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ON FAST NUCLEAR REACTOR MODELING*

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Abstract:

Through an investigation of the physics of fast reactors, the factors affecting fast nuclear reactors are explored and compared with those for thermal reactors. By investigating the appropriate distributional kernel a quick slide-rule method of determining the flux spectrum from the fission spectrum is obtained and refinements discussed.

Contents:

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Prologue: I heard the air though not the undersong,
The fierceness and resolve; but all the same
They're the tradition, and tradition's strong.

J. S. Manifold
The Tomb of Lieut. John Learmonth, A.I.F.
[21, p. 198]

1. Introduction

One of the interesting and practically important open problems of nuclear engineering theory is that of obtaining simple models for quick analysis and preliminary design of fast reactors. This paper reports the beginnings of an applicable study through an investigation of the fundamental physics of fast reactors, obtaining in the end a simple model suitable for quick, rough, but amazingly accurate, calculations of the actual neutron energy spectrum. The description attempts to cover only those basic aspects, suitable for the modeling problem, of the behavior of a fast chain-reacting system.

Section 2 describes the basic processes that are present in such systems giving a quick comparison with thermal reactors. Section 3 discusses some of the parameters used to describe various reactors and discusses their uses and design. Section 4 gives a short preliminary account of general methods under investigation for obtaining rough but relatively accurate models. The discussion of Section 4 leads to a simple model valid for rough calculations by developing suitable straight line approximations for the kernel describing reactor behavior. In the comments at the end of the paper, Section 5, more general methods are outlined while some of the elements which should become available within the context of a simple fast reactor model are discussed in more detail.

2. The Operation of Fast Reactors

Fast reactors are those multiplying nuclear media for which the neutron flux is centered at high energies, or equivalently for which most fissions occur at high energies. By design, and in contrast to thermal reactors, fast reactors are chain reacting systems in which there is essentially no moderating media. As a consequence fast reactors are critical to control and design. In principle neutron scattering is almost exclusively inelastic; neutron energy losses due to elastic scattering (slowing down) common to thermal reactors is virtually eliminated due to the absence of moderating media. Since inelastic scattering by heavy nuclei is present, there is always some neutron slowing down; but this scarcely occurs below about 100 Kev. Therefore, the chain reaction in a fast reactor is established as follows: neutrons produced by fissions at all energies (mainly above 100 Kev) have a flux distribution centered at about 1 Mev; these fission neutrons, as with thermal reactors, either cause new fissions or suffer predominately inelastic scattering with the heavy fuel nuclei (U^{235}, Pu^{239}, U^{233}) thus reducing their energy and causing a shift in the fission spectrum to yield the actual neutron flux distribution centered at about 500 Kev; once reduced in energy these scattered neutrons wander until they are captured, leak from the media, or cause new fissions to maintain the chain reaction.

It should be observed that a certain amount of elastic scattering is always present; this is due to the fact that a small amount of coolant and structural material are always on hand to induce some moderation. The effect depends predominantly upon the size of the reactor. In a fast reactor of small size and very high density almost all neutrons have energies above several hundred electron-volts. In a very large power producing fast reactor a large fraction of the fissions can occur at incident neutron energies between 1 and 100 Kev.

Figure 1(a) shows the fission spectrum, Curve A, for U^{235} [1, p. 113], [2, p. 192] superimposed with the actual neutron flux present in three types of U^{235} reactors. Note how the actual spectrum can be considered as a shifted fission spectrum, the shift being caused by moderation and various types of scattering. Curve A is the fission spectrum χ, the characteristic energy distribution of neutrons produced by fission of a nuclear species; that is, χ is the neutron flux φ when no moderation or scattering is present. Primarily χ depends upon the fuel and can be described (when normalized to unit area) in terms of energy E (in Mev) by [3, p. 528]

$$\varphi(E) = \chi(E) = \frac{2}{\sqrt{\pi}} \frac{E^{1/2}}{\theta^{3/2}} e^{-E/\theta} \qquad (1)$$

