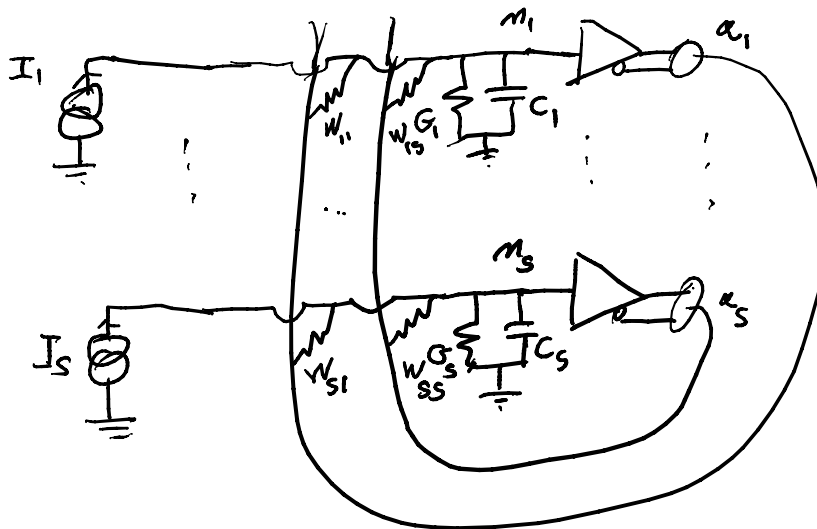


# Hopfield continuous time neural network p. 18-3

EE 434  
02/05/07



$$a = f(n)$$

$n$  = vector of voltages on capacitors an  $S$ -vector

$a$  = vector of voltages out of amplifier

$$C \frac{dn}{dt} = -Gn + I + W(a - n) \quad (eq. (18.1))$$

$$a = f(n)$$

but ignores  $Wn$   
i.e. sets  $Wn = 0$

$$\text{cell } Wa = Ta \Rightarrow W = T$$

↑  
in eq. (18.6)

$$\text{usually } f_i(n_i) = \tanh(n_i)$$

eq. (18.6)

$$C \frac{dn}{dt} = -Gn + I + Wa, \quad a = f(n)$$

here store results in  $n$ , run from  $n(0)$  and end up at an  $n$  that is close.

$I$  &  $G$ ,  $W$  all constants

desire to choose  $W$  &  $I$  so go to an equilibrium.

stored points are solutions of (set  $dn/dt = 0$ )

$$0 = -Gn_e + I + Wa_e, \quad a_e = f(n_e)$$

$\therefore$  choose  $n_e$  desired as stored points; given  $f(\cdot)$  &  $G$ , want  $W$  &  $I$ . Need these to be stable equilibria

take two of  $0 = -Gn_{e_1} + I + W a_{e_1}$  & subtract  
 $0 = -Gn_{e_2} + I + W a_{e_2}$

$$\underline{0} = -G(n_{e_1} - n_{e_2}) + W(a_{e_1} - a_{e_2})$$

$$x_i = n_{e_i} - n_{e_m}; \quad X = [x_1, x_2, \dots, x_{m-1}]$$

$\uparrow$   $\uparrow$   
 1 row & (m-1) columns

for m equilibria  
 $n_{e_1}, n_{e_2}, \dots, n_{e_m}$

$\therefore$  choose  $m-1 = S$  to get X square

$$y_i = a_{e_i} - a_{e_m} \quad Y = [y_1, y_2, \dots, y_{m-1}]$$

$$\underline{0}_{S \times S} = -GX + WY; \quad W = GX Y^{-1}$$

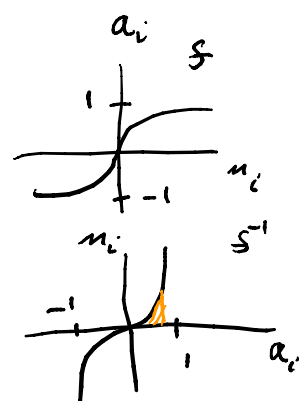
$$I = G n_{e_m} - W a_{e_m}$$

Need stability of equilibria:

Look for a Lyapunov function

use (18.8)

$$V(a) = -\frac{1}{2} \underbrace{a^T W a}_{\text{bounded (by entries in } (-1, 1))} - \underbrace{a^T I}_{\text{bounded}} + \sum_{i=1}^S G_i \int_0^{a_i} \underbrace{f_i^{-1}(u_i)}_{\text{bounded}} du_i$$



$V(a)$  is bounded below for a fixed  $W, G, I, C, f$

next desire  $W \& I$  such that  $\frac{dV(a)}{dt} \leq 0 \Rightarrow V(a(t))$  always decreases (or stops)

$\therefore V(a)$  heads toward its minimum & never runs away. i.e.,

this neural network will be stable.

