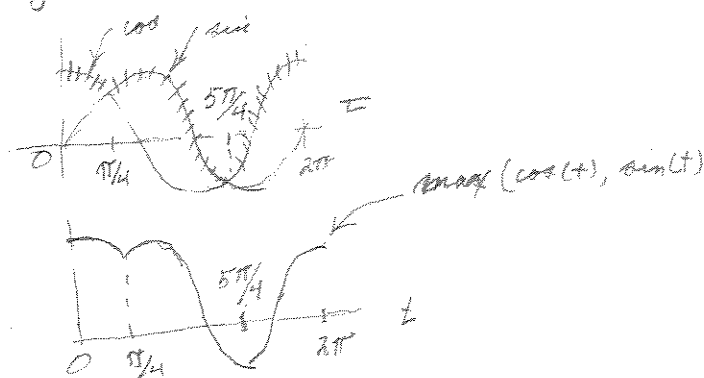


#1 a) $W_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $W_2 = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$, $W_3 = [\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}]$

$$f_2(x) = \begin{bmatrix} x \\ x \cdot 1(x) \\ x \cdot 1(x) \\ x \end{bmatrix}$$

b) This gives the output as the max of the inputs. This seen for positive inputs as it agrees in that case with the Lippmann matrix. For: $P_1 > 0$, $P_2 < 0$, $a_{22} = P_1 - P_2$, $a_{23} = 0$ then $n_{31} = \frac{1}{2}(P_1 + P_2 + P_1 - P_2) = P_1$. For $P_1 < P_2 < 0$ then $a_{22} = 0$, $a_{23} = P_2 - P_1$ then $n_{31} = P_2$. Thus for all real P_1, P_2 this gives the max



here $\cos t = \frac{e^{jt} + e^{-jt}}{2}$
 $\Rightarrow \frac{e^{jt} + e^{-jt}}{2} = \frac{e^{jt} - e^{-jt}}{2j}$
 $\Rightarrow (j-1)e^{j2t} = -(1+j)$
 $\Rightarrow e^{j2t} = j$
 $= e^{j(\frac{\pi}{2} + 2k\pi)}$
 $\Rightarrow t = \frac{1}{2}(\frac{\pi}{2} + 2k\pi)$
 $= \frac{\pi}{4}, \frac{\pi}{4} + \pi, \frac{\pi}{4} + 2\pi, \dots$

$$a_3(t) = \begin{cases} \cos t, & 0 \leq t \leq \pi/4 \\ \sin t, & \pi/4 \leq t \leq 5\pi/4 \\ \cos t, & 5\pi/4 \leq t \leq 2\pi \end{cases}$$

$$\#2, a) \text{ grad } E = \left[\frac{\partial E}{\partial w_1}; \frac{\partial E}{\partial w_2}; \frac{\partial E}{\partial b_1} \right] = 2(y(p) - t_p) \begin{bmatrix} \partial y / \partial w_1 \\ \partial y / \partial w_2 \\ \partial y / \partial b_1 \end{bmatrix}$$

$$\frac{\partial y}{\partial w_1} = r(p, b_1) = e^{-(p/b_1)^2}$$

$$\frac{\partial y}{\partial w_2} = r([p-0.1], 1) = e^{-(p-0.1)^2}$$

$$\frac{\partial y}{\partial b_1} = w_1 \cdot \frac{\partial e^{-(p/b_1)^2}}{\partial b_1} = w_1 \left[-2 \frac{p}{b_1} \cdot \left(\frac{p}{(-b_1^2)} \right) e^{-(p/b_1)^2} \right]$$

$$\therefore \text{grad } E = 2(y(p) - t_p) \begin{bmatrix} r(p, b_1) \\ r([p-0.1], 1) \\ w_1^2 \frac{p^2}{(b_1)^3} r(p, b_1) \end{bmatrix} = 2(y(p) - t_p) \begin{bmatrix} e^{-(p/b_1)^2} \\ e^{-(p-0.1)^2} \\ \frac{2p^2}{b_1^3} e^{-(p/b_1)^2} w_1 \end{bmatrix}$$