where θ is a constant of the fuel (θ = 1.32 Mev for U^{235} and Pu^{239}). As Eq. (1) indicates, it is generally believed that the fission spectrum is independent of the energy of the neutrons causing fission. Curve B is a typical true neutron flux spectrum of a small fast reactor consisting of about 49 kg of U^{235}, spherical and uncooled [4, p. 84], so that it represents a spectrum with the maximum attainable fraction of fast neutrons. The shift of the peak of the distribution from about 2 Mev for the fission spectrum to about 600 Kev for the fast reactor is due almost exclusively to inelastic scattering. For a larger size fast power reactor, as illustrated in Curve C [5, Fig. 16], the shift in the fission spectrum is again almost entirely due to inelastic scattering but the presence of more coolant and of a more complex structure causes the effects of elastic scattering to be felt below about 1 Kev. For comparison purposes a thermal, water-moderated, reactor fueled with enriched Uranium (90% U^{238}, 10% U^{235}) has a spectrum as shown in Curve D [6, p. 15]. In such a reactor most of the fissions, still yielding $\chi(E)$ as the fission spectrum, occur at thermal energies, and the high-energy peak of the spectrum is due both to a decrease in the total cross section at high energies and to the contribution from fissions in U^{238} above its fission threshold (about 1 Mev). The addition of a moderator lowers the flux through elastic scattering below the intermediate region, and very few fissions are induced by neutrons with energies between a few ev and 1 Mev. In a fully enriched thermal U^{235} reactor the high-energy peak is absent.

To understand the difference between thermal and fast reactors it is of interest to note the fission cross section, σ_f (in barns), shown in Fig. 1(b), adapted from [1, pp. 118, 122] [7, p. 115], and defined as the interaction (fission) rate per target nucleus per unit intensity of the incident (neutron) beam [8, p. 18], that is σ_f shows the relative number of fissions that will be induced by neutrons having a given energy. As the energy distributions, Fig. 1(a), differ (according to the amount of moderation present) the two types of reactors differ in characteristics even though σ_f may be identical for the two [σ_f depends only upon the nuclear media].

Note that the same energy scale is used in Fig. 1(a) and (b) so that the two portions of the figure can be compared to show how the various curves of Fig. 1(a) arise.

3. Distinctions of Fast Reactors

a. Neutron Economy

The efficiency with which neutrons are used in a reactor can be described by a single parameter, η, defined as the average number of secondary neutrons produced per neutron absorbed (i.e., captured or fission-inducing) in the fuel [7, p. 96]. If σ_c and σ_f are the microscopic capture and fission cross-sections respectively, the probability of fission per neutron absorbed in the fuel is $\sigma_f/(\sigma_c+\sigma_f)$; if ν is the average number of secondary neutrons produced per fission, then $\eta = \nu\sigma_f/(\sigma_c+\sigma_f)$. This can be written as $\eta = \nu/(1+a)$, where a is the capture-to fission ratio, $a = \sigma_c/\sigma_f$.

The parameter η is a measure of the quality of the fuel; for an isotope to be a nuclear fuel η has to be greater than 1, so that for each neutron absorbed in the fuel, another neutron is produced and leakage and control are still accounted for. Since ν and the σ's are energy-dependent, so is η. As a rule, ν increases (about as $\nu = 2.43 + 0.066E$ [9, p. 55]) and a decreases with increasing energy; the overall effect is an increase in η with energy. This last statement is true when comparing the thermal region (below 1 ev with the fast region (above 10 Kev) for the fissile isotopes U^{235}, U^{233} and Pu^{239} [1, p. 106] [7, p. 112].

b. Breeding [1, p. 124] [10, p. 7]