$\therefore$  need to calculate  $\frac{dV}{dt}$

$$\frac{dV}{dt} = -\frac{1}{2} \left( \frac{da^T}{dt} \right) W a - \frac{1}{2} a^T W \left( \frac{da}{dt} \right) - \left( \frac{da^T}{dt} \right) I + \sum_{i=1}^S G_i \underbrace{f_i^{-1}(a_i)}_{n_i} \cdot \frac{da_i}{dt}$$

$$= -\frac{1}{2} (a^T W^T) \frac{da}{dt} - \frac{1}{2} (a^T W) \frac{da}{dt} - I^T \frac{da}{dt} + (n^T G) \frac{da}{dt}$$

$$= \left[ -\frac{1}{2} a^T (W^T + W) - I^T + n^T G \right] \frac{da}{dt} ; \text{ but } a = f(n)$$

$$\frac{da}{dt} = \frac{\partial f}{\partial n} \cdot \frac{dn}{dt}$$

$$= \left[ -a^T (W_{sym}) - I^T + n^T G^T \right] \left[ \frac{df_i(n_i)}{dn_i} \right] \frac{dn}{dt} \quad \left[ \frac{d^2 f_i(n_i)}{dn_i^2} \right] \frac{dn}{dt}$$

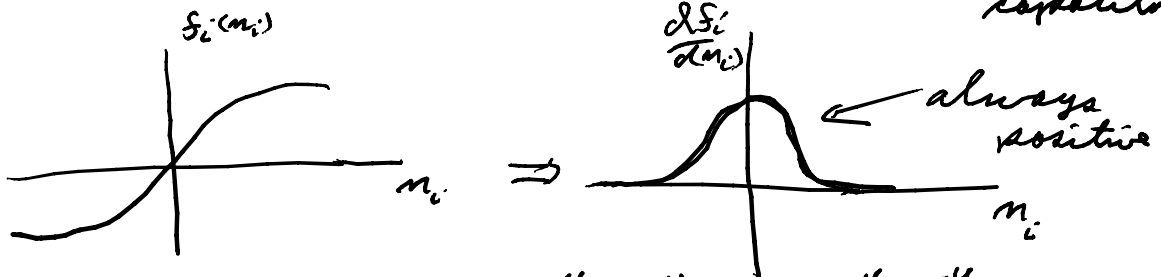
$$= - \left[ a^T W_{sym} + I^T - n^T G^T \right] \underbrace{\left[ \frac{df_i(n_i)}{dn_i} \cdot \frac{1}{C_i} \right]}_{C^{-1}[-Gn + I + wa]} \left[ wa + I - Gm \right]$$

If  $W$  is symmetric  
 $W = \frac{1}{2} (W + W^T)$

$$\frac{dV}{dt} = -[wa + I - Gm]^T \left[ \frac{df_i(n_i)}{dn_i} \cdot \frac{1}{C_i} \right] [wa + I - Gm]$$

$\leq 0$  if  $\left[ \frac{df_i(n_i)}{dn_i} \cdot \frac{1}{C_i} \right]$  is positive definite

But  $\left[ \frac{df_i(n_i)}{dn_i} \cdot \frac{1}{C_i} \right]$  is diagonal entries  $\frac{df_i(n_i)}{dn_i} \cdot \frac{1}{C_i}$ ,  $C_i > 0$  as real capacitor



if  $f(u) = \tanh u$  ;  $\frac{df(u)}{du} = \frac{e^u + e^{-u}}{(e^u + e^{-u})^2} - \frac{(e^u - e^{-u})}{(e^u + e^{-u})^2} (e^u - e^{-u})$

$$= \frac{e^u - e^{-u}}{e^u + e^{-u}} = 1 - \tanh^2(u) \geq 0, \quad -\infty < u < \infty$$

$\therefore \left[ \frac{\partial f_i(n_i)}{\partial n_i} \cdot \frac{1}{C_i} \right] = \text{diagonal } (S \times S) \text{ matrix of positive numbers}$

This means all equilibria are stable (as  $V > \text{some const}$   
if  $W = \frac{1}{2}(W + W^T)$   $\frac{dV}{dt} \leq 0$  )

$$\therefore \text{ need } W = GXY^{-1} = W^T = (Y^{-1})^T X^T G^T$$