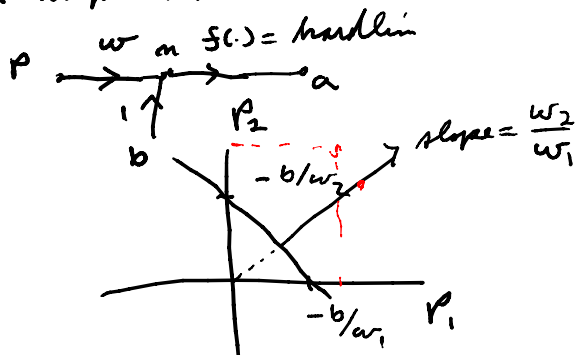


EE434
02/05/07

2- input perceptron



$$\text{hardlin}(x) = 1(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$w p + b = m$$

$$\text{breaks at } w p + b = 0$$

$$w_1 p_1 + w_2 p_2 + b = 0$$

$$\frac{dp_2}{dp_1}: w_1 + w_2 \frac{dp_2}{dp_1} = 0$$

$$\frac{dp_2}{dp_1} = -\frac{w_1}{w_2}$$

$$\text{if } p_1 = 0: p_2 = -b/w_2, \text{ if } p_2 = 0, p_1 = -b/w_1$$

a line $\perp$ to this
break line has

$$\text{slope} = \frac{-1}{\text{slope of the break line}} = \frac{-1}{-(w_1/w_2)} = \frac{w_2}{w_1} \Rightarrow \text{vector of weights is } \perp \text{ to the break line (separating surface giving 1 or 0 out)}$$

adapt vs train p. 2-22 of toolbox manual

batch
or
incremental
training

batch
only

batch $\Rightarrow$ update only after present
all training inputs & targets

incremental $\Rightarrow$ update after each input
& target is presented

Hopfield: 2 neuron one

$$a(k) = \text{sat lin}(w a(k-1) + b)$$

$$\text{at rest} = \text{equilibrium}, a(k) = a(k-1)$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \left\{ \begin{bmatrix} +1 \\ +1 \end{bmatrix}, \begin{bmatrix} +1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ +1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, w a + b \text{ in linear region} \right\}$$

How to choose weights & bias then use a^1, a^2

$$a = w a + b \Rightarrow w a = b - a$$

$$w \begin{bmatrix} a^1, a^2 \end{bmatrix} = \begin{bmatrix} a^1 - b, a^2 - b \end{bmatrix} \quad \text{let } a^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, a^2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

choose b : $\frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = b \Rightarrow W \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -3/2 \end{bmatrix}$

$$\Rightarrow W = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix} \left(\frac{1}{-2} \right) \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} -2 & 0 \\ +2 & -4 \end{bmatrix}$$

$$= -\begin{bmatrix} -1/2 & 0 \\ 1/2 & -1 \end{bmatrix} \quad \text{with } b = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

check:

$$W a^1 + b = -\begin{bmatrix} -1/2 & 0 \\ 1/2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \text{satlin}(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$W a^2 + b = -\begin{bmatrix} -1/2 & 0 \\ 1/2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \text{satlin}(\begin{bmatrix} 1 \\ -1 \end{bmatrix}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$