The process of producing fuel as a nuclear reactor operates is called breeding. Thus, it is possible to obtain Pu^{239} and U^{233} through sets of reactions which start through the capture of a neutron by U^{238} or Th^{232} respectively: (in the reactions γ and e^- stand for gamma and beta ray decay, respectively, while n denotes an incident neutron)

$$Th^{232} + n \rightarrow Th^{233} + \gamma \xrightarrow{23\ min} Pa^{233} + e^- \xrightarrow{27.4\ days} U^{233} + e^- \qquad (2a)$$

$$U^{238} + n \rightarrow U^{239} + \gamma \xrightarrow{23\ min} Np^{239} + e^- \xrightarrow{2.4\ days} Pu^{239} + e^- \qquad (2b)$$

The importance of Eq. (2a), for example, lies in the fact that Th^{232} is not a fuel for nuclear reactors while U^{233} is. Now, U^{233} is not a naturally occurring isotope, but can be produced as above; the objective of the breeding process is to produce more fuel than is consumed in the reactor along its operation (the neutron captured in Th^{232} has to come from a nuclear reactor around which a Thorium blanket is wrapped).

The neutron economy of breeding depends on the value of η, as previously defined. For a breeding reactor, η has to be sufficiently greater than 2, so that on the average one neutron is absorbed in U^{233} to provide fission and still on the average one neutron is captured in Th^{232} to produce more U^{233} than has been consumed. As before, capture and leakage have to be accounted for.

It is clear, then, that the value of η is very important for successful attempts at breeding. Probably a breeder reactor can be either a thermal or a fast one, but from the neutron economy point of view a fast reactor is the more practical breeder since U^{233} typically has $\eta = 2.26$ at thermal energies and 2.31 at high ones. For Pu^{239} there is a remarkable advantage in operating in a fast neutron field, for η goes from about 2.04 to 2.45 with increasing energy; in fact, it seems practically impossible to obtain a thermal Pu^{239}-U^{238} breeder.

A last comment on breeding is that U^{235} cannot be generated in reactors; it does not belong to any reaction scheme as Pu^{239} and U^{233} do. At the same time, since U^{235} is available naturally, and facilities exist for its separation in the natural Uranium, the present breeding schemes use it as an interim fuel until U^{233} or Pu^{239} become available in appreciable quantities to establish a true, complete breeding cycle.

c. <u>General Remarks</u> [11, p. 251] [12, p. 439]

We have seen that fast reactors offer the best neutron economy and thus are able to breed. We now turn to some general characteristics of such systems.

Almost all cross sections are small at high energies [as seen for example in Fig. 1(b)]; thus, large masses of fuel are necessary to maintain a fast chain reaction. On the other hand, due to the absence of a moderator, fast reactors tend to be small. The power density in a fast reactor is much larger than in a thermal one. Since there is not much difference between the cross sections of the elements at high energies, as far as neutrons are concerned the core does have a uniform composition. There are no steep spatial gradients in the flux; not even control rods induce abrupt changes in the flux. To a first approximation, fast reactors are therefore homogeneous; and this is a much better approximation than for thermal reactors. The small volume of fast reactors enhances leakage, but this is not a difficulty, since leaking neutrons can be used for breeding on a blanket surrounding the reactor.

In order to minimize the requirements of fissionable material, high power densities are desirable in fast reactors. This entails the design of finely divided fuel distributions in high environmental temperatures. This design problem, and the peculiar stability and control features of large fast reactors remain some of the most important problems of fast reactor technology [12, p. 439].

A word should be said about the intermediate reactors, where most of the fissions occur between 1 ev and 10 Kev. Since the capture-to-fission ratio α has large values in this region, it seems difficult to operate a breeding reactor in this region. The present engineering attitude is either to design a fast reactor, with as little moderation as possible, or to moderate completely the neutrons, forcing them to cross the intermediate energy region in a few collisions to yield a thermal reactor.

The calculation of the energy distribution of the neutron population in a nuclear reactor is crucial for obtaining integral parameters such as reactivity worths, flux integrals and reaction rates. We proceed now to examine a model for simple calculations of the neutron spectrum.