$$b) \text{ for } (p, t_p) = (0.25, 4), \quad x_0 = [1; 2; 0.2]$$

$$y(p) = 1 \cdot e^{-(0.25/0.2)^2} + 2 \cdot e^{-(0.25-0.1)^2}$$

$$= e^{-(1.25)^2} + 2 \cdot e^{-(0.15)^2} = 0.2096 + 2 \times (0.9778)$$

$$= 2.165$$

$$2(y(p) - t_p) = 2(2.165 - 4) = -3.670$$

$$x_1 = x_0 - \eta \cdot \text{grad} = \begin{bmatrix} 1 \\ 2 \\ 0.2 \end{bmatrix} - \underbrace{0.1 \times (-3.670)}_{+0.367} \begin{bmatrix} e^{-(0.25/0.2)^2} = 0.2096 \\ e^{-(0.15)^2} = 0.9778 \\ 1 \times \frac{2 \times (0.25)^2}{(0.2)^3} e^{-(0.25/0.2)^2} = 3.275 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 0.07692 \\ 0.3589 \\ 1.202 \end{bmatrix}$$

$$= \begin{bmatrix} 1.07692 \\ 2.3589 \\ 2.402 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ b_1 \end{bmatrix}_{\text{new values}}$$

13. a) $c \mathbf{1}_S \frac{dn}{dt} = W a(n) - g \mathbf{1}_S n + I_B$, $\mathbf{1}_S = S \times S$ identity
 $a(n) = \tanh(n)$

dividing by g :

$$\frac{c}{g} \frac{dn}{dt} = \frac{1}{g} W a(n) - n + \frac{1}{g} I_B, \quad a(n) = \text{hardlim}(n)$$

let $(g/c)t = \hat{t}$ gives

$$\begin{aligned} \frac{dn}{d\hat{t}} &= \left(\frac{1}{g} W\right) a(n) - n + \frac{1}{g} I_B, \quad a(n) = \text{hardlim}(n) \\ &= \hat{W} a(n) - n + \frac{1}{g} I_B, \quad \hat{W} = \frac{1}{g} W \end{aligned}$$

as g is a scalar if $W = W^T$ then $(\frac{1}{g} W) = (\frac{1}{g} W)^T$ or $\hat{W} = \hat{W}^T$
 so the symmetry of the weight matrix is preserved

$$\therefore \begin{cases} \frac{dn}{d\hat{t}} = \left(\frac{1}{g} W\right) a(n) - n + \frac{1}{g} I_B \\ a(n) = \text{hardlim}(n) \end{cases}$$

$$b) \quad \hat{V}(a) = -\frac{1}{2} a^T \left(\frac{1}{g} W\right) a - a^T \left(\frac{1}{g} I_B\right) + \sum_{i=1}^S \lim_{T \rightarrow \infty} \int_0^{a_i(T+1)} \frac{1}{2T} \ln\left(\frac{1+y}{1-y}\right) dy$$

since $\lim_{T \rightarrow \infty} \tanh(Tx) = \text{hardlim}(x)$

and $y = \tanh(Tx) = \frac{e^{2Tx} - 1}{e^{2Tx} + 1} \Rightarrow y + 1 = (1 - y) e^{2Tx}$

so $x = \frac{1}{2T} \ln\left(\frac{1+y}{1-y}\right) = \tanh^{-1}(y)$

Here $\hat{V}(a)$ has the same properties as $V(a)$
 which has those in the lim of those
 for $V(a)$ where $a = \tanh(Tx)$, $\therefore$ all equilibria
 in a are stable

c) The entries of $a(n)$ can be $+1$ or -1 of which there
 are S entries, $\therefore 2^S$ possible values for a
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