experimentally a minimum insertion loss of 6 dB at 1.45 GHz and a delay of 0.5 μs, with a bandwidth of 31 MHz.

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