4. Fast Reactor Modeling

Let us now turn to a consideration of the modeling problem. Although a much more general discussion can be given, and to be complete we should consider criticality and dynamical behavior, etc., we give only a brief treatment which allows rough but quick flux calculations. To be somewhat precise we consider the modeling problem as that of determining the functional $f[\ ]$ mapping the fission spectrum $\chi(E)$ into the actual steady state reactor spectrum $\varphi(E)$, $\varphi = f[\chi]$, this in terms of the various reactor parameters (as fuel and composition).

We consider it known [1, p. 221] that a reactor can be described by a linear operator $\mathcal{B}[\ ]$, $\partial\varphi/\partial t = \mathcal{B}[\varphi]$, and hence [13, p. 221] by a distributional kernel $B(E,E',t)$ through

$$\frac{\partial\varphi(E,t)}{\partial t} = \int_{-\infty}^{\infty} B(E,E',t)\varphi(E',t)\,dE' \qquad (3)$$

In actual fact we can decompose the kernel B, writing it as the sum of three parts corresponding to elastic scattering, e, inelastic scattering, i, and fission, f, [14, p. 36]

$$B = B_e + B_i + B_f \qquad (4a)$$

$$= Nv(E)\{[\frac{\sigma_e(E')}{E'(1-\alpha)}1(E'-E)1(E+\alpha-E')] + [P(E)\sigma_i(E')1(E')-\sigma_i(E)\delta(E'-E)] + [\chi(E)\nu(E')\sigma_f(E',t)1(E')-\sigma_a(E',t)\delta(E'-E)]\} \qquad (4b)$$

where v is neutron velocity and N is the average nuclear density; σ_e, σ_i, σ_f and σ_a are the e, i, f and absorption cross sections (in barns); $1(\ )$ is the unit step function and $\delta(\ )$ the unit impulse; α is a constant dependent upon the atomic mass of the medium while ν and χ have been previously discussed. In the evaporation model of the compound nucleus the spectrum of the inelastically scattered neutrons P satisfies [14, p. 36] [15, p. 100]

$$P(E) = \frac{Ee^{-E/T}}{T^2}, \quad E \text{ in Mev} \qquad (5)$$

where T is a constant of the medium ($T \approx 1/3$ for U^{235}).

In the steady state $\partial\varphi/\partial t = 0$, while a reasonable approximation for a typical fast reactor has $B_e \approx 0$. Consequently, with $\sigma_t = \sigma_i + \sigma_a$ the total cross section, Eq. (3) yields

$$\varphi(E) = \frac{P(E)\int_0^\infty \sigma_i(E')\varphi(E')dE' + \chi(E)\int_0^\infty \nu(E')\sigma_f(E')\varphi(E')dE'}{\sigma_t(E)} \tag{6a}$$

Upon making the normalizations

$$\sigma_t^* = \frac{\sigma_t(E)}{\int_0^\infty \nu(E')\sigma_f(E')\varphi(E')dE'}, \qquad \gamma^* = \frac{\int_0^\infty \sigma_i(E')\varphi(E')dE'}{\int_0^\infty \nu(E')\sigma_f(E')\varphi(E')dE'} \tag{6b}$$

we obtain

$$\boxed{\varphi(E) = \frac{\chi(E) + \gamma^* P(E)}{\sigma_t^*(E)}} \tag{6c}$$

Equation (6c) shows the factors affecting the shift of χ into φ. In particular the strong influence of small values of σ_t in any portion of the energy range is evident as well as the complicated dependance upon fuel which influences the various cross sections.

The above discussion is for an infinite reactor, for practical reactors the above flux should be multiplied by a correction factor, which is nearly independent of energy in the fast region, to take into account leakage. If the reactor is large then elastic scattering becomes of interest but for a fast reactor, where elastic scattering effects are small, this can be taken care of by an additive flux term [16, p. 210]

$$\varphi_e(E) = \varphi_o\chi(E \exp 20\xi'), \qquad \xi' = -\ln(\frac{A-1}{A+1}) \tag{7}$$

where φ_o is a constant dependent upon the size of the reactor and A is the nucleus mass. That is, φ_e can be used in the range below about 0.01 Mev while φ of Eq. (6c) suffices for $E > 0.01$; φ_o can be obtained by matching the two curves at $E = 0.01$.

In order to use Eq. (6c) one should know φ at the beginning in order to evaluate γ^* and σ_t^*, but the problem presented is circumvented by using iteration; Eq. (6c) is readily suited for iterative techniques where, for example $\varphi_1 = \chi$ can be initially used in Eq. (6b). The various cross sections are experimentally available, as shown in Fig. 1(b) for U^{235}. If desired these can be appropriately approximated for the problem on hand. In fact the key point we wish to make is that approximation of the cross sections by constants suffices to give reasonable and very quick results.

As an example, let us investigate a small highly enriched U^{235} fast reactor, say of the Godiva type [4, p. 84]. For reasonably good but quick calculations for U^{235} for φ centered in the fast region we can observe, for $\varphi_1 = \chi$, that χ is almost exclusively in the range $0.01 < E < 10$. From Fig. 2, we can take in this range

$$\sigma_i(E) \approx 1.8\times 1(E-1.1)1(7-E) \qquad (8a)$$

$$\sigma_f(E) \approx 1.3\times 1(E-0.01)1(10-E) \qquad (8b)$$

$$\sigma_t(E) \approx 10\times 1(E-0.01)1(10-E) \qquad (8c)$$

These are illustrated in Fig. 2(a). Application of Eq. (6c), using $\nu(E) = 2.5$ [7,p. 115] and χ of Eq. (1) for the φ of Eq. (6b) yields $\gamma^* = 0.376 = 1.8 \int_{1.1}^{7} \chi(E')dE'/[2.5\times 1.3 \times \int_{0.01}^{10} \chi(E')dE']$. Here the unit area of the $\chi(E)$ curve has been used while the integral $\int_{1.1}^{7} \chi(E')dE' = 0.68$ has been quickly evaluated as the area of a triangle of height 0.23 and base (7-1.1). Similarly $\sigma_t^* = 3.08 = 10/[2.5\times 1.3]$. Thus, Eq. (6c),

$$\varphi(E) \approx \frac{\chi(E) + 0.376P(E)}{3.08} = \frac{1}{3.08}[0.745\sqrt{E}\, e^{-0.757E} + 3.38Ee^{-3E}] \qquad (9)$$

Figure 2(b) shows $3.08\varphi(E)$ plotted, the dotted points, by "graphically" using the $P(E)$ and $\chi(E)$ shown also in Fig. 2(b). On the same scale the experimental results (above 0.3 Mev) [17, p. 596] complemented by multigroup calculations (below 0.3 Mev) [4, p. 84], smoothed to obtain a continuous curve, for Godiva are shown to illustrate the close agreement. It should be mentioned that the constants in Eqs. (8) were not juggled after the fact to obtain close agreement but were the first numbers used for the calculation. Of course "improved" results can be obtained by iterating the procedure using the φ of Eq. (9) in Eqs. (6b) subject again to Eqs. (8), but Fig. 2(b) shows that little is to be gained in doing this.

If elastic scattering is of major importance one can replace the B_e term of Eq. (3) by φ_e of Eq. (7), with φ_o arbitrarily chosen, say $\varphi_o = 1$. Iterative techniques can then be used to solve the resulting integral equations in which again appropriate approximations, as for Eq. (8), can be made.

5. Comments and Discussion

The determination of the flux energy distribution for fast reactors having negligible elastic scattering has been shown to be possible in simple form. In particular, straight line approximations for the cross sections are suitable descriptions for a quick calculation. For small fast reactors, neglecting elastic moderation introduces small error in the spectrum; it is apparent that this approximation might not hold so well for large dilute fast assemblies [14, p. 47].

A complete nuclear reactor model includes the spatial and energy description of the neutron population, plus criticality conditions and dynamic behavior. For thermal reactors the simplest, yet complete model, has been formulated in terms of the four factor formula, age-diffusion theory and the Maxwellian distribution [1, p. 378].

No similar formulation has been developed for fast reactors [18, p. 421]. Modal approximations [14, p. 3] are a promising possibility; albedo methods [19, p. 38] provide upper and lower bounds for the critical size of the reactor, at the expense of a fairly crude description of the neutron spectrum. As shown in this paper, the energy dependence can be simply described in an infinite medium. Since the criticality condition is essentially a formulation of the compromise between the geometrical configuration and the mass of fissionable material enclosed, the next step seems to be an examination of the matching between the spatial dependence of the flux and the calculated spectrum.

Multigroup [20, p. 23] techniques and improved cross section measurements [7, p. 112] allow today very accurate fast reactor calculations. These sophisticated techniques, however, leave little room for intuition. There is still room for a simple fast reactor theory.

Epilogue:

And there in that lonely place an ancient swagman,
Traveller, bagman, sundowner, what you will--
Where did he come from, where could he be going?

Nancy Cato
Independence [21, p. 225]

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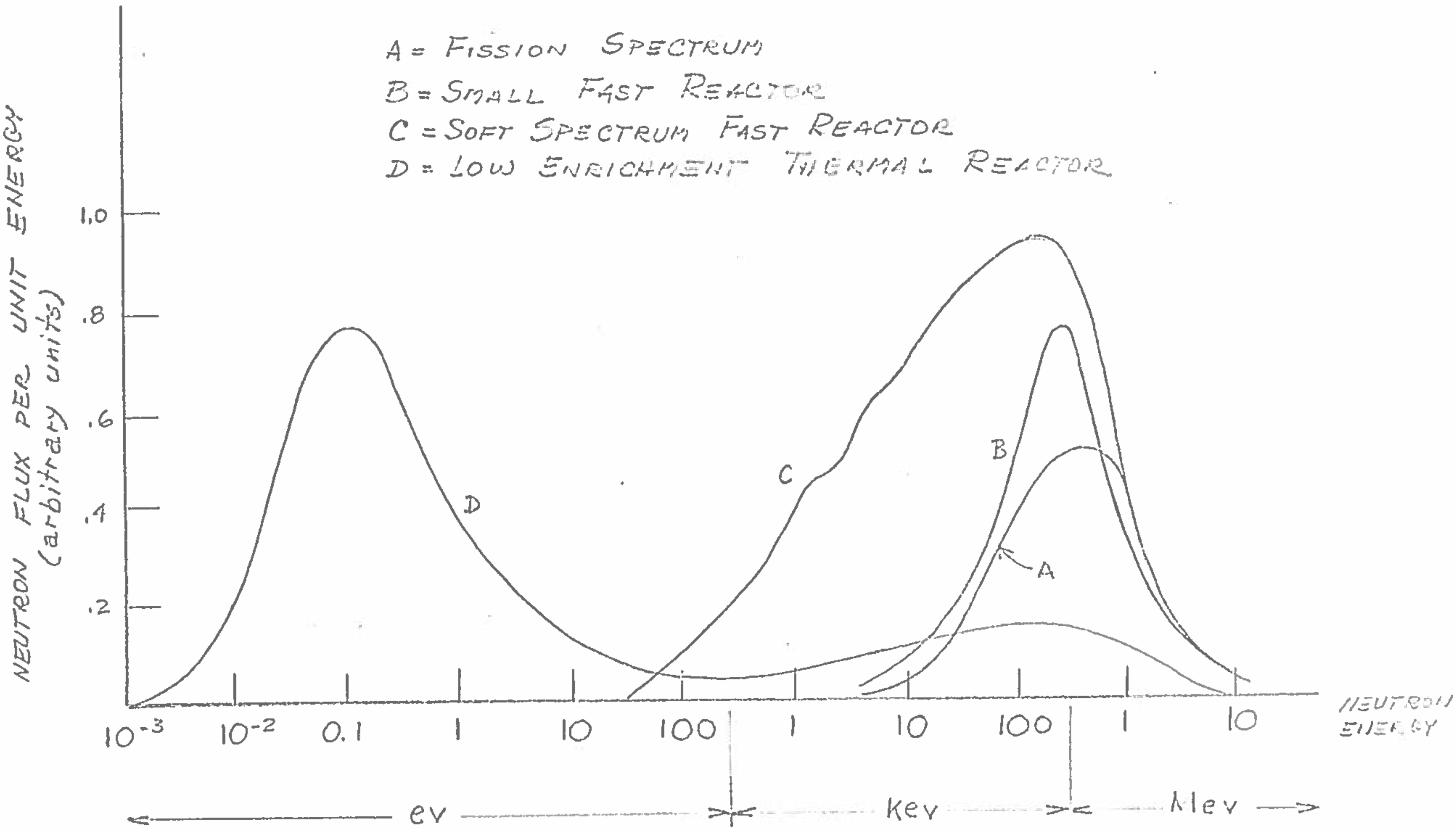
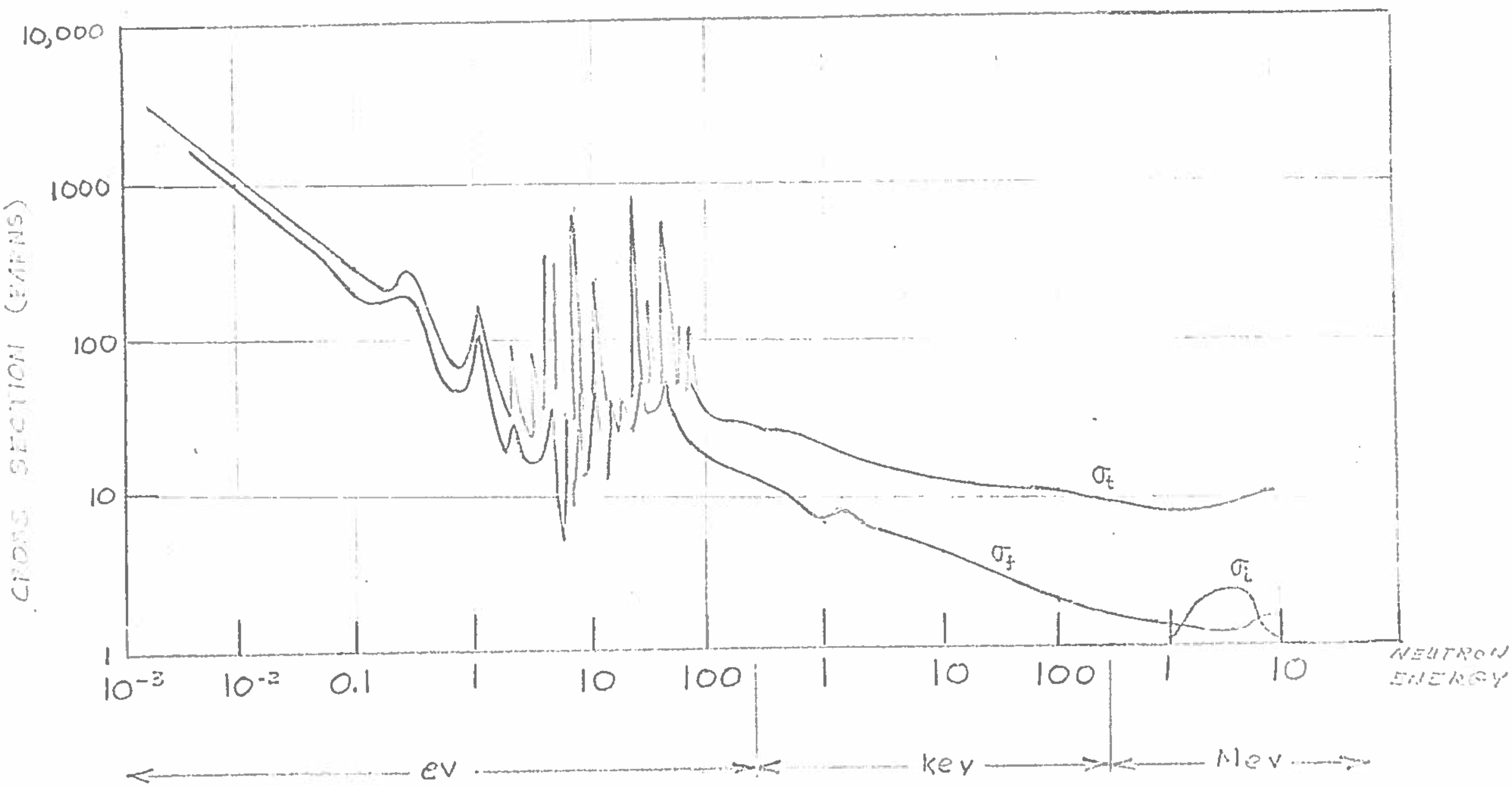


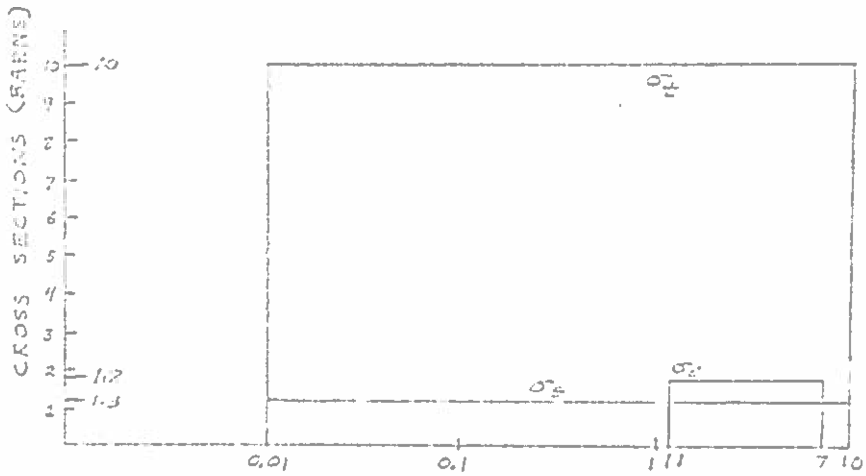
REACTOR NEUTRON SPECTRA

FIG. 1 (a)



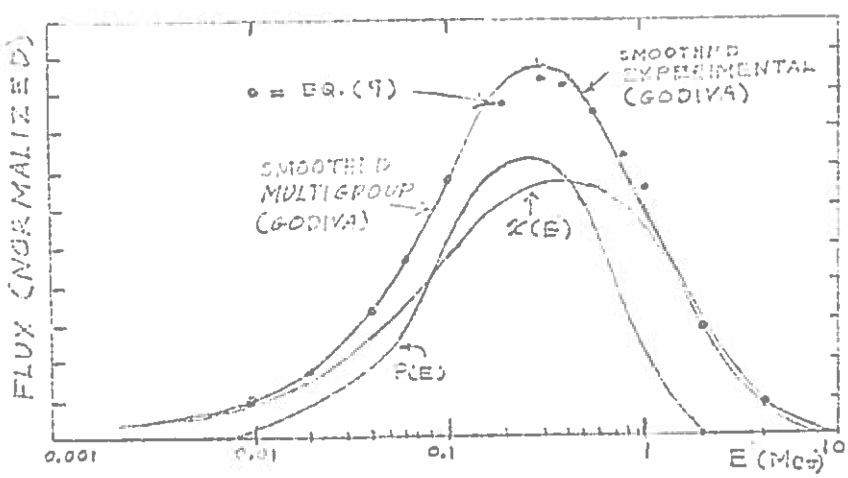
TOTAL (σ_t), FISSION (σ_f) AND INELASTIC (σ_i) MICROSCOPIC CROSS SECTIONS OF U^{235}

FIG. 1 (b)



CROSS SECTION APPROXIMATIONS,
EQS. (8)

FIG. 2(a)



EXPERIMENTAL VERIFICATION
OF EQ. (9)

FIG. 2(b)